2.4 Solving Quadratic Equations Algebraically

Quadratic Equations

A **quadratic equation in** $\boldsymbol{x}$ is an equation that can be written in the general form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers with $a \neq 0$. A quadratic equation in x is also known as a **second-degree polynomial equation in** $\boldsymbol{x}$**.** You should be familiar with the following four methods for solving quadratic equations.

Solving a Quadratic Equation

Factoring: If $ab = 0$, then $a = 0$ or $b = 0$. Zero-Factor Property

Example:

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x - 3 = 0 \Rightarrow x = 3$$

$$x + 2 = 0 \Rightarrow x = -2$$

Extracting Square Roots: If $u^2 = c$, then $u = \pm\sqrt{c}$.

Example: $(x + 3)^2 = 16$

$$x + 3 = \pm 4$$

$$x = -3 \pm 4$$

$$x = 1 \quad \text{or} \quad x = -7$$

Completing the Square: If $x^2 + bx = c$, then

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$$

$$\left(x + \frac{b}{2}\right)^2 = c + \frac{b^2}{4}.$$

Example:

$$x^2 + 6x = 5$$

$$x^2 + 6x + 3^2 = 5 + 3^2$$

$$(x + 3)^2 = 14$$

$$x + 3 = \pm\sqrt{14}$$

$$x = -3 \pm \sqrt{14}$$

Quadratic Formula: If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example: $2x^2 + 3x - 1 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)} = \frac{-3 \pm \sqrt{17}}{4}$$

What you should learn

- Solve quadratic equations by factoring.
- Solve quadratic equations by extracting square roots.
- Solve quadratic equations by completing the square.
- Use the Quadratic Formula to solve quadratic equations.
- Use quadratic equations to model and solve real-life problems.

Why you should learn it

Knowing how to solve quadratic equations algebraically can help you solve real-life problems, such as Exercise 86 on page 207, where you determine the annual per capita consumption of bottled water.

Myrleen Ferguson Cate/PhotoEdit

Example 1 Solving a Quadratic Equation by Factoring

Solve each quadratic equation by factoring.

a. $6x^2 = 3x$ **b.** $9x^2 - 6x + 1 = 0$

Solution

a.

$6x^2 = 3x$		Write original equation.
$6x^2 - 3x = 0$		Write in general form.
$3x(2x - 1) = 0$		Factor.
$3x = 0$	$x = 0$	Set 1st factor equal to 0.
$2x - 1 = 0$	$x = \frac{1}{2}$	Set 2nd factor equal to 0.

b.

$9x^2 - 6x + 1 = 0$		Write original equation.
$(3x - 1)^2 = 0$		Factor.
$3x - 1 = 0$	$x = \frac{1}{3}$	Set repeated factor equal to 0.

Throughout the text, when solving equations, be sure to check your solutions either *algebraically* by substituting in the original equation or *graphically*.

Check

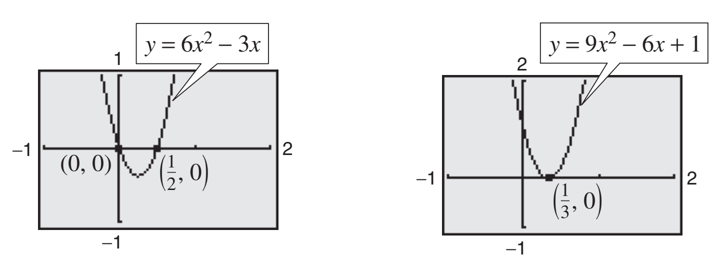
a.

$6x^2 = 3x$	Write original equation.
$6(0)^2 \stackrel{?}{=} 3(0)$	Substitute 0 for x.
$0 = 0$	Solution checks. ✓
$6\left(\frac{1}{2}\right)^2 \stackrel{?}{=} 3\left(\frac{1}{2}\right)$	Substitute $\frac{1}{2}$ for x.
$\frac{6}{4} = \frac{3}{2}$	Solution checks. ✓

b.

$9x^2 - 6x + 1 = 0$	Write original equation.
$9\left(\frac{1}{3}\right)^2 - 6\left(\frac{1}{3}\right) + 1 \stackrel{?}{=} 0$	Substitute $\frac{1}{3}$ for x.
$1 - 2 + 1 \stackrel{?}{=} 0$	Simplify.
$0 = 0$	Solution checks. ✓

Similarly, you can check your solutions graphically using the graphs in Figure 2.26.



(a) (b)

Figure 2.26

✓CHECKPOINT Now try Exercise 7.

Encourage your students to use the Quadratic Formula program at this textbook's *Online Study Center*, as a quick and easy way to check their work when solving quadratic equations.

TECHNOLOGY SUPPORT

Try programming the Quadratic Formula into a computer or graphing calculator. Programs for several graphing calculator models can be found at this textbook's *Online Study Center.*

To use one of the programs, you must first write the equation in general form. Then enter the values of a, b, and c. After the final value has been entered, the program will display either two real solutions *or* the words "NO REAL SOLUTION," *or* the program will give both real and complex solutions.

STUDY TIP

Quadratic equations always have two solutions. From the graph in Figure 2.26(b), it looks like there is only one solution to the equation $9x^2 - 6x + 1 = 0$.

Because the equation is a perfect square trinomial, its two factors are identical. As a result, the equation has two *repeated* solutions.

Solving a quadratic equation by extracting square roots is an efficient method to use when the quadratic equation can be written in the form $ax^2 + c = 0$.

Example 2 Extracting Square Roots

Solve each quadratic equation.

a. $4x^2 = 12$ **b.** $(x - 3)^2 = 7$ **c.** $(2x - 1)^2 = -9$

Solution

a. $4x^2 = 12$ — Write original equation.

$x^2 = 3$ — Divide each side by 4.

$x = \pm\sqrt{3}$ — Take square root of each side.

This equation has two solutions: $x = \pm\sqrt{3} \approx \pm 1.73$.

b. $(x - 3)^2 = 7$ — Write original equation.

$x - 3 = \pm\sqrt{7}$ — Take square root of each side.

$x = 3 \pm \sqrt{7}$ — Add 3 to each side.

This equation has two solutions: $x = 3 + \sqrt{7} \approx 5.65$ and $x = 3 - \sqrt{7} \approx 0.35$.

c. $(2x - 1)^2 = -9$ — Write original equation.

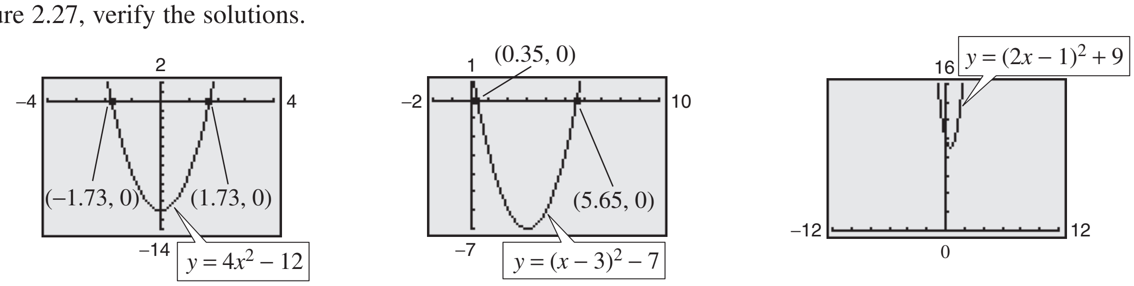
$2x - 1 = \pm\sqrt{-9}$ — Take square root of each side.

$2x - 1 = \pm 3i$ — Write in i-form.

$x = \frac{1}{2} \pm \frac{3}{2}i$ — Solve for x.

This equation has two complex solutions: $x = \frac{1}{2} \pm \frac{3}{2}i$.

The graphs of $y = 4x^2 - 12$, $y = (x - 3)^2 - 7$, and $y = (2x - 1)^2 + 9$, shown in Figure 2.27, verify the solutions.



(a) (b) (c)

Figure 2.27

CHECKPOINT Now try Exercise 19.

STUDY TIP

When you take the square root of a variable expression, you must account for both positive and negative solutions.

TECHNOLOGY TIP

Note that the solutions shown in Example 2 are listed in *exact* form and as decimal approximations. Most graphing utilities produce decimal approximations of solutions rather than exact forms. For instance, if you solve the equations in Example 2 using a graphing utility, you will obtain $x \approx \pm 1.73$ in part (a) and $x \approx 5.65$ and $x \approx 0.35$ in part (b). Some graphing utilities have symbolic algebra programs that *can* list the exact form of a solution.

Completing the square is best suited for quadratic equations in general form $ax^2 + bx + c = 0$ with $a = 1$ and b an even number (see page 195). If the leading coefficient of the quadratic is not 1, divide each side of the equation by this coefficient before completing the square, as shown in Example 4.

Example 3 Completing the Square: Leading Coefficient Is 1

Solve $x^2 + 2x - 6 = 0$ by completing the square.

Solution

$$x^2 + 2x - 6 = 0 \qquad \text{Write original equation.}$$

$$x^2 + 2x = 6 \qquad \text{Add 6 to each side.}$$

$$x^2 + 2x + \underbrace{1^2}_{(\text{Half of } 2)^2} = 6 + 1^2 \qquad \text{Add } 1^2 \text{ to each side.}$$

$$(x + 1)^2 = 7 \qquad \text{Simplify.}$$

$$x + 1 = \pm\sqrt{7} \qquad \text{Take square root of each side.}$$

$$x = -1 \pm \sqrt{7} \qquad \text{Solutions}$$

Using a calculator, the two solutions are $x \approx 1.65$ and $x \approx -3.65$, which agree with the graphical solutions shown in Figure 2.28.

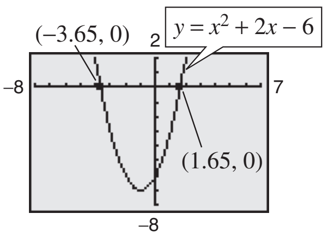


Figure 2.28

CHECKPOINT Now try Exercise 23.

Example 4 Completing the Square: Leading Coefficient Is Not 1

Solve $2x^2 + 8x + 3 = 0$ by completing the square.

Solution

$$2x^2 + 8x + 3 = 0 \qquad \text{Write original equation.}$$

$$2x^2 + 8x = -3 \qquad \text{Subtract 3 from each side.}$$

$$x^2 + 4x = -\frac{3}{2} \qquad \text{Divide each side by 2.}$$

$$x^2 + 4x + \underbrace{2^2}_{(\text{Half of } 4)^2} = -\frac{3}{2} + 2^2 \qquad \text{Add } 2^2 \text{ to each side.}$$

$$(x + 2)^2 = \frac{5}{2} \qquad \text{Simplify.}$$

$$x + 2 = \pm\sqrt{\frac{5}{2}} \qquad \text{Take square root of each side.}$$

$$x + 2 = \pm\frac{\sqrt{10}}{2} \qquad \text{Rationalize denominator.}$$

$$x = -2 \pm \frac{\sqrt{10}}{2} \qquad \text{Solutions}$$

Using a calculator, the two solutions are $x \approx -0.42$ and $x \approx -3.58$, which agree with the graphical solutions shown in Figure 2.29.

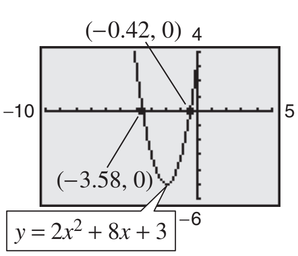


Figure 2.29

CHECKPOINT Now try Exercise 27.

Example 5 Completing the Square: Leading Coefficient Is Not 1

Solve $3x^2 - 4x - 5 = 0$ by completing the square.

Solution

$$3x^2 - 4x - 5 = 0 \qquad \text{Write original equation.}$$

$$3x^2 - 4x = 5 \qquad \text{Add 5 to each side.}$$

$$x^2 - \frac{4}{3}x = \frac{5}{3} \qquad \text{Divide each side by 3.}$$

$$x^2 - \frac{4}{3}x + \left(-\frac{2}{3}\right)^2 = \frac{5}{3} + \left(-\frac{2}{3}\right)^2 \qquad \text{Add } \left(-\tfrac{2}{3}\right)^2 \text{ to each side.}$$

$\left(\text{Half of } -\frac{4}{3}\right)^2$

$$\left(x - \frac{2}{3}\right)^2 = \frac{19}{9} \qquad \text{Simplify.}$$

$$x - \frac{2}{3} = \pm\frac{\sqrt{19}}{3} \qquad \text{Take square root of each side.}$$

$$x = \frac{2}{3} \pm \frac{\sqrt{19}}{3} \qquad \text{Solutions}$$

Using a calculator, the two solutions are $x \approx 2.12$ and $x \approx -0.79$, which agree with the graphical solutions shown in Figure 2.30.

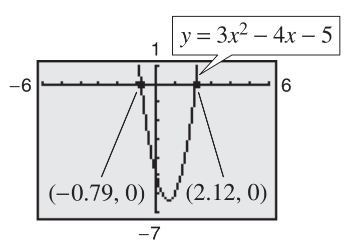


Figure 2.30

CHECKPOINT Now try Exercise 31.

Often in mathematics you are taught the long way of solving a problem first. Then, the longer method is used to develop shorter techniques. The long way stresses understanding and the short way stresses efficiency.

For instance, you can think of completing the square as a "long way" of solving a quadratic equation. When you use the method of completing the square to solve a quadratic equation, you must complete the square for *each* equation separately. In the derivation on the following page, you complete the square *once* in a general setting to obtain the Quadratic Formula, which is a shortcut for solving a quadratic equation.

$$ax^2 + bx + c = 0 \qquad \text{Quadratic equation in general form, } a \neq 0$$

$$ax^2 + bx = -c \qquad \text{Subtract } c \text{ from each side.}$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \qquad \text{Divide each side by } a.$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 \qquad \text{Complete the square.}$$

$$\left(\text{Half of } \frac{b}{a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \qquad \text{Simplify.}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \qquad \text{Extract square roots.}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2|a|} \qquad \text{Solutions}$$

Note that because $\pm 2|a|$ represents the same numbers as $\pm 2a$, you can omit the absolute value sign. So, the formula simplifies to

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Exploration

Use a graphing utility to graph the three quadratic equations

$$y_1 = x^2 - 2x$$

$$y_2 = x^2 - 2x + 1$$

$$y_3 = x^2 - 2x + 2$$

in the same viewing window. Compute the *discriminant* $\sqrt{b^2 - 4ac}$ for each equation and discuss the relationship between the discriminant and the number of zeros of the quadratic function.

Example 6 Quadratic Formula: Two Distinct Solutions

Solve $x^2 + 3x = 9$ using the Quadratic Formula.

Algebraic Solution

$$x^2 + 3x = 9 \qquad \text{Write original equation.}$$

$$x^2 + 3x - 9 = 0 \qquad \text{Write in general form.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic Formula}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-9)}}{2(1)} \qquad \text{Substitute 3 for } b\text{, 1 for } a\text{, and } -9 \text{ for } c.$$

$$x = \frac{-3 \pm \sqrt{45}}{2} \qquad \text{Simplify.}$$

$$x = \frac{-3 \pm 3\sqrt{5}}{2} \qquad \text{Simplify radical.}$$

$$x \approx 1.85 \text{ or } -4.85 \qquad \text{Solutions}$$

The equation has two solutions: $x \approx 1.85$ and $x \approx -4.85$. Check these solutions in the original equation.

CHECKPOINT Now try Exercise 45.

Graphical Solution

Use a graphing utility to graph $y_1 = x^2 + 3x$ and $y_2 = 9$ in the same viewing window. Use the *intersect* feature of the graphing utility to approximate the points where the graphs intersect. In Figure 2.31, it appears that the graphs intersect at $x \approx 1.85$ and $x \approx -4.85$. These x-coordinates of the intersection points are the solutions of the equation $x^2 + 3x = 9$.

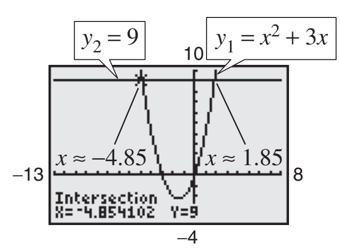


Figure 2.31

Example 7 Quadratic Formula: One Repeated Solution

Solve $8x^2 - 24x + 18 = 0$.

Algebraic Solution

This equation has a common factor of 2. You can simplify the equation by dividing each side of the equation by 2.

$$8x^2 - 24x + 18 = 0 \quad \text{Write original equation.}$$

$$4x^2 - 12x + 9 = 0 \quad \text{Divide each side by 2.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)}$$

$$x = \frac{12 \pm \sqrt{0}}{8} = \frac{3}{2} \quad \text{Repeated solution}$$

This quadratic equation has only one solution: $x = \frac{3}{2}$. Check this solution in the original equation.

CHECKPOINT Now try Exercise 48.

Graphical Solution

Use a graphing utility to graph

$$y = 8x^2 - 24x + 18.$$

Use the *zero* feature of the graphing utility to approximate the value(s) of x for which the function is equal to zero. In the graph in Figure 2.32, it appears that the function is equal to zero when $x = 1.5 = \frac{3}{2}$. This is the only solution of the equation $8x^2 - 24x + 18 = 0$.

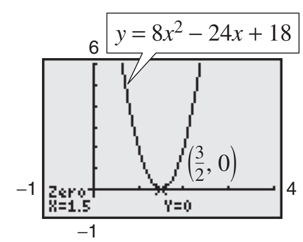


Figure 2.32

Example 8 Complex Solutions of a Quadratic Equation

Solve $3x^2 - 2x + 5 = 0$.

Algebraic Solution

By the Quadratic Formula, you can write the solutions as follows.

$$3x^2 - 2x + 5 = 0 \quad \text{Write original equation.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(5)}}{2(3)} \quad \text{Substitute } -2 \text{ for } b, 3 \text{ for } a, \text{ and } 5 \text{ for } c.$$

$$= \frac{2 \pm \sqrt{-56}}{6} \quad \text{Simplify.}$$

$$= \frac{2 \pm 2\sqrt{14}\,i}{6} \quad \text{Simplify radical.}$$

$$= \frac{1}{3} \pm \frac{\sqrt{14}}{3} i \quad \text{Solutions}$$

The equation has no real solution, but it has two complex solutions: $x = \frac{1}{3}(1 + \sqrt{14}\,i)$ and $x = \frac{1}{3}(1 - \sqrt{14}\,i)$.

CHECKPOINT Now try Exercise 51.

Graphical Solution

Use a graphing utility to graph

$$y = 3x^2 - 2x + 5.$$

Note in Figure 2.33 that the graph of the function appears to have no x-intercept. From this you can conclude that the equation $3x^2 - 2x + 5 = 0$ has no real solution. You can solve the equation algebraically to find the complex solutions.

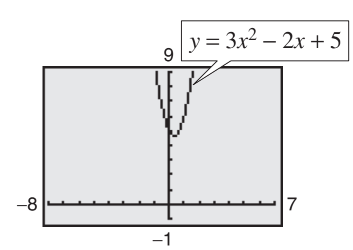


Figure 2.33

Applications

A common application of quadratic equations involves an object that is falling (or projected into the air). The general equation that gives the height of such an object is called a **position equation,** and on *Earth's* surface it has the form

$$s = -16t^2 + v_0t + s_0.$$

In this equation, s represents the height of the object (in feet), v_0 represents the initial velocity of the object (in feet per second), s_0 represents the initial height of the object (in feet), and t represents the time (in seconds). Note that this position equation ignores air resistance.

STUDY TIP

In the position equation

$$s = -16t^2 + v_0t + s_0$$

the initial velocity v_0 is positive when the object is rising and negative when the object is falling.

Example 9 Falling Time

A construction worker on the 24th floor of a building project (see Figure 2.34) accidentally drops a wrench and yells, "Look out below!" Could a person at ground level hear this warning in time to get out of the way?

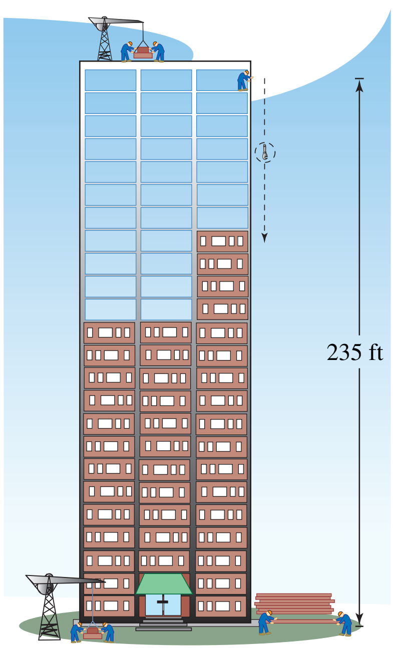


Figure 2.34

Solution

Assume that each floor of the building is 10 feet high, so that the wrench is dropped from a height of 235 feet (the construction worker's hand is 5 feet below the ceiling of the 24th floor). Because sound travels at about 1100 feet per second, it follows that a person at ground level hears the warning within 1 second of the time the wrench is dropped. To set up a mathematical model for the height of the wrench, use the position equation

$$s = -16t^2 + v_0t + s_0. \qquad \text{Position equation}$$

Because the object is dropped rather than thrown, the initial velocity is $v_0 = 0$ feet per second. So, with an initial height of $s_0 = 235$ feet, you have the model

$$s = -16t^2 + (0)t + 235 = -16t^2 + 235.$$

After falling for 1 second, the height of the wrench is $-16(1)^2 + 235 = 219$ feet. After falling for 2 seconds, the height of the wrench is $-16(2)^2 + 235 = 171$ feet. To find the number of seconds it takes the wrench to hit the ground, let the height s be zero and solve the equation for t.

$$s = -16t^2 + 235 \qquad \text{Write position equation.}$$

$$0 = -16t^2 + 235 \qquad \text{Substitute 0 for } s.$$

$$16t^2 = 235 \qquad \text{Add } 16t^2 \text{ to each side.}$$

$$t^2 = \frac{235}{16} \qquad \text{Divide each side by 16.}$$

$$t = \frac{\sqrt{235}}{4} \approx 3.83 \qquad \text{Extract positive square root.}$$

The wrench will take about 3.83 seconds to hit the ground. If the person hears the warning 1 second after the wrench is dropped, the person still has almost 3 more seconds to get out of the way.

CHECKPOINT Now try Exercise 81.

Example 10 Quadratic Modeling: Internet Use

From 1996 to 2003, the numbers of hours h spent annually per person using the Internet in the United States closely followed the quadratic model

$$h = -0.81t^2 + 39.5t - 200, \quad 6 \le t \le 13$$

where t represents the year, with $t = 6$ corresponding to 1996. The numbers of hours per year are shown graphically in Figure 2.35. According to this model, in which year did the number of hours spent per person reach or surpass 150? (Source: Veronis Suhler Stevenson)

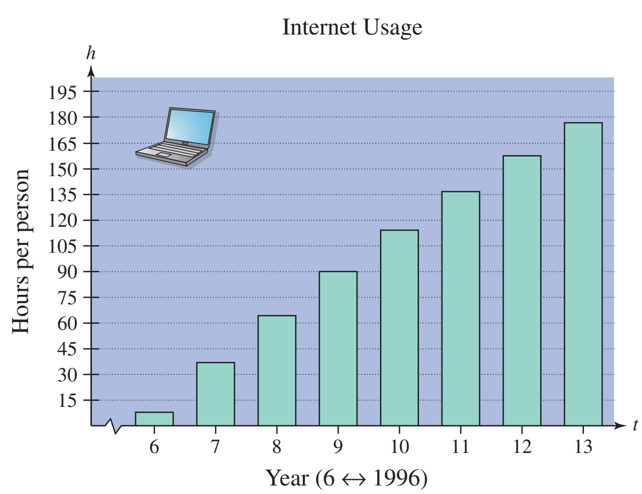


Figure 2.35

Solution

To find when the number of hours spent per person reached 150, you need to solve the equation

$$-0.81t^2 + 39.5t - 200 = 150.$$

To begin, write the equation in general form.

$$-0.81t^2 + 39.5t - 350 = 0$$

Then apply the Quadratic Formula.

$$t = \frac{-39.5 \pm \sqrt{(39.5)^2 - 4(-0.81)(-350)}}{2(-0.81)}$$

$$\approx 11.64 \text{ or } 37.13$$

Choose the smaller value $t = 11.64$. Because $t = 6$ corresponds to 1996, it follows that $t = 11.64$ must correspond to some time in 2001. So, the number of hours spent annually per person using the Internet reached 150 during 2001.

CHECKPOINT Now try Exercise 85.

TECHNOLOGY TIP You can solve Example 10 with your graphing utility by graphing the two functions $y_1 = -0.81x^2 + 39.5x - 200$ and $y_2 = 150$ in the same viewing window and finding their point of intersection. You should obtain $x \approx 11.64$, which verifies the answer obtained algebraically.

Another type of application that often involves a quadratic equation is one dealing with the hypotenuse of a right triangle. These types of applications often use the Pythagorean Theorem, which states that

$$a^2 + b^2 = c^2 \qquad \text{Pythagorean Theorem}$$

where a and b are the legs of a right triangle and c is the hypotenuse, as indicated in Figure 2.36.

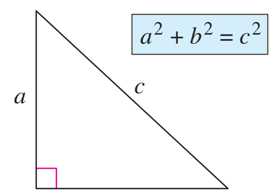


Figure 2.36

Example 11 An Application Involving the Pythagorean Theorem

An L-shaped sidewalk from the athletic center to the library on a college campus is shown in Figure 2.37. The sidewalk was constructed so that the length of one sidewalk forming the L is twice as long as the other. The length of the diagonal sidewalk that cuts across the grounds between the two buildings is 102 feet. How many feet does a person save by walking on the diagonal sidewalk?

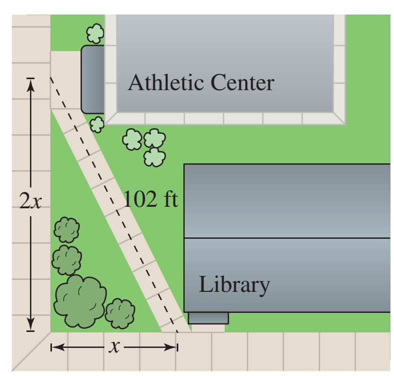


Figure 2.37

Solution

Using the Pythagorean Theorem, you have

$$\begin{aligned}
a^2 + b^2 &= c^2 && \text{Pythagorean Theorem}\\
x^2 + (2x)^2 &= 102^2 && \text{Substitute for } a, b, \text{ and } c.\\
5x^2 &= 10{,}404 && \text{Combine like terms.}\\
x^2 &= 2080.8 && \text{Divide each side by 5.}\\
x &= \pm\sqrt{2080.8} && \text{Take the square root of each side.}\\
x &= \sqrt{2080.8} && \text{Extract positive square root.}
\end{aligned}$$

The total distance covered by walking on the L-shaped sidewalk is

$$\begin{aligned}
x + 2x &= 3x\\
&= 3\sqrt{2080.8}\\
&\approx 136.85 \text{ feet.}
\end{aligned}$$

Walking on the diagonal sidewalk saves a person about $136.85 - 102 = 34.85$ feet.

CHECKPOINT Now try Exercise 89.

2.4 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blanks.

1. An equation of the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$, is a _______ , or a second-degree polynomial equation in x.
2. The four methods that can be used to solve a quadratic equation are _______ , _______ , _______ , and the _______ .
3. The part of the Quadratic Formula $\sqrt{b^2 - 4ac}$, known as the _______ , determines the type of solutions of a quadratic equation.
4. The general equation that gives the height of an object (in feet) in terms of the time t (in seconds) is called the _______ equation, and has the form $s =$ _______ , where v_0 represents the ________ and s_0 represents the _______ .

In Exercises 1–4, write the quadratic equation in general form. Do not solve the equation.

1. $2x^2 = 3 - 5x$
2. $x^2 = 25x + 26$
3. $\frac{1}{5}(3x^2 - 10) = 12x$
4. $x(x + 2) = 3x^2 + 1$

In Exercises 5–14, solve the quadratic equation by factoring. Check your solutions in the original equation.

5. $6x^2 + 3x = 0$
6. $9x^2 - 1 = 0$
7. $x^2 - 2x - 8 = 0$
8. $x^2 - 10x + 9 = 0$
9. $3 + 5x - 2x^2 = 0$
10. $2x^2 = 19x + 33$
11. $x^2 + 4x = 12$
12. $-x^2 + 8x = 12$
13. $(x + a)^2 - b^2 = 0$
14. $x^2 + 2ax + a^2 = 0$

In Exercises 15–22, solve the equation by extracting square roots. List both the exact solutions and the decimal solutions rounded to the nearest hundredth.

15. $x^2 = 49$
16. $x^2 = 144$
17. $(x - 12)^2 = 16$
18. $(x - 5)^2 = 25$
19. $(3x - 1)^2 + 6 = 0$
20. $(2x + 3)^2 + 25 = 0$
21. $(x - 7)^2 = (x + 3)^2$
22. $(x + 5)^2 = (x + 4)^2$

In Exercises 23–32, solve the quadratic equation by completing the square. Verify your answer graphically.

23. $x^2 + 4x - 32 = 0$
24. $x^2 - 2x - 3 = 0$
25. $x^2 + 6x + 2 = 0$
26. $x^2 + 8x + 14 = 0$
27. $9x^2 - 18x + 3 = 0$
28. $4x^2 - 4x - 99 = 0$
29. $-6 + 2x - x^2 = 0$
30. $-x^2 + x - 1 = 0$
31. $2x^2 + 5x - 8 = 0$
32. $9x^2 - 12x - 14 = 0$

Graphical Reasoning **In Exercises 33–38, (a) use a graphing utility to graph the equation, (b) use the graph to approximate any x-intercepts of the graph, and (c) verify your results algebraically.**

33. $y = (x + 3)^2 - 4$
34. $y = 1 - (x - 2)^2$
35. $y = -4x^2 + 4x + 3$
36. $y = x^2 + 3x - 4$
37. $y = \frac{1}{4}(4x^2 - 20x + 25)$
38. $y = -\frac{1}{4}(x^2 - 2x + 9)$

In Exercises 39–44, use a graphing utility to determine the number of real solutions of the quadratic equation.

39. $2x^2 - 5x + 5 = 0$
40. $2x^2 - x - 1 = 0$
41. $\frac{4}{7}x^2 - 8x + 28 = 0$
42. $\frac{1}{3}x^2 - 5x + 25 = 0$
43. $-0.2x^2 + 1.2x - 8 = 0$
44. $9 + 2.4x - 8.3x^2 = 0$

In Exercises 45–52, use the Quadratic Formula to solve the equation. Use a graphing utility to verify your solutions graphically.

45. $2 + 2x - x^2 = 0$
46. $x^2 - 10x + 22 = 0$
47. $2x^2 = 3x - 4$
48. $28x - 49x^2 = 4$
49. $x^2 + 3x = -8$
50. $x^2 + 16 = -5x$
51. $4x^2 + 16x + 17 = 0$
52. $9x^2 - 6x + 37 = 0$

In Exercises 53–60, solve the equation using any convenient method.

53. $x^2 - 2x - 1 = 0$
54. $11x^2 + 33x = 0$
55. $(x + 3)^2 = 81$
56. $(x - 1)^2 = -1$
57. $x^2 - 2x + \frac{13}{4} = 0$
58. $x^2 + 3x - \frac{3}{4} = 0$
59. $(x + 1)^2 = x^2$
60. $a^2x^2 - b^2 = 0,\ a \neq 0$

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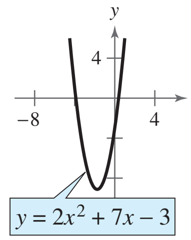
In Exercises 61–66, find algebraically the x-intercept(s), if any, of the graph of the equation.

61.



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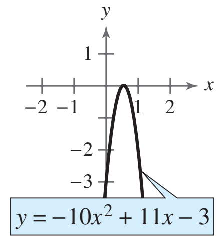

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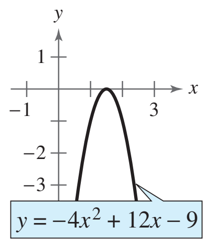

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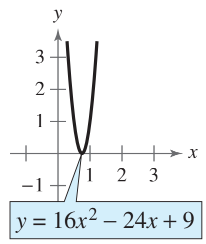

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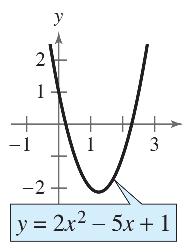

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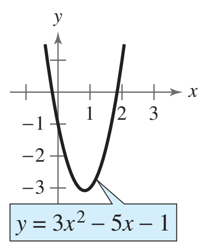


Think About It **In Exercises 67–76, find two quadratic equations having the given solutions. (There are many correct answers.)**

67. $-6, 5$

68. $-2, 1$

69. $-\frac{7}{3}, \frac{6}{7}$

70. $-\frac{2}{3}, \frac{4}{3}$

71. $5\sqrt{3}, -5\sqrt{3}$

72. $2\sqrt{5}, -2\sqrt{5}$

73. $1 + 2\sqrt{3}, 1 - 2\sqrt{3}$

74. $2 + 3\sqrt{5}, 2 - 3\sqrt{5}$

75. $2 + i, 2 - i$

76. $3 + 4i, 3 - 4i$

77. ***Geometry*** The floor of a one-story building is 14 feet longer than it is wide. The building has 1632 square feet of floor space.

(a) Draw a diagram that gives a visual representation of the floor space. Represent the width as w and show the length in terms of w.

(b) Write a quadratic equation in terms of w.

(c) Find the length and width of the building floor.

78. ***Geometry*** An above-ground swimming pool with a square base is to be constructed such that the surface area of the pool is 576 square feet. The height of the pool is to be 4 feet. (See figure.) What should the dimensions of the base be? (*Hint:* The surface area is $S = x^2 + 4xh$.)

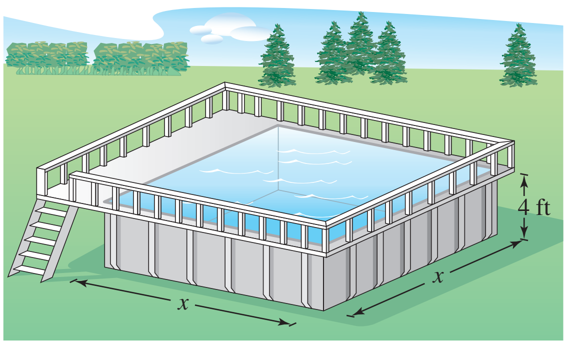
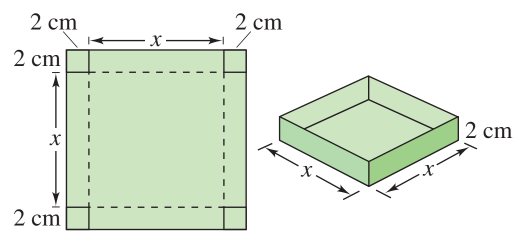
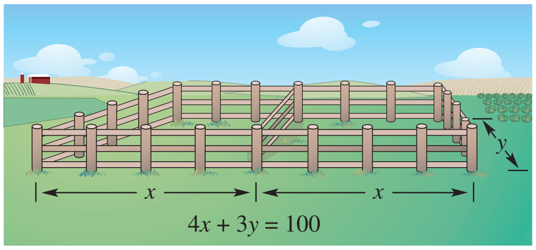


Figure for 78

79. ***Packaging*** An open gift box is to be made from a square piece of material by cutting two-centimeter squares from each corner and turning up the sides (see figure). The volume of the finished gift box is to be 200 cubic centimeters. Find the size of the original piece of material.



80. ***Exploration*** A rancher has 100 meters of fencing to enclose two adjacent rectangular corrals, as shown in the figure.



(a) Write the area A of the enclosed region as a function of x.

(b) Use a graphing utility to generate additional rows of the table. Use the table to estimate the dimensions that will produce a maximum area.

x	y	Area
2	$\frac{92}{3}$	$\frac{368}{3} \approx 123$
4	28	224

(c) Use a graphing utility to graph the area function, and use the graph to estimate the dimensions that will produce a maximum area.

(d) Use the graph to approximate the dimensions such that the enclosed area is 350 square meters.

(e) Find the required dimensions of part (d) algebraically.

In Exercises 81–84, use the position equation given on page 202 as the model for the problem.

81. ***CN Tower*** At 1815 feet tall, the CN Tower in Toronto, Ontario is the world's tallest self-supporting structure. An object is dropped from the top of the tower.

(a) Find the position equation $s = -16t^2 + v_0t + s_0$.

(b) Complete the table.

t	0	2	4	6	8	10	12
s							

(c) From the table in part (b), determine the time interval during which the object reaches the ground. Find the time algebraically.

82. ***Sports*** You throw a baseball straight up into the air at a velocity of 45 feet per second. You release the baseball at a height of 5.5 feet and catch it when it falls back to a height of 6 feet.

(a) Use the position equation to write a mathematical model for the height of the baseball.

(b) Find the height of the baseball after 0.5 second.

(c) How many seconds is the baseball in the air? (Use a graphing utility to verify your answer.)

83. ***Military*** A cargo plane flying at 8000 feet over level terrain drops a 500-pound supply package.

(a) How long will it take the package to strike the ground?

(b) The plane is flying at 600 miles per hour. How far will the package travel horizontally during its descent?

84. ***Rabies Vaccination*** The game comission plane, flying at 1100 feet, drops oral rabies vaccination pellets into a game preserve.

(a) Find the position equation to the model height of the vaccination pellets.

(b) The plane is flying at 95 miles per hour. How far will the pellets travel horizontally during its decent?

85. ***Medical Costs*** The average retail prescription prices P (in dollars) from 1997 through 2004 can be approximated by the model $P = 0.1220t^2 + 1.529t + 18.72$, $7 \le t \le 14$, where t represents the year, with $t = 7$ corresponding to 1997. (Source: National Association of Chain Drug Stores)

(a) Determine algebraically when the average retail price was \$40 and \$50.

(b) Verify your answer to part (a) by creating a table of values for the model.

(c) Use a graphing utility to graph the model.

(d) According to the model, when will the average retail price reach \$75?

(e) Do you believe the model could be used to predict the average retail price for years beyond 2004? Explain your reasoning.

86. ***Water Consumption*** The annual per capita consumptions W of bottled water (in gallons) in the United States from 1995 through 2003 can be approximated by the model $W = 0.045t^2 + 0.50t + 8.1$, $5 \le t \le 13$, where t represents the year, with $t = 5$ corresponding to 1995. (Source: U.S. Department of Agriculture)

(a) Determine algebraically when the per capita consumption was 15 gallons per year and 20 gallons per year.

(b) Verify your answer to part (a) by creating a table of values for the model.

(c) Use a graphing utility to graph the model.

(d) According to the model, when did the per capita consumption reach 25 gallons per year?

(e) Do you believe the model could be used to predict the per capita consumption for years beyond 2003? Explain your reasoning.

87. ***Biology*** The metabolic rate of an ectothermic organism increases with increasing temperature within a certain range. Experimental data for oxygen consumption C (in microliters per gram per hour) of a beetle at certain temperatures yielded the model $C = 0.45x^2 - 1.65x + 50.75$, $10 \le x \le 25$, where x is the air temperature (in degrees Celsius).

(a) Use a graphing utility to graph the consumption model over the specified domain.

(b) Use the graph to approximate the air temperature resulting in oxygen consumption of 150 microliters per gram per hour.

(c) If the temperature is increased from 10°C to 20°C, the oxygen consumption will be increased by approximately what factor?

88. ***Fuel Efficiency*** The distance d (in miles) a car can travel on one tank of fuel is approximated by $d = -0.024s^2 + 1.455s + 431.5$, $0 < s \le 75$, where s is the average speed of the car (in miles per hour).

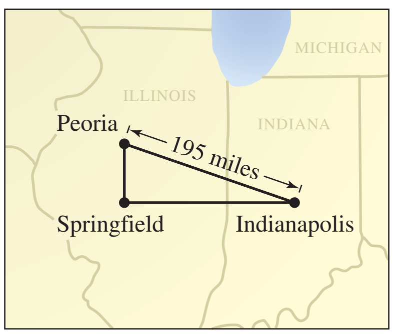
(a) Use a graphing utility to graph the function over the specified domain.

(b) Use the graph to determine the greatest distance that can be traveled on a tank of fuel. How long will the trip take?

(c) Determine the greatest distance that can be traveled in this car in 8 hours with no refueling. How fast should the car be driven? [*Hint:* The distance traveled in 8 hours is $8s$. Graph this expression in the same viewing window as the graph in part (a) and approximate the point of intersection.]

89. ***Flying Speed*** Two planes leave simultaneously from Chicago's O'Hare Airport, one flying due north and the other due east. The northbound plane is flying 50 miles per hour faster than the eastbound plane. After 3 hours the planes are 2440 miles apart. Find the speed of each plane. (*Hint:* Draw a diagram.)

90. ***Flying Distance*** A chartered airplane flies to three cities whose locations form the vertices of a right triangle (see figure). The total flight distance (from Indianapolis to Peoria to Springfield and back to Indianapolis) is approximately 448 miles. It is 195 miles between Indianapolis and Peoria. Approximate the other two distances.



Synthesis

True or False? **In Exercises 91–94, determine whether the statement is true or false. Justify your answer.**

91. The quadratic equation $-3x^2 - x = 10$ has two real solutions.

92. If $(2x - 3)(x + 5) = 8$, then $2x - 3 = 8$ or $x + 5 = 8$.

93. A quadratic equation with real coefficients can have one real solution and one imaginary solution.

94. A quadratic equation with real coefficients can have one repeated imaginary solution.

95. ***Exploration*** Solve $3(x + 4)^2 + (x + 4) - 2 = 0$ in two ways.

(a) Let $u = x + 4$, and solve the resulting equation for u. Then find the corresponding values of x that are the solutions of the original equation.

(b) Expand and collect like terms in the original equation, and solve the resulting equation for x.

(c) Which method is easier? Explain.

96. ***Exploration*** Given that a and b are nonzero real numbers, determine the solutions of the equations.

(a) $ax^2 + bx = 0$ (b) $ax^2 - ax = 0$

97. ***Proof*** Given that the solutions of a quadratic equation are $x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/(2a)$, show that the sum of the solutions is $S = -b/a$.

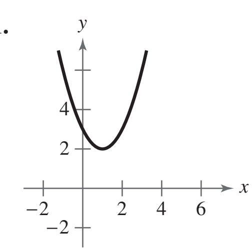
98. ***Proof*** Given that the solutions of a quadratic equation are $x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/(2a)$, show that the product of the solutions is $P = c/a$.

99. ***Writing*** On a graphing utility, store the value 5 in A, -2 in B, and 1 in C. Use the graphing utility to graph $y = C(x - A)(x - B)$. Explain how the values of A and B can be determined from the graph. Now store any other nonzero value in C. Does the value of C affect the x-intercepts of the graph? Explain. Find values of A, B, and C such that the graph opens downward and has x-intercepts at $(-5, 0)$ and $(0, 0)$. Summarize your findings.

100. ***Exploration*** Is it possible for a quadratic equation to have only one x-intercept? Explain.

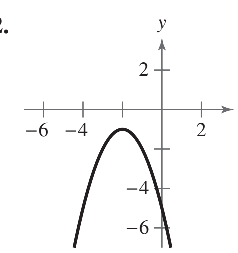
Library of Parent Functions In Exercises 101 and 102, determine which function the graph represents.

101.



(a) $y = (x + 1)^2 + 2$
(b) $y = (x - 2)^2 + 1$
(c) $y = (x + 2)^2 + 1$
(d) $y = x^2 - x + 2$
(e) $y = (x - 1)^2 + 2$

102.



(a) $y = -(x - 2)^2 - 1$
(b) $y = -(x + 2)^2 + 1$
(c) $y = -x^2 - 2x + 1$
(d) $y = -(x + 2)^2 - 1$
(e) $y = -(x - 1)^2 + 2$

Skills Review

In Exercises 103–106, completely factor the expression over the real numbers.

103. $x^5 - 27x^2$

104. $x^3 - 5x^2 - 14x$

105. $x^3 + 5x^2 - 2x - 10$

106. $5(x + 5)x^{1/3} + 4x^{4/3}$

In Exercises 107–112, determine whether y is a function of x.

107. $5x + 8y = -1$

108. $-x^2 + y^2 = 2$

109. $x + y^2 = 10$

110. $-2y = \sqrt{x + 6}$

111. $y = |x - 3|$

112. $|y| = 1 - x$

113. ***Make a Decision*** To work an extended application analyzing the population of the United States, visit this textbooks's *Online Study Center.* (Data Source: U.S. Census Bureau)