1.4 Graphs of Functions

The Graph of a Function

In Section 1.3, functions were represented graphically by points on a graph in a coordinate plane in which the input values are represented by the horizontal axis and the output values are represented by the vertical axis. The **graph of a function** f is the collection of ordered pairs $(x, f(x))$ such that x is in the domain of f. As you study this section, remember the geometric interpretations of x and $f(x)$.

$x = $ the directed distance from the y-axis

$f(x) = $ the directed distance from the x-axis

Example 1 shows how to use the graph of a function to find the domain and range of the function.

Example 1   Finding the Domain and Range of a Function

Use the graph of the function f shown in Figure 1.34 to find (a) the domain of f, (b) the function values $f(-1)$ and $f(2)$, and (c) the range of f.
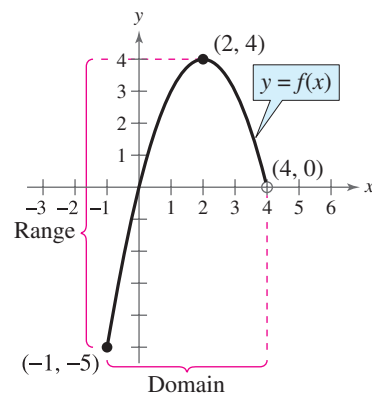

Figure 1.34

Solution

a. The closed dot at $(-1, -5)$ indicates that $x = -1$ is in the domain of f, whereas the open dot at $(4, 0)$ indicates that $x = 4$ is not in the domain. So, the domain of f is all x in the interval $[-1, 4)$.

b. Because $(-1, -5)$ is a point on the graph of f, it follows that

$$f(-1) = -5.$$

Similarly, because $(2, 4)$ is a point on the graph of f, it follows that

$$f(2) = 4.$$

c. Because the graph does not extend below $f(-1) = -5$ or above $f(2) = 4$, the range of f is the interval $[-5, 4]$.

✓CHECKPOINT   Now try Exercise 3.

What you should learn

■ Find the domains and ranges of functions and use the Vertical Line Test for functions.
■ Determine intervals on which functions are increasing, decreasing, or constant.
■ Determine relative maximum and relative minimum values of functions.
■ Identify and graph step functions and other piecewise-defined functions.
■ Identify even and odd functions.

Why you should learn it

Graphs of functions provide a visual relationship between two variables. For example, in Exercise 88 on page 125, you will use the graph of a step function to model the cost of sending a package.

Stephen Chernin/Getty Images

STUDY TIP

The use of dots (open or closed) at the extreme left and right points of a graph indicates that the graph does not extend beyond these points. If no such dots are shown, assume that the graph extends beyond these points.

Example 2   Finding the Domain and Range of a Function

Find the domain and range of

$$f(x) = \sqrt{x - 4}.$$

Algebraic Solution

Because the expression under a radical cannot be negative, the domain of $f(x) = \sqrt{x - 4}$ is the set of all real numbers such that $x - 4 \geq 0$. Solve this linear inequality for x as follows. (For help with solving linear inequalities, see Appendix D.)

$x - 4 \geq 0$ Write original inequality.

$x \geq 4$ Add 4 to each side.

So, the domain is the set of all real numbers greater than or equal to 4. Because the value of a radical expression is never negative, the range of $f(x) = \sqrt{x - 4}$ is the set of all nonnegative real numbers.

✔CHECKPOINT   Now try Exercise 7.

Graphical Solution

Use a graphing utility to graph the equation $y = \sqrt{x - 4}$, as shown in Figure 1.35. Use the *trace* feature to determine that the x-coordinates of points on the graph extend from 4 to the right. When x is greater than or equal to 4, the expression under the radical is nonnegative. So, you can conclude that the domain is the set of all real numbers greater than or equal to 4. From the graph, you can see that the y-coordinates of points on the graph extend from 0 upwards. So you can estimate the range to be the set of all nonnegative real numbers.
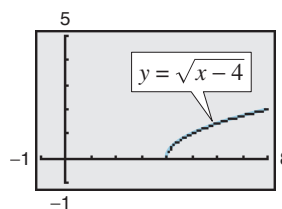

Figure 1.35

By the definition of a function, at most one y-value corresponds to a given x-value. It follows, then, that a vertical line can intersect the graph of a function at most once. This leads to the **Vertical Line Test** for functions.

> ### Vertical Line Test for Functions
>
> A set of points in a coordinate plane is the graph of y as a function of x if and only if no vertical line intersects the graph at more than one point.

Example 3   Vertical Line Test for Functions

Use the Vertical Line Test to decide whether the graphs in Figure 1.36 represent y as a function of x.
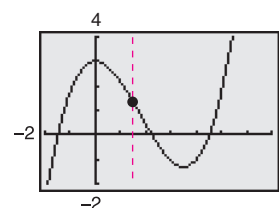
Solution

a. This is *not* a graph of y as a function of x because you can find a vertical line that intersects the graph twice.

b. This *is* a graph of y as a function of x because every vertical line intersects the graph at most once.

✔CHECKPOINT   Now try Exercise 17.
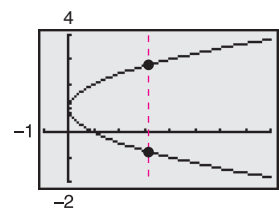
(a)

(b)

Figure 1.36

Increasing and Decreasing Functions

The more you know about the graph of a function, the more you know about the function itself. Consider the graph shown in Figure 1.37. Moving from *left to right*, this graph falls from $x = -2$ to $x = 0$, is constant from $x = 0$ to $x = 2$, and rises from $x = 2$ to $x = 4$.

Increasing, Decreasing, and Constant Functions

A function f is **increasing** on an interval if, for any x_1 and x_2 in the interval,

$$x_1 < x_2 \text{ implies } f(x_1) < f(x_2).$$

A function f is **decreasing** on an interval if, for any x_1 and x_2 in the interval,

$$x_1 < x_2 \text{ implies } f(x_1) > f(x_2).$$

A function f is **constant** on an interval if, for any x_1 and x_2 in the interval,

$$f(x_1) = f(x_2).$$

Example 4   Increasing and Decreasing Functions

In Figure 1.38, determine the open intervals on which each function is increasing, decreasing, or constant.

Solution

a. Although it might appear that there is an interval in which this function is constant, you can see that if $x_1 < x_2$, then $(x_1)^3 < (x_2)^3$, which implies that $f(x_1) < f(x_2)$. So, the function is increasing over the entire real line.

b. This function is increasing on the interval $(-\infty, -1)$, decreasing on the interval $(-1, 1)$, and increasing on the interval $(1, \infty)$.

c. This function is increasing on the interval $(-\infty, 0)$, constant on the interval $(0, 2)$, and decreasing on the interval $(2, \infty)$.

Figure 1.37
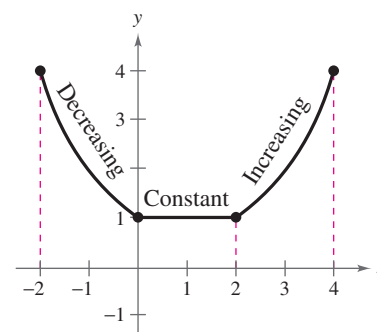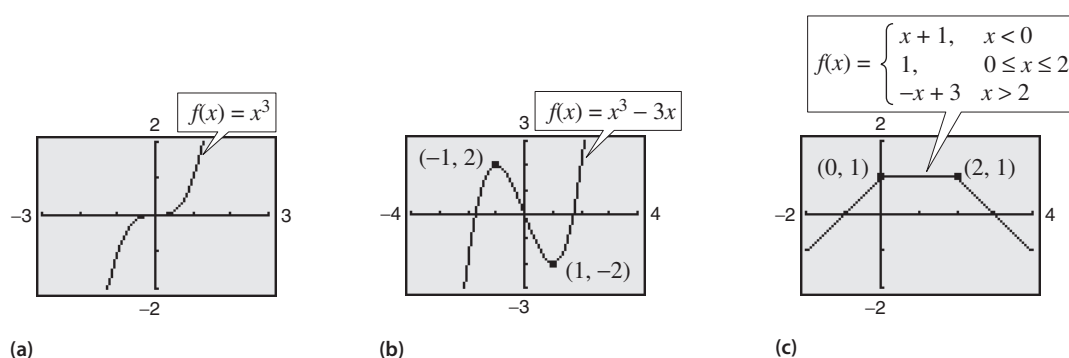
(a)   (b)   (c)

Figure 1.38

✔**CHECKPOINT**   Now try Exercise 21.

Relative Minimum and Maximum Values

The points at which a function changes its increasing, decreasing, or constant behavior are helpful in determining the relative maximum or relative minimum values of the function.

Definitions of Relative Minimum and Relative Maximum

A function value $f(a)$ is called a **relative minimum** of f if there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \le f(x).$$

A function value $f(a)$ is called a **relative maximum** of f if there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \ge f(x).$$
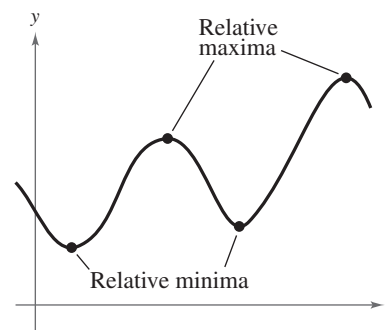

Figure 1.39

Figure 1.39 shows several different examples of relative minima and relative maxima. In Section 3.1, you will study a technique for finding the *exact points* at which a second-degree polynomial function has a relative minimum or relative maximum. For the time being, however, you can use a graphing utility to find reasonable approximations of these points.

Example 5 Approximating a Relative Minimum

Use a graphing utility to approximate the relative minimum of the function given by $f(x) = 3x^2 - 4x - 2$.

Solution

The graph of f is shown in Figure 1.40. By using the *zoom* and *trace* features of a graphing utility, you can estimate that the function has a relative minimum at the point

$(0.67, -3.33)$.   See Figure 1.41.

Later, in Section 3.1, you will be able to determine that the exact point at which the relative minimum occurs is $\left(\frac{2}{3}, -\frac{10}{3}\right)$.
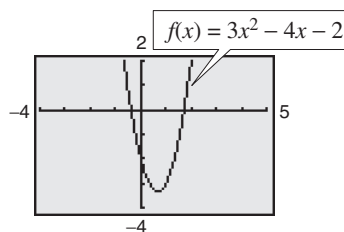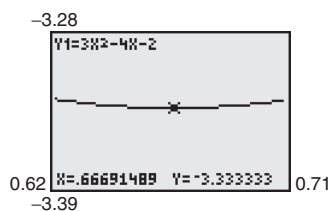

Figure 1.40

Figure 1.41

✓CHECKPOINT   Now try Exercise 31.

TECHNOLOGY TIP   Some graphing utilities have built-in programs that will find minimum or maximum values. These features are demonstrated in Example 6.

TECHNOLOGY TIP

When you use a graphing utility to estimate the *x*- and *y*-values of a relative minimum or relative maximum, the *zoom* feature will often produce graphs that are nearly flat, as shown in Figure 1.41. To overcome this problem, you can manually change the vertical setting of the viewing window. The graph will stretch vertically if the values of Ymin and Ymax are closer together.

Example 6   Approximating Relative Minima and Maxima

Use a graphing utility to approximate the relative minimum and relative maximum of the function given by $f(x) = -x^3 + x$.

Solution

The graph of f is shown in Figure 1.42. By using the *zoom* and *trace* features or the *minimum* and *maximum* features of the graphing utility, you can estimate that the function has a relative minimum at the point

$(-0.58, -0.38)$ See Figure 1.43.

and a relative maximum at the point

$(0.58, 0.38)$.   See Figure 1.44.

If you take a course in calculus, you will learn a technique for finding the exact points at which this function has a relative minimum and a relative maximum.

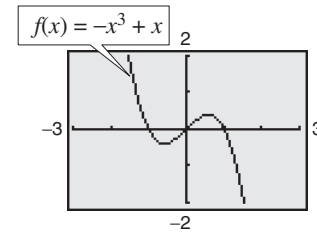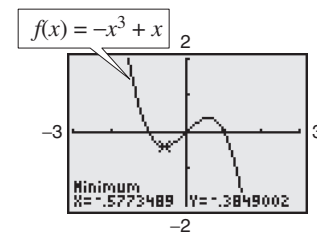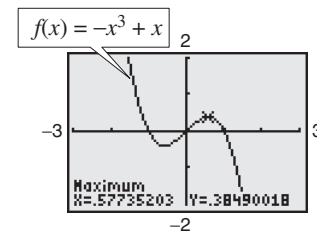✓**CHECKPOINT**   Now try Exercise 33.

Figure 1.42

Figure 1.43

Example 7   Temperature

During a 24-hour period, the temperature y (in degrees Fahrenheit) of a certain city can be approximated by the model

$$y = 0.026x^3 - 1.03x^2 + 10.2x + 34, \qquad 0 \le x \le 24$$

where x represents the time of day, with $x = 0$ corresponding to 6 A.M. Approximate the maximum and minimum temperatures during this 24-hour period.

Solution

To solve this problem, graph the function as shown in Figure 1.45. Using the *zoom* and *trace* features or the *maximum* feature of a graphing utility, you can determine that the maximum temperature during the 24-hour period was approximately 64°F. This temperature occurred at about 12:36 P.M. ($x \approx 6.6$), as shown in Figure 1.46. Using the *zoom* and *trace* features or the *minimum* feature, you can determine that the minimum temperature during the 24-hour period was approximately 34°F, which occurred at about 1:48 A.M. ($x \approx 19.8$), as shown in Figure 1.47.
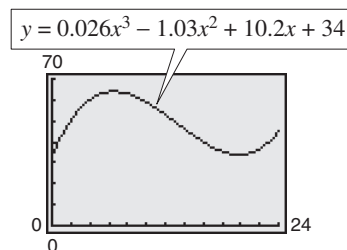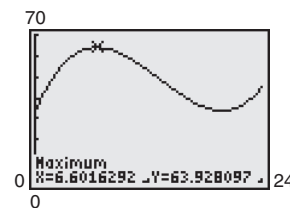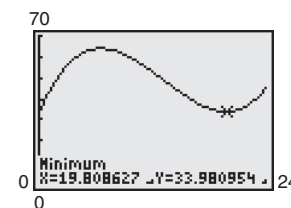

Figure 1.44

> **TECHNOLOGY SUPPORT**
>
> For instructions on how to use the *minimum* and *maximum* features, see Appendix A; for specific keystrokes, go to this textbook's *Online Study Center*.

Figure 1.45

Figure 1.46

Figure 1.47

✓**CHECKPOINT**   Now try Exercise 91.

Graphing Step Functions and Piecewise–Defined Functions

Library of Parent Functions: Greatest Integer Function

The *greatest integer function*, denoted by $[\![x]\!]$ and defined as the greatest integer less than or equal to x, has an infinite number of breaks or steps— one at each integer value in its domain. The basic characteristics of the greatest integer function are summarized below. A review of the greatest integer function can be found in the *Study Capsules*.

Graph of $f(x) = [\![x]\!]$
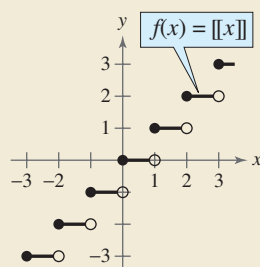
Domain: $(-\infty, \infty)$
Range: the set of integers
x-intercepts: in the interval $[0, 1)$
y-intercept: $(0, 0)$
Constant between each pair of consecutive integers
Jumps vertically one unit at each integer value

Could you describe the greatest integer function using a piecewise-defined function? How does the graph of the greatest integer function differ from the graph of a line with a slope of zero?

Most graphing utilities display graphs in *connected mode*, which means that the graph has no breaks. When you are sketching graphs that do have breaks, it is better to use *dot mode*. Graph the greatest integer function [often called Int (x)] in *connected* and *dot modes*, and compare the two results.

Because of the vertical jumps described above, the greatest integer function is an example of a **step function** whose graph resembles a set of stairsteps. Some values of the greatest integer function are as follows.

$$[\![-1]\!] = (\text{greatest integer} \le -1) = -1$$

$$\left[\!\left[\tfrac{1}{10}\right]\!\right] = \left(\text{greatest integer} \le \tfrac{1}{10}\right) = 0$$

$$[\![1.5]\!] = (\text{greatest integer} \le 1.5) = 1$$

In Section 1.3, you learned that a piecewise-defined function is a function that is defined by two or more equations over a specified domain. To sketch the graph of a piecewise-defined function, you need to sketch the graph of each equation on the appropriate portion of the domain.

Demonstrate the real-life nature of step functions by discussing Exercises 87 and 88 in this section. If writing is a part of your course, this section provides a good opportunity for students to find other examples of step functions and write brief essays on their applications.
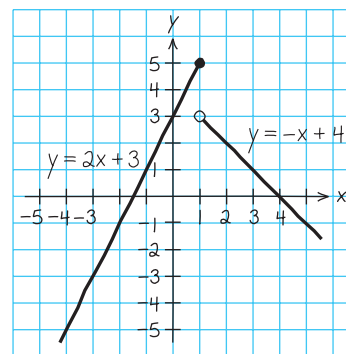
Example 8   Graphing a Piecewise-Defined Function

Sketch the graph of $f(x) = \begin{cases} 2x + 3, & x \le 1 \\ -x + 4, & x > 1 \end{cases}$ by hand.

Solution

This piecewise-defined function is composed of two linear functions. At and to the left of $x = 1$, the graph is the line given by $y = 2x + 3$. To the right of $x = 1$, the graph is the line given by $y = -x + 4$ (see Figure 1.48). Notice that the point $(1, 5)$ is a solid dot and the point $(1, 3)$ is an open dot. This is because $f(1) = 5$.

✔CHECKPOINT   Now try Exercise 43.

Figure 1.48

Even and Odd Functions

A graph has *symmetry with respect to the y-axis* if whenever (x, y) is on the graph, so is the point $(-x, y)$. A graph has *symmetry with respect to the origin* if whenever (x, y) is on the graph, so is the point $(-x, -y)$. A graph has *symmetry with respect to the x-axis* if whenever (x, y) is on the graph, so is the point $(x, -y)$. A function whose graph is symmetric with respect to the *y*-axis is an **even function.** A function whose graph is symmetric with respect to the origin is an **odd function.** A graph that is symmetric with respect to the *x*-axis is not the graph of a function (except for the graph of $y = 0$). These three types of symmetry are illustrated in Figure 1.49.
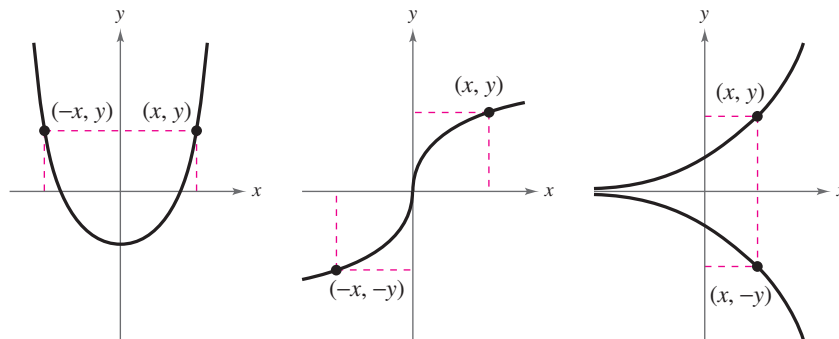
Symmetric to y-axis
Even function

Symmetric to origin
Odd function

Symmetric to x-axis
Not a function

Figure 1.49

Test for Even and Odd Functions

A function f is **even** if, for each x in the domain of f, $f(-x) = f(x)$.
A function f is **odd** if, for each x in the domain of f, $f(-x) = -f(x)$.

Example 9   Testing for Evenness and Oddness

Is the function given by $f(x) = |x|$ even, odd, or neither?

Algebraic Solution

This function is even because

$$f(-x) = |-x|$$

$$= |x|$$

$$= f(x).$$

✔CHECKPOINT   Now try Exercise 59.

Graphical Solution

Use a graphing utility to enter $y = |x|$ in the *equation editor*, as shown in Figure 1.50. Then graph the function using a standard viewing window, as shown in Figure 1.51. You can see that the graph appears to be symmetric about the *y*-axis. So, the function is even.
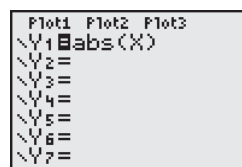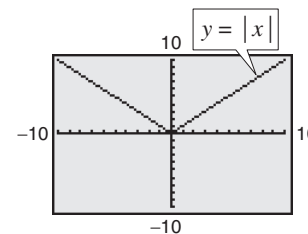
Ploti Plot2 Plot3
\Y1■abs(X)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=

$y = |x|$

Figure 1.50   **Figure 1.51**

Example 10   Even and Odd Functions

Determine whether each function is even, odd, or neither.

a. $g(x) = x^3 - x$

b. $h(x) = x^2 + 1$

c. $f(x) = x^3 - 1$

Algebraic Solution

a. This function is odd because

$$g(-x) = (-x)^3 - (-x)$$
$$= -x^3 + x$$
$$= -(x^3 - x)$$
$$= -g(x).$$

b. This function is even because

$$h(-x) = (-x)^2 + 1$$
$$= x^2 + 1$$
$$= h(x).$$

c. Substituting $-x$ for x produces

$$f(-x) = (-x)^3 - 1$$
$$= -x^3 - 1.$$

Because $f(x) = x^3 - 1$ and $-f(x) = -x^3 + 1$, you can conclude that $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$. So, the function is neither even nor odd.

Graphical Solution

a. In Figure 1.52, the graph is symmetric with respect to the origin. So, this function is odd.
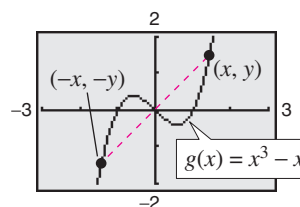

Figure 1.52

b. In Figure 1.53, the graph is symmetric with respect to the y-axis. So, this function is even.
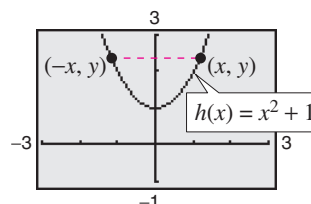

Figure 1.53

c. In Figure 1.54, the graph is neither symmetric with respect to the origin nor with respect to the y-axis. So, this function is neither even nor odd.
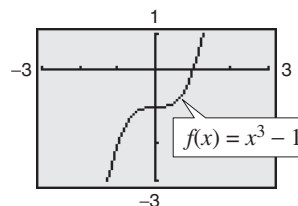

Figure 1.54

✓**CHECKPOINT**   Now try Exercise 61.

To help visualize symmetry with respect to the origin, place a pin at the origin of a graph and rotate the graph 180°. If the result after rotation coincides with the original graph, the graph is symmetric with respect to the origin.
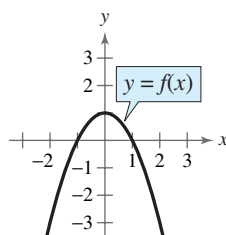
1.4 Exercises

### Vocabulary Check

Fill in the blanks.

1. The graph of a function f is a collection of _____ (x, y) such that x is in the domain of f.

2. The _____ is used to determine whether the graph of an equation is a function of y in terms of x.

3. A function f is _____ on an interval if, for any x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

4. A function value $f(a)$ is a relative _____ of f if there exists an interval (x_1, x_2) containing a such that $x_1 < x < x_2$ implies $f(a) \le f(x)$.

5. The function $f(x) = [\![x]\!]$ is called the _____ function, and is an example of a step function.

6. A function f is _____ if, for each x in the domain of f, $f(-x) = f(x)$.

In Exercises 1–4, use the graph of the function to find the domain and range of f. Then find $f(0)$.
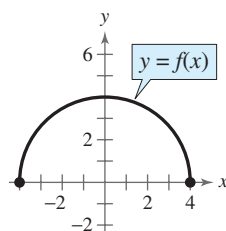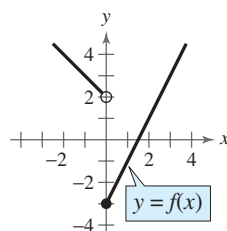
1.

2.
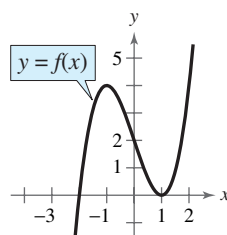
3.

4.

In Exercises 5–10, use a graphing utility to graph the function and estimate its domain and range. Then find the domain and range algebraically.

5. $f(x) = 2x^2 + 3$

6. $f(x) = -x^2 - 1$

7. $f(x) = \sqrt{x - 1}$

8. $h(t) = \sqrt{4 - t^2}$

9. $f(x) = |x + 3|$

10. $f(x) = -\frac{1}{4}|x - 5|$

In Exercises 11–14, use the given function to answer the questions.

(a) Determine the domain of the function.

(b) Find the value(s) of x such that $f(x) = 0$.

(c) The values of x from part (b) are referred to as what graphically?

(d) Find $f(0)$, if possible.

(e) The value from part (d) is referred to as what graphically?

(f) What is the value of f at $x = 1$? What are the coordinates of the point?
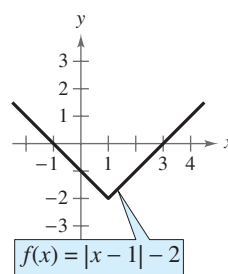
(g) What is the value of f at $x = -1$? What are the coordinates of the point?

(h) The coordinates of the point on the graph of f at which $x = -3$. can be labeled $(-3, f(-3))$ or $(-3, \quad)$.
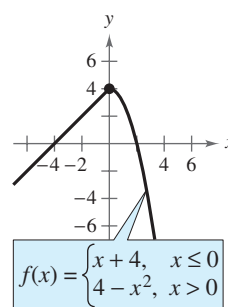
11. $f(x) = x^2 - x - 6$

12. $f(x) = x^3 - 4x$

13.

$f(x) = |x - 1| - 2$

14.

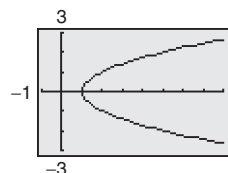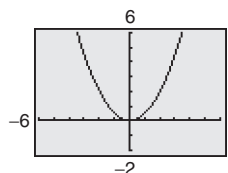$f(x) = \begin{cases} x + 4, & x \le 0 \\ 4 - x^2, & x > 0 \end{cases}$

In Exercises 15–18, use the Vertical Line Test to determine whether y is a function of x. Describe how you can use a graphing utility to produce the given graph.
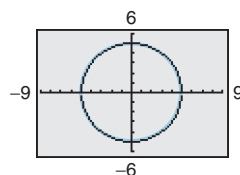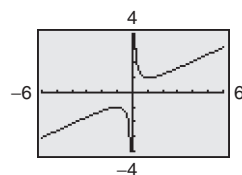
15. $y = \frac{1}{2}x^2$

16. $x - y^2 = 1$
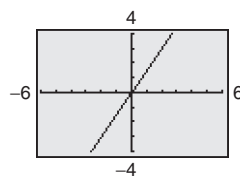
17. $x^2 + y^2 = 25$
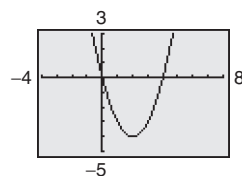
18. $x^2 = 2xy - 1$

In Exercises 19–22, determine the open intervals over which the function is increasing, decreasing, or constant.
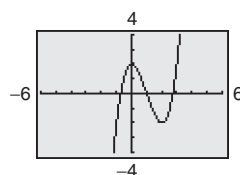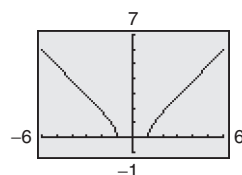
19. $f(x) = \frac{3}{2}x$

20. $f(x) = x^2 - 4x$

21. $f(x) = x^3 - 3x^2 + 2$

22. $f(x) = \sqrt{x^2 - 1}$

In Exercises 23–30, (a) use a graphing utility to graph the function and (b) determine the open intervals on which the function is increasing, decreasing, or constant.

23. $f(x) = 3$

24. $f(x) = x$

25. $f(x) = x^{2/3}$

26. $f(x) = -x^{3/4}$

27. $f(x) = x\sqrt{x + 3}$

28. $f(x) = \sqrt{1 - x}$

29. $f(x) = |x + 1| + |x - 1|$

30. $f(x) = -|x + 4| - |x + 1|$

In Exercises 31–36, use a graphing utility to approximate any relative minimum or relative maximum values of the function.

31. $f(x) = x^2 - 6x$

32. $f(x) = 3x^2 - 2x - 5$

33. $y = 2x^3 + 3x^2 - 12x$

34. $y = x^3 - 6x^2 + 15$

35. $h(x) = (x - 1)\sqrt{x}$

36. $g(x) = x\sqrt{4 - x}$

In Exercises 37–42, (a) approximate the relative minimum or relative maximum values of the function by sketching its graph using the point-plotting method, (b) use a graphing utility to approximate any relative minimum or relative maximum values, and (c) compare your answers from parts (a) and (b).

37. $f(x) = x^2 - 4x - 5$

38. $f(x) = 3x^2 - 12x$

39. $f(x) = x^3 - 3x$

40. $f(x) = -x^3 + 3x^2$

41. $f(x) = 3x^2 - 6x + 1$

42. $f(x) = 8x - 4x^2$

In Exercises 43–50, sketch the graph of the piecewise-defined function by hand.

43. $f(x) = \begin{cases} 2x + 3, & x < 0 \\ 3 - x, & x \geq 0 \end{cases}$

44. $f(x) = \begin{cases} x + 6, & x \leq -4 \\ 2x - 4, & x > -4 \end{cases}$

45. $f(x) = \begin{cases} \sqrt{4 + x}, & x < 0 \\ \sqrt{4 - x}, & x \geq 0 \end{cases}$

46. $f(x) = \begin{cases} 1 - (x - 1)^2, & x \leq 2 \\ \sqrt{x - 2}, & x > 2 \end{cases}$

47. $f(x) = \begin{cases} x + 3, & x \leq 0 \\ 3, & 0 < x \leq 2 \\ 2x - 1, & x > 2 \end{cases}$

48. $g(x) = \begin{cases} x + 5, & x \leq -3 \\ -2, & -3 < x < 1 \\ 5x - 4, & x \geq 1 \end{cases}$

49. $f(x) = \begin{cases} 2x + 1, & x \leq -1 \\ x^2 - 2, & x > -1 \end{cases}$

50. $h(x) = \begin{cases} 3 + x, & x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$

Library of Parent Functions   In Exercises 51–56, sketch the graph of the function by hand. Then use a graphing utility to verify the graph.

51. $f(x) = [\![x]\!] + 2$

52. $f(x) = [\![x]\!] - 3$

53. $f(x) = [\![x - 1]\!] + 2$

54. $f(x) = [\![x - 2]\!] + 1$

55. $f(x) = [\![2x]\!]$

56. $f(x) = [\![4x]\!]$

In Exercises 57 and 58, use a graphing utility to graph the function. State the domain and range of the function. Describe the pattern of the graph.

57. $s(x) = 2\left(\frac{1}{4}x - \left[\!\left[\frac{1}{4}x\right]\!\right]\right)$

58. $g(x) = 2\left(\frac{1}{4}x - \left[\!\left[\frac{1}{4}x\right]\!\right]\right)^2$

In Exercises 59–66, algebraically determine whether the function is even, odd, or neither. Verify your answer using a graphing utility.

59. $f(t) = t^2 + 2t - 3$

60. $f(x) = x^6 - 2x^2 + 3$

61. $g(x) = x^3 - 5x$

62. $h(x) = x^3 - 5$

63. $f(x) = x\sqrt{1 - x^2}$

64. $f(x) = x\sqrt{x + 5}$

65. $g(s) = 4s^{2/3}$

66. $f(s) = 4s^{3/2}$

Think About It   **In Exercises 67–72, find the coordinates of a second point on the graph of a function f if the given point is on the graph and the function is (a) even and (b) odd.**

67. $\left(-\frac{3}{2}, 4\right)$

68. $\left(-\frac{5}{3}, -7\right)$

69. $(4, 9)$

70. $(5, -1)$

71. $(x, -y)$

72. $(2a, 2c)$

In Exercises 73–82, use a graphing utility to graph the function and determine whether it is even, odd, or neither. Verify your answer algebraically.

73. $f(x) = 5$

74. $f(x) = -9$

75. $f(x) = 3x - 2$

76. $f(x) = 5 - 3x$

77. $h(x) = x^2 - 4$

78. $f(x) = -x^2 - 8$

79. $f(x) = \sqrt{1 - x}$

80. $g(t) = \sqrt[3]{t - 1}$

81. $f(x) = |x + 2|$

82. $f(x) = -|x - 5|$

In Exercises 83–86, graph the function and determine the interval(s) (if any) on the real axis for which $f(x) \geq 0$. Use a graphing utility to verify your results.

83. $f(x) = 4 - x$

84. $f(x) = 4x + 2$

85. $f(x) = x^2 - 9$

86. $f(x) = x^2 - 4x$

87. *Communications*   The cost of using a telephone calling card is $1.05 for the first minute and $0.38 for each additional minute or portion of a minute.

(a) A customer needs a model for the cost C of using the calling card for a call lasting t minutes. Which of the following is the appropriate model?

$C_1(t) = 1.05 + 0.38[\![t - 1]\!]$

$C_2(t) = 1.05 - 0.38[\![-(t - 1)]\!]$

(b) Use a graphing utility to graph the appropriate model. Use the *value* feature or the *zoom* and *trace* features to estimate the cost of a call lasting 18 minutes and 45 seconds.

88. *Delivery Charges*   The cost of sending an overnight package from New York to Atlanta is $9.80 for a package weighing up to but not including 1 pound and $2.50 for each additional pound or portion of a pound. Use the greatest integer function to create a model for the cost C of overnight delivery of a package weighing x pounds, where $x > 0$. Sketch the graph of the function.

∫ In Exercises 89 and 90, write the height h of the rectangle as a function of x.

89.

90.
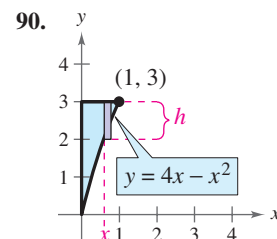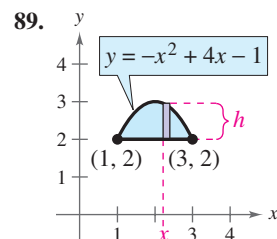
91. *Population*   During a 14 year period from 1990 to 2004, the population P (in thousands) of West Virginia fluctuated according to the model

$P = 0.0108t^4 - 0.211t^3 + 0.40t^2 + 7.9t + 1791,$

$0 \leq t \leq 14$

where t represents the year, with $t = 0$ corresponding to 1990.   (Source: U.S. Census Bureau)

(a) Use a graphing utility to graph the model over the appropriate domain.

(b) Use the graph from part (a) to determine during which years the population was increasing. During which years was the population decreasing?

(c) Approximate the maximum population between 1990 and 2004.

92. *Fluid Flow*   The intake pipe of a 100-gallon tank has a flow rate of 10 gallons per minute, and two drain pipes have a flow rate of 5 gallons per minute each. The graph shows the volume V of fluid in the tank as a function of time t. Determine in which pipes the fluid is flowing in specific subintervals of the one-hour interval of time shown on the graph. (There are many correct answers.)
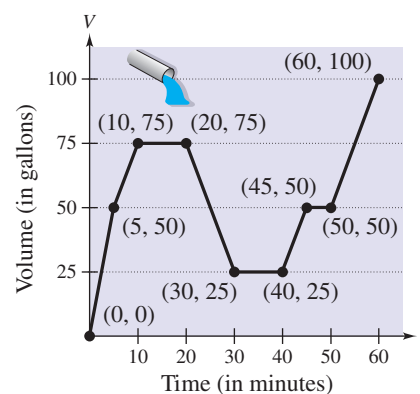
Synthesis

True or False?   **In Exercises 93 and 94, determine whether the statement is true or false. Justify your answer.**

93. A function with a square root cannot have a domain that is the set of all real numbers.

94. It is possible for an odd function to have the interval $[0, \infty)$ as its domain.

Think About It **In Exercises 95–100, match the graph of the function with the best choice that describes the situation.**

(a) The air temperature at a beach on a sunny day

(b) The height of a football kicked in a field goal attempt

(c) The number of children in a family over time

(d) The population of California as a function of time

(e) The depth of the tide at a beach over a 24-hour period

(f) The number of cupcakes on a tray at a party

95.

96.

97.

98.

99.

100.
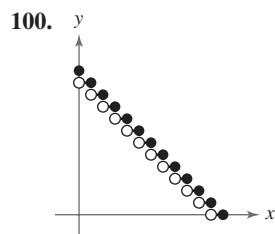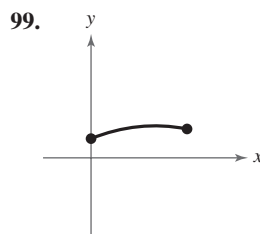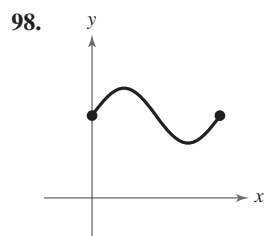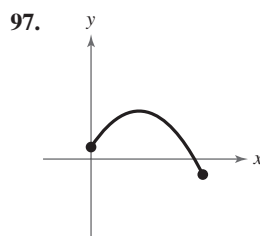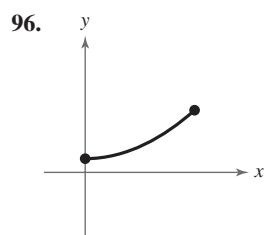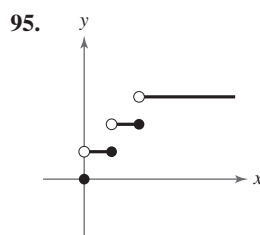
101. *Proof*   Prove that a function of the following form is odd.

$$y = a_{2n+1}x^{2n+1} + a_{2n-1}x^{2n-1} + \cdots + a_3x^3 + a_1x$$

102. *Proof*   Prove that a function of the following form is even.

$$y = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$$

103. If f is an even function, determine if g is even, odd, or neither. Explain.

(a) $g(x) = -f(x)$ (b) $g(x) = f(-x)$

(c) $g(x) = f(x) - 2$ (d) $g(x) = -f(x - 2)$

104. *Think About It*   Does the graph in Exercise 16 represent x as a function of y? Explain.

105. *Think About It*   Does the graph in Exercise 17 represent x as a function of y? Explain.

106. *Writing*   Write a short paragraph describing three different functions that represent the behaviors of quantities between 1995 and 2006. Describe one quantity that decreased during this time, one that increased, and one that was constant. Present your results graphically.

Skills Review

In Exercises 107–110, identify the terms. Then identify the coefficients of the variable terms of the expression.

107. $-2x^2 + 8x$ **108.** $10 + 3x$

109. $\dfrac{x}{3} - 5x^2 + x^3$ **110.** $7x^4 + \sqrt{2}x^2$

In Exercises 111–114, find (a) the distance between the two points and (b) the midpoint of the line segment joining the points.

111. $(-2, 7), (6, 3)$

112. $(-5, 0), (3, 6)$

113. $\left(\frac{5}{2}, -1\right), \left(-\frac{3}{2}, 4\right)$

114. $\left(-6, \frac{2}{3}\right), \left(\frac{3}{4}, \frac{1}{6}\right)$

In Exercises 115–118, evaluate the function at each specified value of the independent variable and simplify.

115. $f(x) = 5x - 1$

(a) $f(6)$ (b) $f(-1)$ (c) $f(x - 3)$

116. $f(x) = -x^2 - x + 3$

(a) $f(4)$ (b) $f(-2)$ (c) $f(x - 2)$

117. $f(x) = x\sqrt{x - 3}$

(a) $f(3)$ (b) $f(12)$ (c) $f(6)$

118. $f(x) = -\frac{1}{2}x|x + 1|$

(a) $f(-4)$ (b) $f(10)$ (c) $f\left(-\frac{2}{3}\right)$

∫ **In Exercises 119 and 120, find the difference quotient and simplify your answer.**

119. $f(x) = x^2 - 2x + 9, \dfrac{f(3 + h) - f(3)}{h}, h \neq 0$

120. $f(x) = 5 + 6x - x^2, \dfrac{f(6 + h) - f(6)}{h}, h \neq 0$