1.7 Inverse Functions

Inverse Functions

Recall from Section 1.3 that a function can be represented by a set of ordered pairs. For instance, the function $f(x) = x + 4$ from the set $A = \{1, 2, 3, 4\}$ to the set $B = \{5, 6, 7, 8\}$ can be written as follows.

$$f(x) = x + 4\text{:} \quad \{(1, 5), (2, 6), (3, 7), (4, 8)\}$$

In this case, by interchanging the first and second coordinates of each of these ordered pairs, you can form the **inverse function** of f, which is denoted by f^{-1}. It is a function from the set B to the set A, and can be written as follows.

$$f^{-1}(x) = x - 4\text{:} \quad \{(5, 1), (6, 2), (7, 3), (8, 4)\}$$

Note that the domain of f is equal to the range of f^{-1}, and vice versa, as shown in Figure 1.82. Also note that the functions f and f^{-1} have the effect of "undoing" each other. In other words, when you form the composition of f with f^{-1} or the composition of f^{-1} with f, you obtain the identity function.

$$f(f^{-1}(x)) = f(x - 4) = (x - 4) + 4 = x$$

$$f^{-1}(f(x)) = f^{-1}(x + 4) = (x + 4) - 4 = x$$

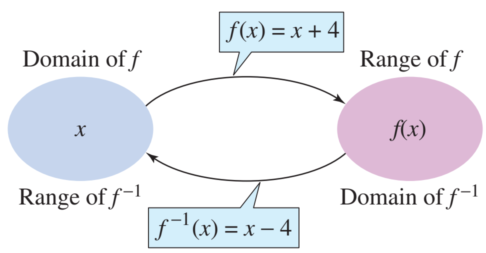

Figure 1.82

What you should learn

- Find inverse functions informally and verify that two functions are inverse functions of each other.
- Use graphs of functions to decide whether functions have inverse functions.
- Determine if functions are one-to-one.
- Find inverse functions algebraically.

Why you should learn it

Inverse functions can be helpful in further exploring how two variables relate to each other. For example, in Exercises 103 and 104 on page 156, you will use inverse functions to find the European shoe sizes from the corresponding U.S. shoe sizes.

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Example 1 Finding Inverse Functions Informally

Find the inverse function of $f(x) = 4x$. Then verify that both $f(f^{-1}(x))$ and $f^{-1}(f(x))$ are equal to the identity function.

Solution

The function f *multiplies* each input by 4. To "undo" this function, you need to *divide* each input by 4. So, the inverse function of $f(x) = 4x$ is given by

$$f^{-1}(x) = \frac{x}{4}.$$

You can verify that both $f(f^{-1}(x))$ and $f^{-1}(f(x))$ are equal to the identity function as follows.

$$f(f^{-1}(x)) = f\left(\frac{x}{4}\right) = 4\left(\frac{x}{4}\right) = x \qquad f^{-1}(f(x)) = f^{-1}(4x) = \frac{4x}{4} = x$$

CHECKPOINT Now try Exercise 1.

STUDY TIP

Don't be confused by the use of the exponent -1 to denote the inverse function f^{-1}. In this text, whenever f^{-1} is written, it always refers to the inverse function of the function f and not to the reciprocal of $f(x)$, which is given by

$$\frac{1}{f(x)}.$$

Example 2 Finding Inverse Functions Informally

Find the inverse function of $f(x) = x - 6$. Then verify that both $f(f^{-1}(x))$ and $f^{-1}(f(x))$ are equal to the identity function.

Solution

The function f *subtracts* 6 from each input. To "undo" this function, you need to *add* 6 to each input. So, the inverse function of $f(x) = x - 6$ is given by

$$f^{-1}(x) = x + 6.$$

You can verify that both $f(f^{-1}(x))$ and $f^{-1}(f(x))$ are equal to the identity function as follows.

$$f(f^{-1}(x)) = f(x + 6) = (x + 6) - 6 = x$$

$$f^{-1}(f(x)) = f^{-1}(x - 6) = (x - 6) + 6 = x$$

CHECKPOINT Now try Exercise 3.

A table of values can help you understand inverse functions. For instance, the following table shows several values of the function in Example 2. Interchange the rows of this table to obtain values of the inverse function.

x	-2	-1	0	1	2
$f(x)$	-8	-7	-6	-5	-4

x	-8	-7	-6	-5	-4
$f^{-1}(x)$	-2	-1	0	1	2

In the table at the left, each output is 6 less than the input, and in the table at the right, each output is 6 more than the input.

The formal definition of an inverse function is as follows.

Definition of Inverse Function

Let f and g be two functions such that

$$f(g(x)) = x \qquad \text{for every } x \text{ in the domain of } g$$

and

$$g(f(x)) = x \qquad \text{for every } x \text{ in the domain of } f.$$

Under these conditions, the function g is the **inverse function** of the function f. The function g is denoted by f^{-1} (read "f-inverse"). So,

$$f(f^{-1}(x)) = x \qquad \text{and} \qquad f^{-1}(f(x)) = x.$$

The domain of f must be equal to the range of f^{-1}, and the range of f must be equal to the domain of f^{-1}.

If the function g is the inverse function of the function f, it must also be true that the function f is the inverse function of the function g. For this reason, you can say that the functions f and g are *inverse functions of each other*.

Example 3 Verifying Inverse Functions Algebraically

Show that the functions are inverse functions of each other.

$$f(x) = 2x^3 - 1 \qquad \text{and} \qquad g(x) = \sqrt[3]{\frac{x+1}{2}}$$

Solution

$$f(g(x)) = f\left(\sqrt[3]{\frac{x+1}{2}}\right) = 2\left(\sqrt[3]{\frac{x+1}{2}}\right)^3 - 1$$

$$= 2\left(\frac{x+1}{2}\right) - 1$$

$$= x + 1 - 1$$

$$= x$$

$$g(f(x)) = g(2x^3 - 1) = \sqrt[3]{\frac{(2x^3 - 1) + 1}{2}}$$

$$= \sqrt[3]{\frac{2x^3}{2}}$$

$$= \sqrt[3]{x^3}$$

$$= x$$

CHECKPOINT Now try Exercise 15.

Example 4 Verifying Inverse Functions Algebraically

Which of the functions is the inverse function of $f(x) = \dfrac{5}{x-2}$?

$$g(x) = \frac{x-2}{5} \qquad \text{or} \qquad h(x) = \frac{5}{x} + 2$$

Solution

By forming the composition of f with g, you have

$$f(g(x)) = f\left(\frac{x-2}{5}\right) = \frac{5}{\left(\frac{x-2}{5}\right) - 2} = \frac{25}{x - 12} \neq x.$$

Because this composition is not equal to the identity function x, it follows that g *is not* the inverse function of f. By forming the composition of f with h, you have

$$f(h(x)) = f\left(\frac{5}{x} + 2\right) = \frac{5}{\left(\frac{5}{x} + 2\right) - 2} = \frac{5}{\left(\frac{5}{x}\right)} = x.$$

So, it appears that h is the inverse function of f. You can confirm this by showing that the composition of h with f is also equal to the identity function.

CHECKPOINT Now try Exercise 19.

Point out to students that when using a graphing utility, it is important to know a function's behavior because the graphing utility may show an incomplete function. For instance, it is important to know that the domain of $f(x) = x^{2/3}$ is all real numbers, because a graphing utility may show an incomplete graph of the function, depending on how the function was entered.

TECHNOLOGY TIP

Most graphing utilities can graph $y = x^{1/3}$ in two ways:

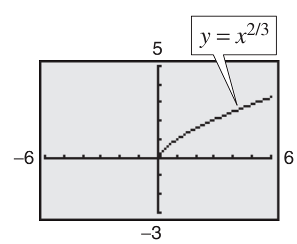
$y_1 = x \wedge (1/3)$ or

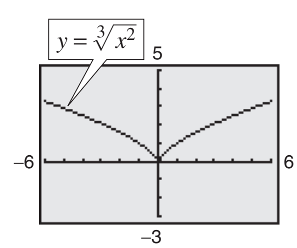
$y_1 = \sqrt[3]{x}$.

However, you may not be able to obtain the complete graph of $y = x^{2/3}$ by entering $y_1 = x \wedge (2/3)$. If not, you should use

$y_1 = (x \wedge (1/3))^2$ or

$y_1 = \sqrt[3]{x^2}$.





The Graph of an Inverse Function

The graphs of a function f and its inverse function f^{-1} are related to each other in the following way. If the point (a, b) lies on the graph of f, then the point (b, a) must lie on the graph of f^{-1}, and vice versa. This means that the graph of f^{-1} is a *reflection* of the graph of f in the line $y = x$, as shown in Figure 1.83.

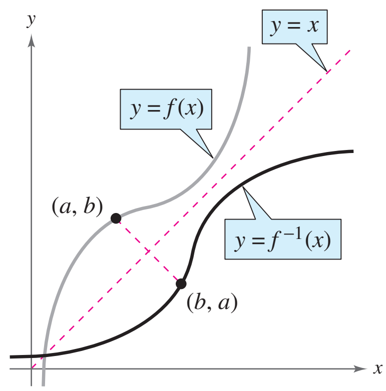


Figure 1.83

TECHNOLOGY TIP

In Examples 3 and 4, inverse functions were verified algebraically. A graphing utility can also be helpful in checking whether one function is the inverse function of another function. Use the Graph Reflection Program found at this textbook's *Online Study Center* to verify Example 4 graphically.

Example 5 Verifying Inverse Functions Graphically and Numerically

Verify that the functions f and g from Example 3 are inverse functions of each other graphically and numerically.

Graphical Solution

You can verify that f and g are inverse functions of each other *graphically* by using a graphing utility to graph f and g in the same viewing window. (Be sure to use a *square setting*.) From the graph in Figure 1.84, you can verify that the graph of g is the reflection of the graph of f in the line $y = x$.

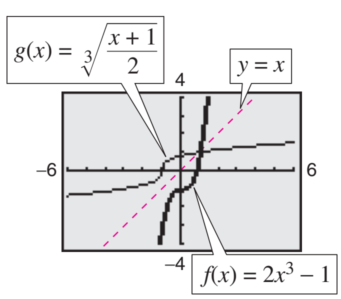


Figure 1.84

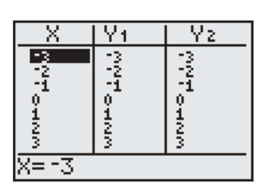
Numerical Solution

You can verify that f and g are inverse functions of each other *numerically*. Begin by entering the compositions $f(g(x))$ and $g(f(x))$ into a graphing utility as follows.

$$y_1 = f(g(x)) = 2\left(\sqrt[3]{\frac{x+1}{2}}\right)^3 - 1$$

$$y_2 = g(f(x)) = \sqrt[3]{\frac{(2x^3 - 1) + 1}{2}}$$

Then use the *table* feature of the graphing utility to create a table, as shown in Figure 1.85. Note that the entries for x, y_1, and y_2 are the same. So, $f(g(x)) = x$ and $g(f(x)) = x$. You can now conclude that f and g are inverse functions of each other.



X	Y1	Y2
-3	-3	-3
-2	-2	-2
-1	-1	-1
0	0	0
1	1	1
2	2	2
3	3	3

X= -3

Figure 1.85

CHECKPOINT Now try Exercise 25.

The Existence of an Inverse Function

Consider the function $f(x) = x^2$. The first table at the right is a table of values for $f(x) = x^2$. The second table was created by interchanging the rows of the first table. The second table does not represent a function because the input $x = 4$ is matched with two different outputs: $y = -2$ and $y = 2$. So, $f(x) = x^2$ does not have an inverse function.

x	-2	-1	0	1	2
$f(x)$	4	1	0	1	4

↓

x	4	1	0	1	4
$g(x)$	-2	-1	0	1	2

To have an inverse function, a function must be **one-to-one,** which means that no two elements in the domain of f correspond to the same element in the range of f.

Definition of a One-to-One Function

A function f is **one-to-one** if, for a and b in its domain, $f(a) = f(b)$ implies that $a = b$.

Existence of an Inverse Function

A function f has an inverse function f^{-1} if and only if f is one-to-one.

From its graph, it is easy to tell whether a function of x is one-to-one. Simply check to see that every horizontal line intersects the graph of the function at most once. This is called the **Horizontal Line Test.** For instance, Figure 1.86 shows the graph of $y = x^2$. On the graph, you can find a horizontal line that intersects the graph twice.

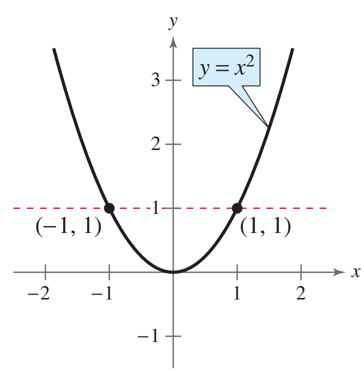


Figure 1.86 $f(x) = x^2$ **is not one-to-one.**

Two special types of functions that pass the Horizontal Line Test are those that are increasing or decreasing on their entire domains.

1. If f is *increasing* on its entire domain, then f is one-to-one.
2. If f is *decreasing* on its entire domain, then f is one-to-one.

Example 6 Testing for One-to-One Functions

Is the function $f(x) = \sqrt{x} + 1$ one-to-one?

Algebraic Solution

Let a and b be nonnegative real numbers with $f(a) = f(b)$.

$$\sqrt{a} + 1 = \sqrt{b} + 1 \qquad \text{Set } f(a) = f(b).$$

$$\sqrt{a} = \sqrt{b}$$

$$a = b$$

So, $f(a) = f(b)$ implies that $a = b$. You can conclude that f *is* one-to-one and *does* have an inverse function.

CHECKPOINT Now try Exercise 55.

Graphical Solution

Use a graphing utility to graph the function $y = \sqrt{x} + 1$. From Figure 1.87, you can see that a horizontal line will intersect the graph at most once and the function is increasing. So, f *is* one-to-one and *does* have an inverse function.

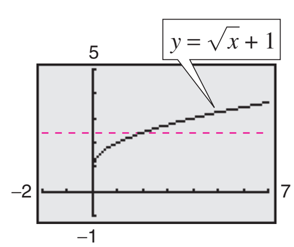


Figure 1.87

Finding Inverse Functions Algebraically

For simple functions, you can find inverse functions by inspection. For more complicated functions, however, it is best to use the following guidelines.

> **Finding an Inverse Function**
>
> 1. Use the Horizontal Line Test to decide whether f has an inverse function.
> 2. In the equation for $f(x)$, replace $f(x)$ by y.
> 3. Interchange the roles of x and y, and solve for y.
> 4. Replace y by $f^{-1}(x)$ in the new equation.
> 5. Verify that f and f^{-1} are inverse functions of each other by showing that the domain of f is equal to the range of f^{-1}, the range of f is equal to the domain of f^{-1}, and $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

The function f with an implied domain of all real numbers may not pass the Horizontal Line Test. In this case, the domain of f may be restricted so that f does have an inverse function. For instance, if the domain of $f(x) = x^2$ is restricted to the nonnegative real numbers, then f does have an inverse function.

Example 7 Finding an Inverse Function Algebraically

Find the inverse function of $f(x) = \dfrac{5 - 3x}{2}$.

Solution

The graph of f in Figure 1.88 passes the Horizontal Line Test. So you know that f is one-to-one and has an inverse function.

$$f(x) = \frac{5 - 3x}{2} \qquad \text{Write original function.}$$

$$y = \frac{5 - 3x}{2} \qquad \text{Replace } f(x) \text{ by } y.$$

$$x = \frac{5 - 3y}{2} \qquad \text{Interchange } x \text{ and } y.$$

$$2x = 5 - 3y \qquad \text{Multiply each side by 2.}$$

$$3y = 5 - 2x \qquad \text{Isolate the } y\text{-term.}$$

$$y = \frac{5 - 2x}{3} \qquad \text{Solve for } y.$$

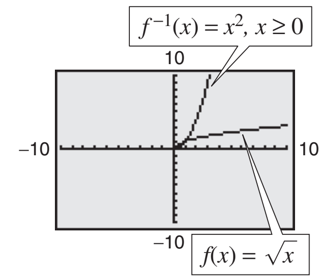
$$f^{-1}(x) = \frac{5 - 2x}{3} \qquad \text{Replace } y \text{ by } f^{-1}(x).$$

The domains and ranges of f and f^{-1} consist of all real numbers. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

CHECKPOINT Now try Exercise 59.

TECHNOLOGY TIP

Many graphing utilities have a built-in feature for drawing an inverse function. To see how this works, consider the function $f(x) = \sqrt{x}$. The inverse function of f is given by $f^{-1}(x) = x^2$, $x \geq 0$. Enter the function $y_1 = \sqrt{x}$. Then graph it in the standard viewing window and use the *draw inverse* feature. You should obtain the figure below, which shows both f and its inverse function f^{-1}. For instructions on how to use the *draw inverse* feature, see Appendix A; for specific keystrokes, go to this textbook's *Online Study Center*.



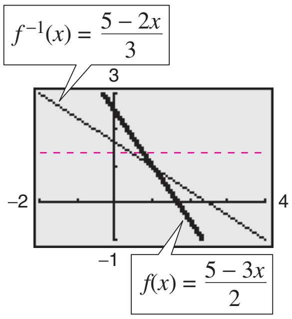


Figure 1.88

The *draw inverse* feature is particularly useful if you cannot find an expression for the inverse function of a given function. For example, it would be very difficult to determine the equation for the inverse function of the one-to-one function

$$f(x) = \tfrac{1}{4}x^5 + \tfrac{1}{4}x^3 + \tfrac{1}{2}x - 1.$$

However, it is easy to use the technique outlined above to obtain the *graph* of the inverse function.

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Example 8 Finding an Inverse Function Algebraically

Find the inverse function of $f(x) = x^3 - 4$ and use a graphing utility to graph f and f^{-1} in the same viewing window.

Solution

$f(x) = x^3 - 4$	Write original function.
$y = x^3 - 4$	Replace $f(x)$ by y.
$x = y^3 - 4$	Interchange x and y.
$y^3 = x + 4$	Isolate y.
$y = \sqrt[3]{x+4}$	Solve for y.
$f^{-1}(x) = \sqrt[3]{x+4}$	Replace y by $f^{-1}(x)$.

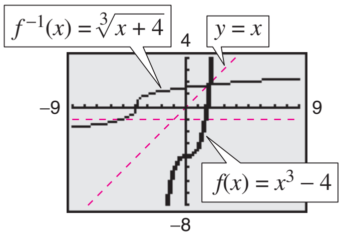

Figure 1.89

The graph of f in Figure 1.89 passes the Horizontal Line Test. So, you know that f is one-to-one and has an inverse function. The graph of f^{-1} in Figure 1.89 is the reflection of the graph of f in the line $y = x$. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

CHECKPOINT Now try Exercise 61.

Example 9 Finding an Inverse Function Algebraically

Find the inverse function of $f(x) = \sqrt{2x - 3}$ and use a graphing utility to graph f and f^{-1} in the same viewing window.

Solution

$f(x) = \sqrt{2x-3}$	Write original function.
$y = \sqrt{2x-3}$	Replace $f(x)$ by y.
$x = \sqrt{2y-3}$	Interchange x and y.
$x^2 = 2y - 3$	Square each side.
$2y = x^2 + 3$	Isolate y.
$y = \dfrac{x^2+3}{2}$	Solve for y.
$f^{-1}(x) = \dfrac{x^2+3}{2}, \quad x \ge 0$	Replace y by $f^{-1}(x)$.

The graph of f in Figure 1.90 passes the Horizontal Line Test. So you know that f is one-to-one and has an inverse function. The graph of f^{-1} in Figure 1.90 is the reflection of the graph of f in the line $y = x$. Note that the range of f is the interval $[0, \infty)$, which implies that the domain of f^{-1} is the interval $[0, \infty)$. Moreover, the domain of f is the interval $\left[\frac{3}{2}, \infty\right)$, which implies that the range of f^{-1} is the interval $\left[\frac{3}{2}, \infty\right)$. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

CHECKPOINT Now try Exercise 65.

Activities

1. Given $f(x) = 5x - 7$, find $f^{-1}(x)$.

   Answer: $f^{-1}(x) = \dfrac{x+7}{5}$

2. Show that f and g are inverse functions by showing that $f(g(x)) = x$ and $g(f(x)) = x$.

   $f(x) = 3x^3 + 1$

   $g(x) = \sqrt[3]{\dfrac{x-1}{3}}$

3. Describe the graphs of functions that have inverse functions and show how the graph of a function and its inverse function are related.

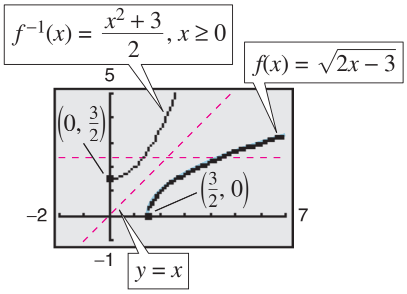

Figure 1.90

1.7 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blanks.

1. If the composite functions $f(g(x)) = x$ and $g(f(x)) = x$, then the function g is the _______ function of f, and is denoted by _______ .
2. The domain of f is the _______ of f^{-1}, and the _______ of f^{-1} is the range of f.
3. The graphs of f and f^{-1} are reflections of each other in the line _______ .
4. To have an inverse function, a function f must be _______ ; that is, $f(a) = f(b)$ implies $a = b$.
5. A graphical test for the existence of an inverse function is called the _______ Line Test.

In Exercises 1–8, find the inverse function of f informally. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

1. $f(x) = 6x$
2. $f(x) = \frac{1}{3}x$
3. $f(x) = x + 7$
4. $f(x) = x - 3$
5. $f(x) = 2x + 1$
6. $f(x) = \dfrac{x - 1}{4}$
7. $f(x) = \sqrt[3]{x}$
8. $f(x) = x^5$

In Exercises 9–14, (a) show that f and g are inverse functions algebraically and (b) use a graphing utility to create a table of values for each function to numerically show that f and g are inverse functions.

9. $f(x) = -\dfrac{7}{2}x - 3, \quad g(x) = -\dfrac{2x + 6}{7}$
10. $f(x) = \dfrac{x - 9}{4}, \quad g(x) = 4x + 9$
11. $f(x) = x^3 + 5, \quad g(x) = \sqrt[3]{x - 5}$
12. $f(x) = \dfrac{x^3}{2}, \quad g(x) = \sqrt[3]{2x}$
13. $f(x) = -\sqrt{x - 8}, \quad g(x) = 8 + x^2, \quad x \le 0$
14. $f(x) = \sqrt[3]{3x - 10}, \quad g(x) = \dfrac{x^3 + 10}{3}$

In Exercises 15–20, show that f and g are inverse functions algebraically. Use a graphing utility to graph f and g in the same viewing window. Describe the relationship between the graphs.

15. $f(x) = x^3, \quad g(x) = \sqrt[3]{x}$
16. $f(x) = \dfrac{1}{x}, \quad g(x) = \dfrac{1}{x}$
17. $f(x) = \sqrt{x - 4}; \quad g(x) = x^2 + 4, \ x \ge 0$
18. $f(x) = 9 - x^2, \quad x \ge 0; \quad g(x) = \sqrt{9 - x}$
19. $f(x) = 1 - x^3, \quad g(x) = \sqrt[3]{1 - x}$
20. $f(x) = \dfrac{1}{1 + x}, \ x \ge 0; \quad g(x) = \dfrac{1 - x}{x}, \ 0 < x \le 1$

In Exercises 21–24, match the graph of the function with the graph of its inverse function. [The graphs of the inverse functions are labeled (a), (b), (c), and (d).]

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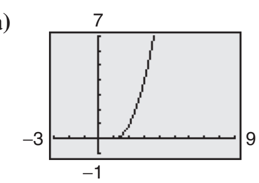

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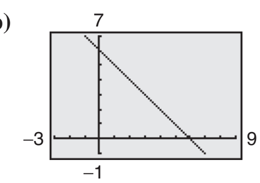

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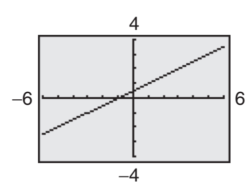

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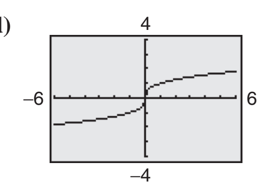

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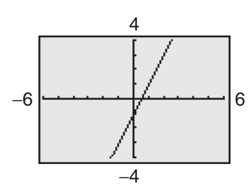

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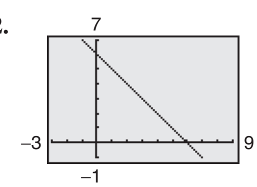

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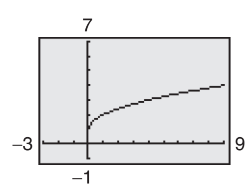

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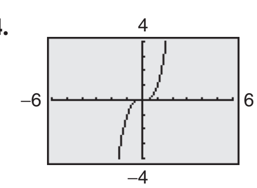


In Exercises 25–28, show that f and g are inverse functions (a) graphically and (b) numerically.

25. $f(x) = 2x, \quad g(x) = \dfrac{x}{2}$

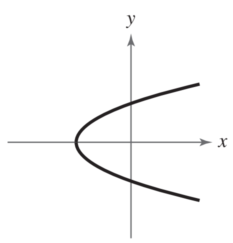
26. $f(x) = x - 5, \quad g(x) = x + 5$

27. $f(x) = \dfrac{x - 1}{x + 5}, \quad g(x) = -\dfrac{5x + 1}{x - 1}$

28. $f(x) = \dfrac{x + 3}{x - 2}, \quad g(x) = \dfrac{2x + 3}{x - 1}$

In Exercises 29–34, determine if the graph is that of a function. If so, determine if the function is one-to-one.

29.



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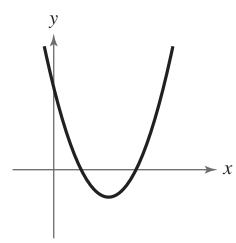

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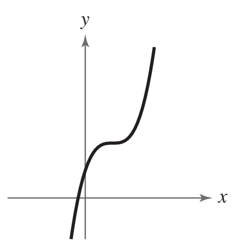

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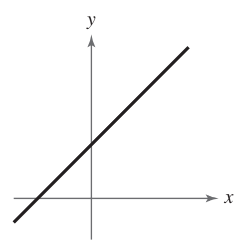

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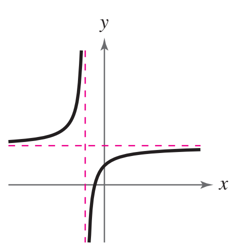

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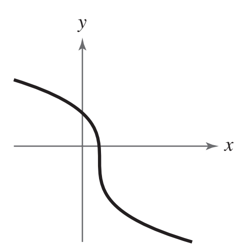


In Exercises 35–46, use a graphing utility to graph the function and use the Horizontal Line Test to determine whether the function is one-to-one and so has an inverse function exists.

35. $f(x) = 3 - \frac{1}{2}x$

36. $f(x) = \frac{1}{4}(x + 2)^2 - 1$

37. $h(x) = \dfrac{x^2}{x^2 + 1}$

38. $g(x) = \dfrac{4 - x}{6x^2}$

39. $h(x) = \sqrt{16 - x^2}$

40. $f(x) = -2x\sqrt{16 - x^2}$

41. $f(x) = 10$

42. $f(x) = -0.65$

43. $g(x) = (x + 5)^3$

44. $f(x) = x^5 - 7$

45. $h(x) = |x + 4| - |x - 4|$

46. $f(x) = -\dfrac{|x - 6|}{|x + 6|}$

In Exercises 47–58, determine algebraically whether the function is one-to-one. Verify your answer graphically.

47. $f(x) = x^4$

48. $g(x) = x^2 - x^4$

49. $f(x) = \dfrac{3x + 4}{5}$

50. $f(x) = 3x + 5$

51. $f(x) = \dfrac{1}{x^2}$

52. $h(x) = \dfrac{4}{x^2}$

53. $f(x) = (x + 3)^2, \quad x \geq -3$

54. $q(x) = (x - 5)^2, \quad x \leq 5$

55. $f(x) = \sqrt{2x + 3}$

56. $f(x) = \sqrt{x - 2}$

57. $f(x) = |x - 2|, \quad x \leq 2$

58. $f(x) = \dfrac{x^2}{x^2 + 1}$

In Exercises 59–68, find the inverse function of f algebraically. Use a graphing utility to graph both f and f^{-1} in the same viewing window. Describe the relationship between the graphs.

59. $f(x) = 2x - 3$

60. $f(x) = 3x$

61. $f(x) = x^5$

62. $f(x) = x^3 + 1$

63. $f(x) = x^{3/5}$

64. $f(x) = x^2, \quad x \geq 0$

65. $f(x) = \sqrt{4 - x^2}, \quad 0 \leq x \leq 2$

66. $f(x) = \sqrt{16 - x^2}, \quad -4 \leq x \leq 0$

67. $f(x) = \dfrac{4}{x}$

68. $f(x) = \dfrac{6}{\sqrt{x}}$

Think About It **In Exercises 69–78, restrict the domain of the function f so that the function is one-to-one and has an inverse function. Then find the inverse function f^{-1}. State the domains and ranges of f and f^{-1}. Explain your results. (There are many correct answers.)**

69. $f(x) = (x - 2)^2$

70. $f(x) = 1 - x^4$

71. $f(x) = |x + 2|$

72. $f(x) = |x - 2|$

73. $f(x) = (x + 3)^2$

74. $f(x) = (x - 4)^2$

75. $f(x) = -2x^2 + 5$

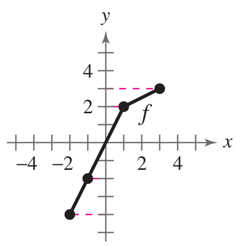
76. $f(x) = \frac{1}{2}x^2 - 1$

77. $f(x) = |x - 4| + 1$

78. $f(x) = -|x - 1| - 2$

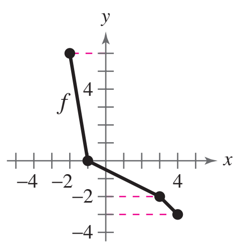
In Exercises 79 and 80, use the graph of the function f to complete the table and sketch the graph of f^{-1}.

79.



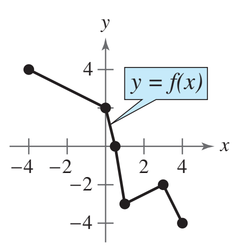
x	$f^{-1}(x)$
-4	
-2	
2	
3	

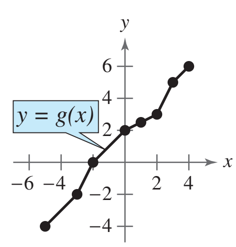
80.



x	$f^{-1}(x)$
-3	
-2	
0	
6	

In Exercises 81–88, use the graphs of $y = f(x)$ and $y = g(x)$ to evaluate the function.





81. $f^{-1}(0)$

82. $g^{-1}(0)$

83. $(f \circ g)(2)$

84. $g(f(-4))$

85. $f^{-1}(g(0))$

86. $(g^{-1} \circ f)(3)$

87. $(g \circ f^{-1})(2)$

88. $(f^{-1} \circ g^{-1})(-2)$

Graphical Reasoning **In Exercises 89–92, (a) use a graphing utility to graph the function, (b) use the *draw inverse* feature of the graphing utility to draw the inverse of the function, and (c) determine whether the graph of the inverse relation is an inverse function, explaining your reasoning.**

89. $f(x) = x^3 + x + 1$

90. $h(x) = x\sqrt{4 - x^2}$

91. $g(x) = \dfrac{3x^2}{x^2 + 1}$

92. $f(x) = \dfrac{4x}{\sqrt{x^2 + 15}}$

In Exercises 93–98, use the functions $f(x) = \frac{1}{8}x - 3$ and $g(x) = x^3$ to find the indicated value or function.

93. $(f^{-1} \circ g^{-1})(1)$

94. $(g^{-1} \circ f^{-1})(-3)$

95. $(f^{-1} \circ f^{-1})(6)$

96. $(g^{-1} \circ g^{-1})(-4)$

97. $(f \circ g)^{-1}$

98. $g^{-1} \circ f^{-1}$

In Exercises 99–102, use the functions $f(x) = x + 4$ and $g(x) = 2x - 5$ to find the specified function.

99. $g^{-1} \circ f^{-1}$

100. $f^{-1} \circ g^{-1}$

101. $(f \circ g)^{-1}$

102. $(g \circ f)^{-1}$

103. ***Shoe Sizes*** The table shows men's shoe sizes in the United States and the corresponding European shoe sizes. Let $y = f(x)$ represent the function that gives the men's European shoe size in terms of x, the men's U.S. size.

Men's U.S. shoe size	Men's European shoe size
8	41
9	42
10	43
11	45
12	46
13	47

(a) Is f one-to-one? Explain.

(b) Find $f(11)$.

(c) Find $f^{-1}(43)$, if possible.

(d) Find $f(f^{-1}(41))$.

(e) Find $f^{-1}(f(13))$.

104. ***Shoe Sizes*** The table shows women's shoe sizes in the United States and the corresponding European shoe sizes. Let $y = g(x)$ epresent the function that gives the women's European shoe size in terms of x, the women's U.S. size.

Women's U.S. shoe size	Women's European shoe size
4	35
5	37
6	38
7	39
8	40
9	42

(a) Is g one-to-one? Explain.

(b) Find $g(6)$.

(c) Find $g^{-1}(42)$.

(d) Find $g(g^{-1}(39))$.

(e) Find $g^{-1}(g(5))$.

105. ***Transportation*** The total values of new car sales f (in billions of dollars) in the United States from 1995 through 2004 are shown in the table. The time (in years) is given by t, with $t = 5$ corresponding to 1995. (Source: National Automobile Dealers Association)

Year, t	Sales, $f(t)$
5	456.2
6	490.0
7	507.5
8	546.3
9	606.5
10	650.3
11	690.4
12	679.5
13	699.2
14	714.3

(a) Does f^{-1} exist?

(b) If f^{-1} exists, what does it mean in the context of the problem?

(c) If f^{-1} exists, find $f^{-1}(650.3)$.

(d) If the table above were extended to 2005 and if the total value of new car sales for that year were \$690.4 billion, would f^{-1} exist? Explain.

106. ***Hourly Wage*** Your wage is \$8.00 per hour plus \$0.75 for each unit produced per hour. So, your hourly wage y in terms of the number of units produced x is $y = 8 + 0.75x$.

(a) Find the inverse function. What does each variable in the inverse function represent?

(b) Use a graphing utility to graph the function and its inverse function.

(c) Use the *trace* feature of a graphing utility to find the hourly wage when 10 units are produced per hour.

(d) Use the *trace* feature of a graphing utility to find the number of units produced when your hourly wage is \$22.25.

Synthesis

True or False? **In Exercises 107 and 108, determine whether the statement is true or false. Justify your answer.**

107. If f is an even function, f^{-1} exists.

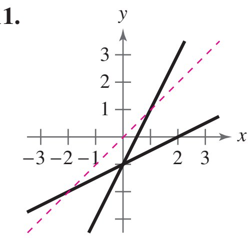
108. If the inverse function of f exists, and the graph of f has a y-intercept, the y-intercept of f is an x-intercept of f^{-1}.

109. ***Proof*** Prove that if f and g are one-to-one functions, $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$.

110. ***Proof*** Prove that if f is a one-to-one odd function, f^{-1} is an odd function.

In Exercises 111–114, decide whether the two functions shown in the graph appear to be inverse functions of each other. Explain your reasoning.

111.



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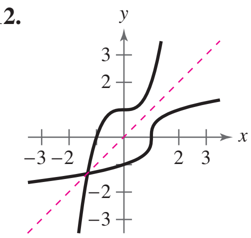

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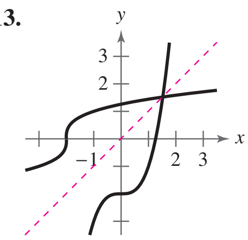

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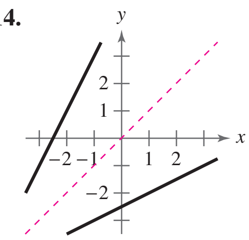

In Exercises 115–118, determine if the situation could be represented by a one-to-one function. If so, write a statement that describes the inverse function.

115. The number of miles n a marathon runner has completed in terms of the time t in hours

116. The population p of South Carolina in terms of the year t from 1960 to 2005

117. The depth of the tide d at a beach in terms of the time t over a 24-hour period

118. The height h in inches of a human born in the year 2000 in terms of his or her age n in years

Skills Review

In Exercises 119–122, write the rational expression in simplest form.

119. $\dfrac{27x^3}{3x^2}$

120. $\dfrac{5x^2y}{xy + 5x}$

121. $\dfrac{x^2 - 36}{6 - x}$

122. $\dfrac{x^2 + 3x - 40}{x^2 - 3x - 10}$

In Exercises 123–128, determine whether the equation represents y as a function of x.

123. $4x - y = 3$

124. $x = 5$

125. $x^2 + y^2 = 9$

126. $x^2 + y = 8$

127. $y = \sqrt{x + 2}$

128. $x - y^2 = 0$