1.1 Graphs of Equations

The Graph of an Equation

News magazines often show graphs comparing the rate of inflation, the federal deficit, or the unemployment rate to the time of year. Businesses use graphs to report monthly sales statistics. Such graphs provide geometric pictures of the way one quantity changes with respect to another. Frequently, the relationship between two quantities is expressed as an **equation.** This section introduces the basic procedure for determining the geometric picture associated with an equation.

For an equation in the variables x and y, a point (a, b) is a **solution point** if substitution of a for x and b for y satisfies the equation. Most equations have *infinitely many* solution points. For example, the equation $3x + y = 5$ has solution points $(0, 5)$, $(1, 2)$, $(2, -1)$, $(3, -4)$, and so on. The set of all solution points of an equation is the **graph of the equation.**

What you should learn

- Sketch graphs of equations by point plotting.
- Graph equations using a graphing utility.
- Use graphs of equations to solve real-life problems.

Why you should learn it

The graph of an equation can help you see relationships between real-life quantities. For example, in Exercise 74 on page 87, a graph can be used to estimate the life expectancies of children born in the years 1948 and 2010.

Bruce Avres/Getty Images

Example 1 Determining Solution Points

Determine whether (a) $(2, 13)$ and (b) $(-1, -3)$ lie on the graph of $y = 10x - 7$.

Solution

a. $y = 10x - 7$ — Write original equation.

$13 \stackrel{?}{=} 10(2) - 7$ — Substitute 2 for x and 13 for y.

$13 = 13$ — $(2, 13)$ is a solution. ✓

The point $(2, 13)$ *does* lie on the graph of $y = 10x - 7$ because it is a solution point of the equation.

b. $y = 10x - 7$ — Write original equation.

$-3 \stackrel{?}{=} 10(-1) - 7$ — Substitute -1 for x and -3 for y.

$-3 \neq -17$ — $(-1, -3)$ is not a solution.

The point $(-1, -3)$ *does not* lie on the graph of $y = 10x - 7$ because it is not a solution point of the equation.

✓CHECKPOINT Now try Exercise 3.

Prerequisite Skills

When evaluating an expression or an equation, remember to follow the Basic Rules of Algebra. To review the Basic Rules of Algebra, see Section P.1.

The basic technique used for sketching the graph of an equation is the point-plotting method.

Sketching the Graph of an Equation by Point Plotting

1. If possible, rewrite the equation so that one of the variables is isolated on one side of the equation.
2. Make a table of values showing several solution points.
3. Plot these points on a rectangular coordinate system.
4. Connect the points with a smooth curve or line.

Example 2 Sketching a Graph by Point Plotting

Use point plotting and graph paper to sketch the graph of $3x + y = 6$.

Solution

In this case you can isolate the variable y.

$y = 6 - 3x$ Solve equation for y.

Using negative, zero, and positive values for x, you can obtain the following table of values (solution points).

x	-1	0	1	2	3
$y = 6 - 3x$	9	6	3	0	-3
Solution point	$(-1, 9)$	$(0, 6)$	$(1, 3)$	$(2, 0)$	$(3, -3)$

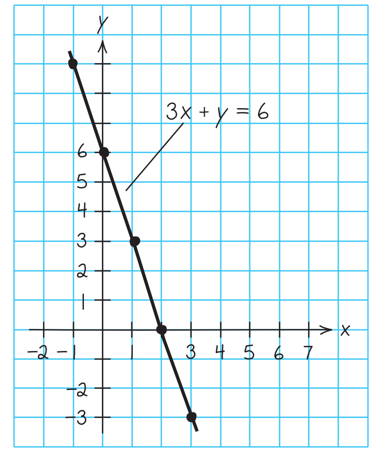


Figure 1.1

Next, plot these points and connect them, as shown in Figure 1.1. It appears that the graph is a straight line. You will study lines extensively in Section 1.2.

CHECKPOINT Now try Exercise 7.

The points at which a graph touches or crosses an axis are called the **intercepts** of the graph. For instance, in Example 2 the point $(0, 6)$ is the y-intercept of the graph because the graph crosses the y-axis at that point. The point $(2, 0)$ is the x-intercept of the graph because the graph crosses the x-axis at that point.

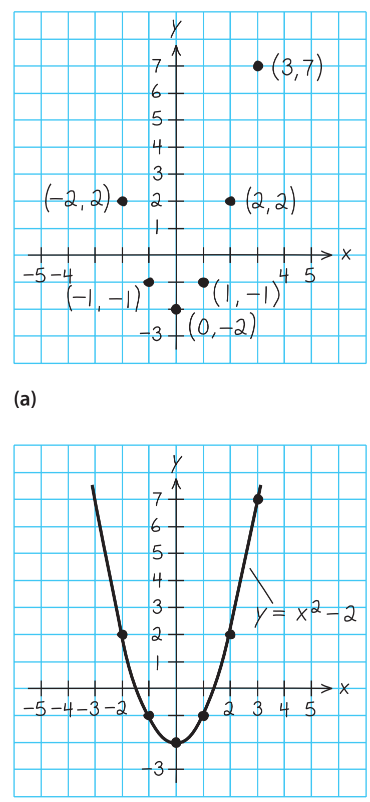
Example 3 Sketching a Graph by Point Plotting

Use point plotting and graph paper to sketch the graph of $y = x^2 - 2$.

Solution

Because the equation is already solved for y, make a table of values by choosing several convenient values of x and calculating the corresponding values of y.

x	-2	-1	0	1	2	3
$y = x^2 - 2$	2	-1	-2	-1	2	7
Solution point	$(-2, 2)$	$(-1, -1)$	$(0, -2)$	$(1, -1)$	$(2, 2)$	$(3, 7)$



(a)

(b)

Figure 1.2

Next, plot the corresponding solution points, as shown in Figure 1.2(a). Finally, connect the points with a smooth curve, as shown in Figure 1.2(b). This graph is called a *parabola*. You will study parabolas in Section 3.1.

CHECKPOINT Now try Exercise 8.

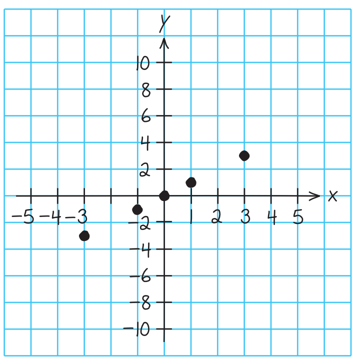
In this text, you will study two basic ways to create graphs: *by hand* and *using a graphing utility*. For instance, the graphs in Figures 1.1 and 1.2 were sketched by hand and the graph in Figure 1.6 (on page 80) was created using a graphing utility.

Using a Graphing Utility

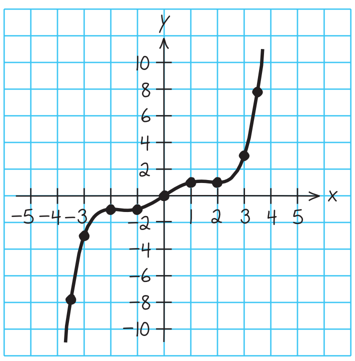
One of the disadvantages of the point-plotting method is that to get a good idea about the shape of a graph, you need to plot *many* points. With only a few points, you could misrepresent the graph of an equation. For instance, consider the equation

$$y = \frac{1}{30}x(x^4 - 10x^2 + 39).$$

Suppose you plotted only five points: $(-3, -3)$, $(-1, -1)$, $(0, 0)$, $(1, 1)$, and $(3, 3)$, as shown in Figure 1.3(a). From these five points, you might assume that the graph of the equation is a line. That, however, is not correct. By plotting several more points and connecting the points with a smooth curve, you can see that the actual graph is not a line at all, as shown in Figure 1.3(b).



(a)



(b)

Figure 1.3

From this, you can see that the point-plotting method leaves you with a dilemma. This method can be very inaccurate if only a few points are plotted, and it is very time-consuming to plot a dozen (or more) points. Technology can help solve this dilemma. Plotting several (even several hundred) points on a rectangular coordinate system is something that a computer or calculator can do easily.

TECHNOLOGY TIP The point-plotting method is the method used by *all* graphing utilities. Each computer or calculator screen is made up of a grid of hundreds or thousands of small areas called *pixels*. Screens that have many pixels per square inch are said to have a higher *resolution* than screens with fewer pixels.

Using a Graphing Utility to Graph an Equation

To graph an equation involving x and y on a graphing utility, use the following procedure.

1. Rewrite the equation so that y is isolated on the left side.
2. Enter the equation in the graphing utility.
3. Determine a *viewing window* that shows all important features of the graph.
4. Graph the equation.

TECHNOLOGY SUPPORT

This section presents a brief overview of how to use a graphing utility to graph an equation. For more extensive coverage of this topic, see Appendix A and the *Graphing Technology Guide* at this textbook's *Online Study Center*.

Example 4 Using a Graphing Utility to Graph an Equation

Use a graphing utility to graph $2y + x^3 = 4x$.

Solution

To begin, solve the equation for y in terms of x.

$$2y + x^3 = 4x \qquad \text{Write original equation.}$$

$$2y = -x^3 + 4x \qquad \text{Subtract } x^3 \text{ from each side.}$$

$$y = -\frac{x^3}{2} + 2x \qquad \text{Divide each side by 2.}$$

Enter this equation in a graphing utility (see Figure 1.4). Using a standard viewing window (see Figure 1.5), you can obtain the graph shown in Figure 1.6.

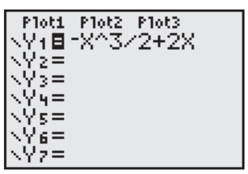


Figure 1.4

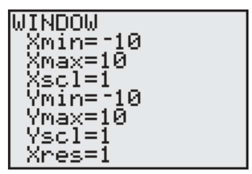


Figure 1.5

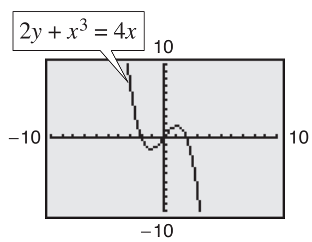
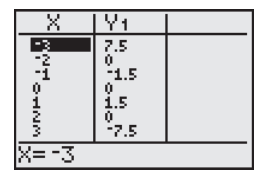


Figure 1.6

CHECKPOINT Now try Exercise 41.

TECHNOLOGY TIP

Many graphing utilities are capable of creating a table of values such as the following, which shows some points of the graph in Figure 1.6. For instructions on how to use the *table* feature, see Appendix A; for specific keystrokes, go to this textbook's *Online Study Center*.



Additional Examples

a. A viewing window on a graphing utility that shows all the important characteristics of the graph of

$$y = x^4 - 5x^2 + 6$$

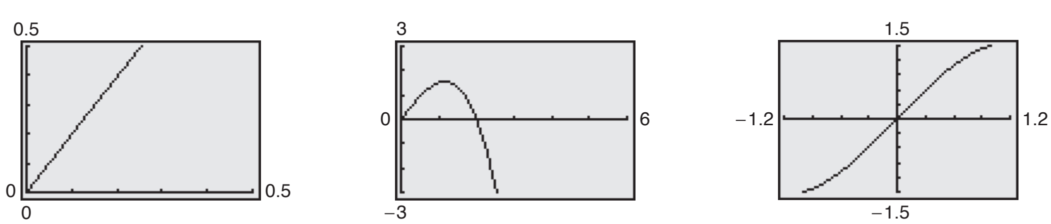
is $-3 \le x \le 3$, $-1 \le y \le 7$.

b. A viewing window on a graphing utility that shows all the important characteristics of the graph of

$$y = -x^5 + 5x^3 - 4x + 10$$

is $-3 \le x \le 3$, $-5 \le y \le 15$.

TECHNOLOGY TIP By choosing different viewing windows for a graph, it is possible to obtain very different impressions of the graph's shape. For instance, Figure 1.7 shows three different viewing windows for the graph of the equation in Example 4. However, none of these views shows *all* of the important features of the graph as does Figure 1.6. For instructions on how to set up a viewing window, see Appendix A; for specific keystrokes, go to this textbook's *Online Study Center*.



(a) (b) (c)

Figure 1.7

TECHNOLOGY TIP The standard viewing window on many graphing utilities does not give a true geometric perspective because the screen is rectangular, which distorts the image. That is, perpendicular lines will not appear to be perpendicular and circles will not appear to be circular. To overcome this, you can use a *square setting*, as demonstrated in Example 5.

Example 5 Using a Graphing Utility to Graph a Circle

Use a graphing utility to graph $x^2 + y^2 = 9$.

Solution

The graph of $x^2 + y^2 = 9$ is a circle whose center is the origin and whose radius is 3. To graph the equation, begin by solving the equation for y.

$$x^2 + y^2 = 9 \qquad \text{Write original equation.}$$

$$y^2 = 9 - x^2 \qquad \text{Subtract } x^2 \text{ from each side.}$$

$$y = \pm\sqrt{9 - x^2} \qquad \text{Take the square root of each side.}$$

Remember that when you take the square root of a variable expression, you must account for both the positive and negative solutions. The graph of

$$y = \sqrt{9 - x^2} \qquad \text{Upper semicircle}$$

is the upper semicircle. The graph of

$$y = -\sqrt{9 - x^2} \qquad \text{Lower semicircle}$$

is the lower semicircle. Enter *both* equations in your graphing utility and generate the resulting graphs. In Figure 1.8, note that if you use a standard viewing window, the two graphs do not appear to form a circle. You can overcome this problem by using a *square setting*, in which the horizontal and vertical tick marks have equal spacing, as shown in Figure 1.9. On many graphing utilities, a square setting can be obtained by using a y to x ratio of 2 to 3. For instance, in Figure 1.9, the y to x ratio is

$$\frac{\text{Y}_{\text{max}} - \text{Y}_{\text{min}}}{\text{X}_{\text{max}} - \text{X}_{\text{min}}} = \frac{4 - (-4)}{6 - (-6)} = \frac{8}{12} = \frac{2}{3}.$$

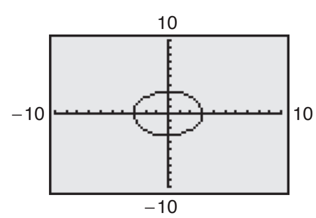


Figure 1.8

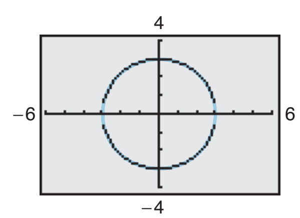


Figure 1.9

✓CHECKPOINT Now try Exercise 63.

Prerequisite Skills

To review the equation of a circle, see Section P.5.

You may wish to point out to your students that some graphing utilities have commands that allow the user to draw complete circles by specifying the radius and the coordinates of the center.

TECHNOLOGY TIP

Notice that when you graph a circle by graphing two separate equations for y, your graphing utility may not connect the two semicircles. This is because some graphing utilities are limited in their resolution. So, in this text, a blue curve is placed behind the graphing utility's display to indicate where the graph should appear.

Applications

Throughout this course, you will learn that there are many ways to approach a problem. Two of the three common approaches are illustrated in Example 6.

An Algebraic Approach: Use the rules of algebra.
A Graphical Approach: Draw and use a graph.
A Numerical Approach: Construct and use a table.

You should develop the habit of using at least two approaches to solve every problem in order to build your intuition and to check that your answer is reasonable.

The following two applications show how to develop mathematical models to represent real-world situations. You will see that both a graphing utility and algebra can be used to understand and solve the problems posed.

TECHNOLOGY SUPPORT

For instructions on how to use the *value* feature, the *zoom* and *trace* features, and the *table* feature of a graphing utility, see Appendix A; for specific keystrokes, go to this textbook's *Online Study Center*.

Example 6 Running a Marathon

A runner runs at a constant rate of 4.9 miles per hour. The verbal model and algebraic equation relating distance run and elapsed time are as follows.

Verbal Model: Distance = Rate · Time   *Equation:* $d = 4.9t$

a. Determine how far the runner can run in 3.1 hours.

b. Determine how long it will take to run a 26.2-mile marathon.

Algebraic Solution

a. To begin, find how far the runner can run in 3.1 hours by substituting 3.1 for t in the equation.

$$\begin{aligned} d &= 4.9t && \text{Write original equation.} \\ &= 4.9(3.1) && \text{Substitute 3.1 for } t. \\ &\approx 15.2 && \text{Use a calculator.} \end{aligned}$$

So, the runner can run about 15.2 miles in 3.1 hours. Use estimation to check your answer. Because 4.9 is about 5 and 3.1 is about 3, the distance is about $5(3) = 15$. So, 15.2 is reasonable.

b. You can find how long it will take to run a 26.2-mile marathon as follows. (For help with solving linear equations, see Appendix D.)

$$\begin{aligned} d &= 4.9t && \text{Write original equation.} \\ 26.2 &= 4.9t && \text{Substitute 26.2 for } d. \\ \frac{26.2}{4.9} &= t && \text{Divide each side by 4.9.} \\ 5.3 &\approx t && \text{Use a calculator.} \end{aligned}$$

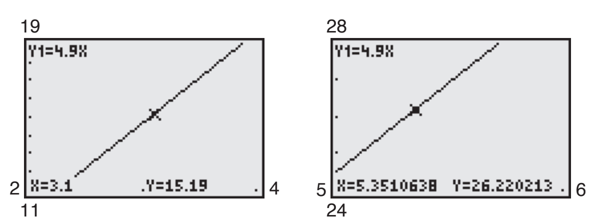
So, it will take about 5.3 hours to run 26.2 miles.

CHECKPOINT Now try Exercise 71.

Graphical Solution

a. Use a graphing utility to graph the equation $d = 4.9t$. (Represent d by y and t by x.) Be sure to use a viewing window that shows the graph at $x = 3.1$. Then use the *value* feature or the *zoom* and *trace* features of the graphing utility to estimate that when $x = 3.1$, the distance is $y \approx 15.2$ miles, as shown in Figure 1.10(a).

b. Adjust the viewing window so that it shows the graph at $y = 26.2$. Use the *zoom* and *trace* features to estimate that when $y \approx 26.2$, the time is $x \approx 5.3$ hours, as shown in Figure 1.10(b).



(a) (b)

Figure 1.10

Note that the viewing window on your graphing utility may differ slightly from those shown in Figure 1.10.

Example 7 Monthly Wage

You receive a monthly salary of \$2000 plus a commission of 10% of sales. The verbal model and algebraic equation relating the wages, the salary, and the commission are as follows.

Verbal Model: Wages = Salary + Commission on sales

Equation: $y = 2000 + 0.1x$

a. Sales are \$1480 in August. What are your wages for that month?

b. You receive \$2225 for September. What are your sales for that month?

Numerical Solution

a. To find the wages in August, evaluate the equation when $x = 1480$.

$$y = 2000 + 0.1x \qquad \text{Write original equation.}$$

$$= 2000 + 0.1(1480) \qquad \text{Substitute 1480 for } x.$$

$$= 2148 \qquad \text{Simplify.}$$

So, your wages in August are \$2148.

b. You can use the *table* feature of a graphing utility to create a table that shows the wages for different sales amounts. First enter the equation in the graphing utility. Then set up a table, as shown in Figure 1.11. The graphing utility produces the table shown in Figure 1.12.

Figure 1.11

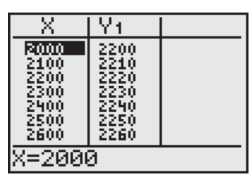


Figure 1.12

From the table, you can see that wages of \$2225 result from sales between \$2200 and \$2300. You can improve this estimate by setting up the table shown in Figure 1.13. The graphing utility produces the table shown in Figure 1.14.

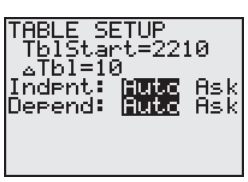


Figure 1.13

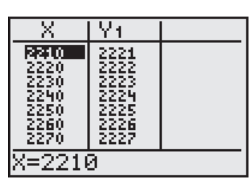


Figure 1.14

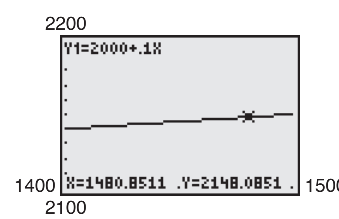
From the table, you can see that wages of \$2225 result from sales of \$2250.

CHECKPOINT Now try Exercise 73.

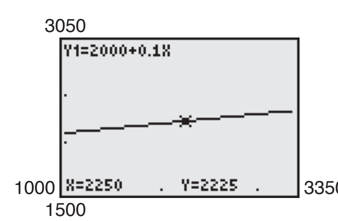
Graphical Solution

a. You can use a graphing utility to graph $y = 2000 + 0.1x$ and then estimate the wages when $x = 1480$. Be sure to use a viewing window that shows the graph for $x \geq 0$ and $y > 2000$. Then, by using the *value* feature or the *zoom* and *trace* features near $x = 1480$, you can estimate that the wages are about \$2148, as shown in Figure 1.15(a).

b. Use the graphing utility to find the value along the x-axis (sales) that corresponds to a y-value of 2225 (wages). Using the *zoom* and *trace* features, you can estimate the sales to be about \$2250, as shown in Figure 1.15(b).



(a) Zoom near $x = 1480$



(b) Zoom near $y = 2225$

Figure 1.15

1.1 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blanks.

1. For an equation in x and y, if substitution of a for x and b for y satisfies the equation, then the point (a, b) is a _______ .

2. The set of all solution points of an equation is the _______ of the equation.

3. The points at which a graph touches or crosses an axis are called the _______ of the graph.

In Exercises 1–6, determine whether each point lies on the graph of the equation.

Equation	*Points*	
1. $y = \sqrt{x + 4}$	(a) $(0, 2)$	(b) $(5, 3)$
2. $y = x^2 - 3x + 2$	(a) $(2, 0)$	(b) $(-2, 8)$
3. $y = 4 - \lvert x - 2 \rvert$	(a) $(1, 5)$	(b) $(1.2, 3.2)$
4. $2x - y - 3 = 0$	(a) $(1, 2)$	(b) $(1, -1)$
5. $x^2 + y^2 = 20$	(a) $(3, -2)$	(b) $(-4, 2)$
6. $y = \frac{1}{3}x^3 - 2x^2$	(a) $\left(2, -\frac{16}{3}\right)$	(b) $(-3, 9)$

In Exercises 7 and 8, complete the table. Use the resulting solution points to sketch the graph of the equation. Use a graphing utility to verify the graph.

7. $3x - 2y = 2$

x	-2	0	$\frac{2}{3}$	1	2
y					
Solution point					

8. $2x + y = x^2$

x	-1	0	1	2	3
y					
Solution point					

9. ***Exploration***

(a) Complete the table for the equation $y = \frac{1}{4}x - 3$.

x	-2	-1	0	1	2
y					

(b) Use the solution points to sketch the graph. Then use a graphing utility to verify the graph.

(c) Repeat parts (a) and (b) for the equation $y = -\frac{1}{4}x - 3$. Describe any differences between the graphs.

10. ***Exploration***

(a) Complete the table for the equation

$$y = \frac{6x}{x^2 + 1}.$$

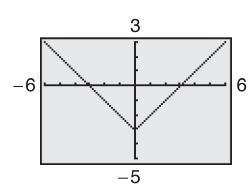
x	-2	-1	0	1	2
y					

(b) Use the solution points to sketch the graph. Then use a graphing utility to verify the graph.

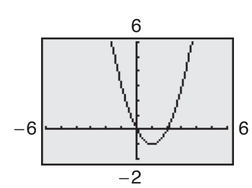
(c) Continue the table in part (a) for x-values of 5, 10, 20, and 40. What is the value of y approaching? Can y be negative for positive values of x? Explain.

In Exercises 11–16, match the equation with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

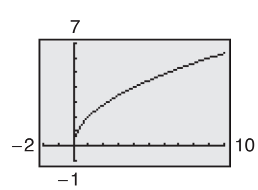
(a)



(b)



(c)



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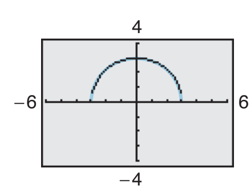

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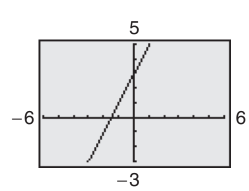
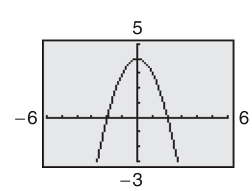


(f)



11. $y = 2x + 3$

12. $y = 4 - x^2$

13. $y = x^2 - 2x$

14. $y = \sqrt{9 - x^2}$

15. $y = 2\sqrt{x}$

16. $y = \lvert x \rvert - 3$

In Exercises 17–30, sketch the graph of the equation.

17. $y = -4x + 1$

18. $y = 2x - 3$

19. $y = 2 - x^2$

20. $y = x^2 - 1$

21. $y = x^2 - 3x$

22. $y = -x^2 - 4x$

23. $y = x^3 + 2$

24. $y = x^3 - 3$

25. $y = \sqrt{x - 3}$

26. $y = \sqrt{1 - x}$

27. $y = |x - 2|$

28. $y = 5 - |x|$

29. $x = y^2 - 1$

30. $x = y^2 + 4$

In Exercises 31–44, use a graphing utility to graph the equation. Use a standard viewing window. Approximate any *x*- or *y*-intercepts of the graph.

31. $y = x - 7$

32. $y = x + 1$

33. $y = 3 - \frac{1}{2}x$

34. $y = \frac{2}{3}x - 1$

35. $y = \dfrac{2x}{x - 1}$

36. $y = \dfrac{4}{x}$

37. $y = x\sqrt{x + 3}$

38. $y = (6 - x)\sqrt{x}$

39. $y = \sqrt[3]{x - 8}$

40. $y = \sqrt[3]{x + 1}$

41. $x^2 - y = 4x - 3$

42. $2y - x^2 + 8 = 2x$

43. $y - 4x = x^2(x - 4)$

44. $x^3 + y = 1$

In Exercises 45–48, use a graphing utility to graph the equation. Begin by using a standard viewing window. Then graph the equation a second time using the specified viewing window. Which viewing window is better? Explain.

45. $y = \frac{5}{2}x + 5$

Xmin = 0
Xmax = 6
Xscl = 1
Ymin = 0
Ymax = 10
Yscl = 1

46. $y = -3x + 50$

Xmin = -1
Xmax = 4
Xscl = 1
Ymin = -5
Ymax = 60
Yscl = 5

47. $y = -x^2 + 10x - 5$

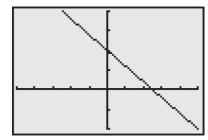
Xmin = -1
Xmax = 11
Xscl = 1
Ymin = -5
Ymax = 25
Yscl = 5

48. $y = 4(x + 5)\sqrt{4 - x}$

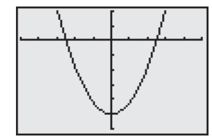
Xmin = -6
Xmax = 6
Xscl = 1
Ymin = -5
Ymax = 50
Yscl = 5

In Exercises 49–54, describe the viewing window of the graph shown.

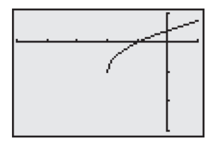
49. $y = -10x + 50$



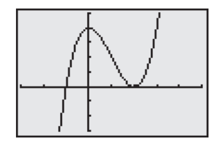
50. $y = 4x^2 - 25$



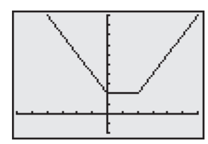
51. $y = \sqrt{x + 2} - 1$



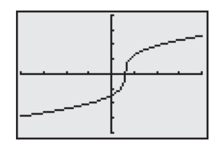
52. $y = x^3 - 3x^2 + 4$



53. $y = |x| + |x - 10|$



54. $y = 8\sqrt[3]{x - 6}$



In Exercises 55–58, explain how to use a graphing utility to verify that $y_1 = y_2$. Identify the rule of algebra that is illustrated.

55. $y_1 = \frac{1}{4}(x^2 - 8)$

$y_2 = \frac{1}{4}x^2 - 2$

56. $y_1 = \frac{1}{2}x + (x + 1)$

$y_2 = \frac{3}{2}x + 1$

57. $y_1 = \dfrac{1}{5}[10(x^2 - 1)]$

$y_2 = 2(x^2 - 1)$

58. $y_1 = (x - 3) \cdot \dfrac{1}{x - 3}$

$y_2 = 1$

In Exercises 59–62, use a graphing utility to graph the equation. Use the *trace* feature of the graphing utility to approximate the unknown coordinate of each solution point accurate to two decimal places. (*Hint:* You may need to use the *zoom* feature of the graphing utility to obtain the required accuracy.)

59. $y = \sqrt{5 - x}$
(a) $(2, y)$
(b) $(x, 3)$

60. $y = x^3(x - 3)$
(a) $(2.25, y)$
(b) $(x, 20)$

61. $y = x^5 - 5x$
(a) $(-0.5, y)$
(b) $(x, -4)$

62. $y = |x^2 - 6x + 5|$
(a) $(2, y)$
(b) $(x, 1.5)$

In Exercises 63–66, solve for y and use a graphing utility to graph each of the resulting equations in the same viewing window. (Adjust the viewing window so that the circle appears circular.)

63. $x^2 + y^2 = 16$

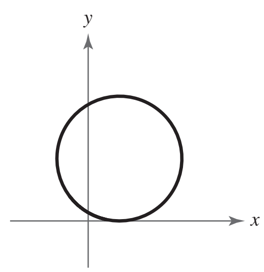
64. $x^2 + y^2 = 36$

65. $(x - 1)^2 + (y - 2)^2 = 4$

66. $(x - 3)^2 + (y - 1)^2 = 25$

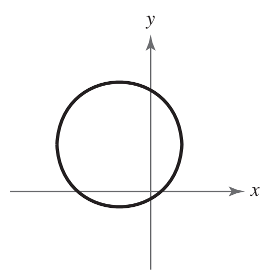
In Exercises 67 and 68, determine which equation is the best choice for the graph of the circle shown.

67.



(a) $(x - 1)^2 + (y - 2)^2 = 4$
(b) $(x + 1)^2 + (y - 2)^2 = 4$
(c) $(x - 1)^2 + (y - 2)^2 = 16$
(d) $(x + 1)^2 + (y + 2)^2 = 4$

68.



(a) $(x - 2)^2 + (y - 3)^2 = 4$
(b) $(x - 2)^2 + (y - 3)^2 = 16$
(c) $(x + 2)^2 + (y - 3)^2 = 16$
(d) $(x + 2)^2 + (y - 3)^2 = 4$

In Exercises 69 and 70, determine whether each point lies on the graph of the circle. (There may be more than one correct answer.)

69. $(x - 1)^2 + (y - 2)^2 = 25$
(a) $(1, 2)$ (b) $(-2, 6)$
(c) $(5, -1)$ (d) $(0, 2 + 2\sqrt{6})$

70. $(x + 2)^2 + (y - 3)^2 = 25$
(a) $(-2, 3)$ (b) $(0, 0)$
(c) $(1, -1)$ (d) $(-1, 3 - 2\sqrt{6})$

71. ***Depreciation*** A manufacturing plant purchases a new molding machine for \$225,000. The depreciated value (decreased value) y after t years is $y = 225{,}000 - 20{,}000t$, for $0 \le t \le 8$.

(a) Use the constraints of the model to graph the equation using an appropriate viewing window.

(b) Use the *value* feature or the *zoom* and *trace* features of a graphing utility to determine the value of y when $t = 5.8$. Verify your answer algebraically.

(c) Use the *value* feature or the *zoom* and *trace* features of a graphing utility to determine the value of y when $t = 2.35$. Verify your answer algebraically.

72. ***Consumerism*** You buy a personal watercraft for \$8100. The depreciated value y after t years is $y = 8100 - 929t$, for $0 \le t \le 6$.

(a) Use the constraints of the model to graph the equation using an appropriate viewing window.

(b) Use the *zoom* and *trace* features of a graphing utility to determine the value of t when $y = 5545.25$. Verify your answer algebraically.

(c) Use the *value* feature or the *zoom* and *trace* features of a graphing utility to determine the value of y when $t = 5.5$. Verify your answer algebraically.

73. ***Data Analysis*** The table shows the median (middle) sales prices (in thousands of dollars) of new one-family homes in the southern United States from 1995 to 2004. (Sources: U.S. Census Bureau and U.S. Department of Housing and Urban Development)

Year	Median sales price, y
1995	124.5
1996	126.2
1997	129.6
1998	135.8
1999	145.9
2000	148.0
2001	155.4
2002	163.4
2003	168.1
2004	181.1

A model for the median sales price during this period is given by

$$y = -0.0049t^3 + 0.443t^2 - 0.75t + 116.7, \quad 5 \le t \le 14$$

where y represents the sales price and t represents the year, with $t = 5$ corresponding to 1995.

(a) Use the model and the *table* feature of a graphing utility to find the median sales prices from 1995 to 2004. How well does the model fit the data? Explain.

(b) Use a graphing utility to graph the data from the table and the model in the same viewing window. How well does the model fit the data? Explain.

(c) Use the model to estimate the median sales prices in 2008 and 2010. Do the values seem reasonable? Explain.

(d) Use the *zoom* and *trace* features of a graphing utility to determine during which year(s) the median sales price was approximately $150,000.

74. ***Population Statistics*** The table shows the life expectancies of a child (at birth) in the United States for selected years from 1930 to 2000. (Source: U.S. National Center for Health Statistics)

Year	Life expectancy, y
1930	59.7
1940	62.9
1950	68.2
1960	69.7
1970	70.8
1980	73.7
1990	75.4
2000	77.0

A model for the life expectancy during this period is given by

$$y = \frac{59.617 + 1.18t}{1 + 0.012t}, \quad 0 \le t \le 70$$

where y represents the life expectancy and t is the time in years, with $t = 0$ corresponding to 1930.

(a) Use a graphing utility to graph the data from the table above and the model in the same viewing window. How well does the model fit the data? Explain.

(b) What does the y-intercept of the graph of the model represent?

(c) Use the *zoom* and *trace* features of a graphing utility to determine the year when the life expectancy was 73.2. Verify your answer algebraically.

(d) Determine the life expectancy in 1948 both graphically and algebraically.

(e) Use the model to estimate the life expectancy of a child born in 2010.

75. ***Geometry*** A rectangle of length x and width w has a perimeter of 12 meters.

(a) Draw a diagram that represents the rectangle. Use the specified variables to label its sides.

(b) Show that the width of the rectangle is $w = 6 - x$ and that its area is $A = x(6 - x)$.

(c) Use a graphing utility to graph the area equation.

(d) Use the *zoom* and *trace* features of a graphing utility to determine the value of A when $w = 4.9$ meters. Verify your answer algebraically.

(e) From the graph in part (c), estimate the dimensions of the rectangle that yield a maximum area.

76. Find the standard form of the equation of the circle for which the endpoints of a diameter are $(0, 0)$ and $(4, -6)$.

Synthesis

True or False? **In Exercises 77 and 78, determine whether the statement is true or false. Justify your answer.**

77. A parabola can have only one x-intercept.

78. The graph of a linear equation can have either no x-intercepts or only one x-intercept.

79. ***Writing*** Explain how to find an appropriate viewing window for the graph of an equation.

80. ***Writing*** Your employer offers you a choice of wage scales: a monthly salary of $3000 plus commission of 7% of sales or a salary of $3400 plus a 5% commission. Write a short paragraph discussing how you would choose your option. At what sales level would the options yield the same salary?

81. ***Writing*** Given the equation $y = 250x + 1000$, write a possible explanation of what the equation could represent in real life.

82. ***Writing*** Given the equation $y = -0.1x + 10$, write a possible explanation of what the equation could represent in real life.

Skills Review

In Exercises 83–86, perform the operation and simplify.

83. $7\sqrt{72} - 5\sqrt{18}$

84. $-10\sqrt{25y} - \sqrt{y}$

85. $7^{3/2} \cdot 7^{11/2}$

86. $\dfrac{10^{17/4}}{10^{5/4}}$

In Exercises 87 and 88, perform the operation and write the result in standard form.

87. $(9x - 4) + (2x^2 - x + 15)$

88. $(3x^2 - 5)(-x^2 + 1)$