

## 3.1 Quadratic Functions

### The Graph of a Quadratic Function

In this and the next section, you will study the graphs of polynomial functions.

#### Definition of Polynomial Function

Let  $n$  be a nonnegative integer and let  $a_n, a_{n-1}, \dots, a_2, a_1, a_0$  be real numbers with  $a_n \neq 0$ . The function given by

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

is called a **polynomial function in  $x$  of degree  $n$** .

Polynomial functions are classified by degree. For instance, the polynomial function

$$f(x) = a \quad \text{Constant function}$$

has degree 0 and is called a **constant function**. In Chapter 1, you learned that the graph of this type of function is a horizontal line. The polynomial function

$$f(x) = mx + b, \quad m \neq 0 \quad \text{Linear function}$$

has degree 1 and is called a **linear function**. You also learned in Chapter 1 that the graph of the linear function  $f(x) = mx + b$  is a line whose slope is  $m$  and whose  $y$ -intercept is  $(0, b)$ . In this section, you will study second-degree polynomial functions, which are called **quadratic functions**.

#### Definition of Quadratic Function

Let  $a, b$ , and  $c$  be real numbers with  $a \neq 0$ . The function given by

$$f(x) = ax^2 + bx + c \quad \text{Quadratic function}$$

is called a **quadratic function**.

Often real-life data can be modeled by quadratic functions. For instance, the table at the right shows the height  $h$  (in feet) of a projectile fired from a height of 6 feet with an initial velocity of 256 feet per second at any time  $t$  (in seconds). A quadratic model for the data in the table is  $h(t) = -16t^2 + 256t + 6$  for  $0 \leq t \leq 16$ .

The graph of a quadratic function is a special type of U-shaped curve called a **parabola**. Parabolas occur in many real-life applications, especially those involving reflective properties, such as satellite dishes or flashlight reflectors. You will study these properties in a later chapter.

All parabolas are symmetric with respect to a line called the **axis of symmetry**, or simply the **axis** of the parabola. The point where the axis intersects the parabola is called the **vertex** of the parabola.

#### What you should learn


- Analyze graphs of quadratic functions.
- Write quadratic functions in standard form and use the results to sketch graphs of functions.
- Find minimum and maximum values of quadratic functions in real-life applications.

#### Why you should learn it

Quadratic functions can be used to model the design of a room. For instance, Exercise 53 on page 260 shows how the size of an indoor fitness room with a running track can be modeled.



Dwight Cendrowski

 $t$	$h$
0	6
2	454
4	774
6	966
8	1030
10	966
12	774
14	454
16	6

### Library of Parent Functions: Quadratic Function

The simplest type of *quadratic function* is  $f(x) = ax^2$ , also known as the *squaring function* when  $a = 1$ . The basic characteristics of a quadratic function are summarized below. A review of quadratic functions can be found in the *Study Capsules*.

Graph of  $f(x) = ax^2$ ,  $a > 0$

Domain:  $(-\infty, \infty)$

Range:  $[0, \infty)$

Intercept:  $(0, 0)$

Decreasing on  $(-\infty, 0)$

Increasing on  $(0, \infty)$

Even function

Axis of symmetry:  $x = 0$

Relative minimum or vertex:  $(0, 0)$

Graph of  $f(x) = ax^2$ ,  $a < 0$

Domain:  $(-\infty, \infty)$

Range:  $(-\infty, 0]$

Intercept:  $(0, 0)$

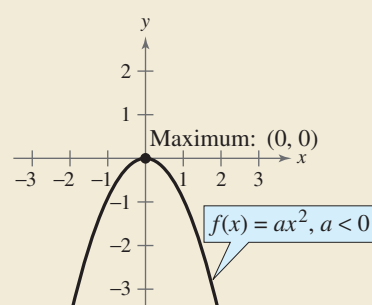
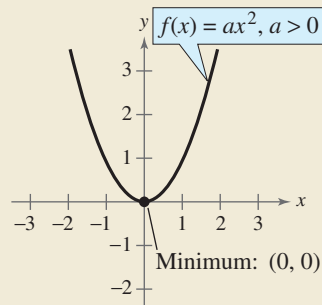
Increasing on  $(-\infty, 0)$

Decreasing on  $(0, \infty)$

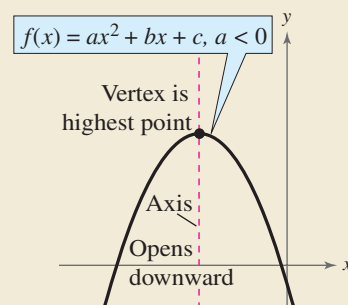
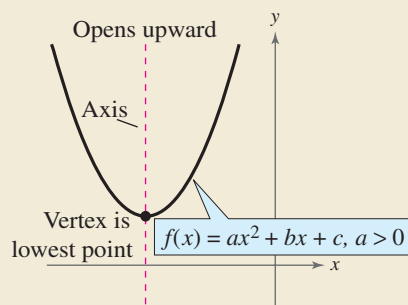
Even function

Axis of symmetry:  $x = 0$

Relative maximum or vertex:  $(0, 0)$



For the general quadratic form  $f(x) = ax^2 + bx + c$ , if the leading coefficient  $a$  is positive, the parabola opens upward; and if the leading coefficient  $a$  is negative, the parabola opens downward. Later in this section you will learn ways to find the coordinates of the vertex of a parabola.



When sketching the graph of  $f(x) = ax^2$ , it is helpful to use the graph of  $y = x^2$  as a reference, as discussed in Section 1.5. There you saw that when  $a > 1$ , the graph of  $y = af(x)$  is a vertical stretch of the graph of  $y = f(x)$ . When  $0 < a < 1$ , the graph of  $y = af(x)$  is a vertical shrink of the graph of  $y = f(x)$ . This is demonstrated again in Example 1.

**Example 1** Graphing Simple Quadratic Functions

Describe how the graph of each function is related to the graph of  $y = x^2$ .

- a.  $f(x) = \frac{1}{3}x^2$       b.  $g(x) = 2x^2$   
 c.  $h(x) = -x^2 + 1$       d.  $k(x) = (x + 2)^2 - 3$

**Solution**

- a. Compared with  $y = x^2$ , each output of  $f$  “shrinks” by a factor of  $\frac{1}{3}$ . The result is a parabola that opens upward and is broader than the parabola represented by  $y = x^2$ , as shown in Figure 3.1.
- b. Compared with  $y = x^2$ , each output of  $g$  “stretches” by a factor of 2, creating a narrower parabola, as shown in Figure 3.2.
- c. With respect to the graph of  $y = x^2$ , the graph of  $h$  is obtained by a *reflection* in the  $x$ -axis and a vertical shift one unit *upward*, as shown in Figure 3.3.
- d. With respect to the graph of  $y = x^2$ , the graph of  $k$  is obtained by a horizontal shift two units *to the left* and a vertical shift three units *downward*, as shown in Figure 3.4.

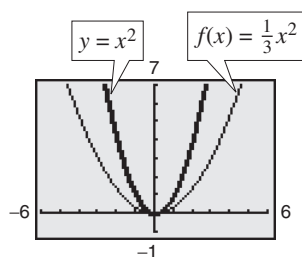


Figure 3.1

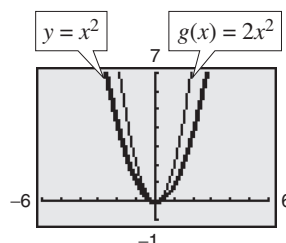


Figure 3.2

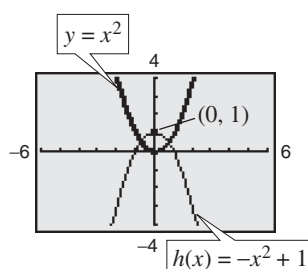


Figure 3.3

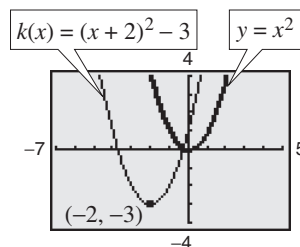


Figure 3.4

**CHECKPOINT** Now try Exercise 5.

Recall from Section 1.5 that the graphs of  $y = f(x \pm c)$ ,  $y = f(x) \pm c$ ,  $y = -f(x)$ , and  $y = f(-x)$  are rigid transformations of the graph of  $y = f(x)$ .

$y = f(x \pm c)$	Horizontal shift	$y = -f(x)$	Reflection in $x$ -axis
$y = f(x) \pm c$	Vertical shift	$y = f(-x)$	Reflection in $y$ -axis

Emphasize that the technique illustrated in Example 1 (comparing functions with the parent function  $y = x^2$ ) is very useful when analyzing functions and preparing to graph them. This kind of exercise (see Exercises 5 and 6 in the exercise set) also helps build conceptual understanding. You may want to cover extra examples of this technique during class for practice.

**STUDY TIP**

In Example 1, note that the coefficient  $a$  determines how widely the parabola given by  $f(x) = ax^2$  opens. If  $|a|$  is small, the parabola opens more widely than if  $|a|$  is large.

**Prerequisite Skills**

If you have difficulty with this example, review shifting, reflecting, and stretching of graphs in Section 1.5.

## The Standard Form of a Quadratic Function

The equation in Example 1(d) is written in the **standard form**

$$f(x) = a(x - h)^2 + k.$$

This form is especially convenient for sketching a parabola because it identifies the vertex of the parabola as  $(h, k)$ .

### Standard Form of a Quadratic Function

The quadratic function given by

$$f(x) = a(x - h)^2 + k, \quad a \neq 0$$

is in **standard form**. The graph of  $f$  is a parabola whose axis is the vertical line  $x = h$  and whose vertex is the point  $(h, k)$ . If  $a > 0$ , the parabola opens upward, and if  $a < 0$ , the parabola opens downward.

## Example 2 Identifying the Vertex of a Quadratic Function

Describe the graph of  $f(x) = 2x^2 + 8x + 7$  and identify the vertex.

### Solution

Write the quadratic function in standard form by completing the square. Recall that the first step is to factor out any coefficient of  $x^2$  that is not 1.

$$f(x) = 2x^2 + 8x + 7$$

Write original function.

$$= (2x^2 + 8x) + 7$$

Group  $x$ -terms.

$$= 2(x^2 + 4x) + 7$$

Factor 2 out of  $x$ -terms.

$$= 2(x^2 + 4x + 4 - 4) + 7$$

Add and subtract  $(4/2)^2 = 4$  within parentheses to complete the square.

$$\underbrace{\phantom{+ 4 - 4}}_{\left(\frac{4}{2}\right)^2}$$

$$= 2(x^2 + 4x + 4) - 2(4) + 7$$

Regroup terms.

$$= 2(x + 2)^2 - 1$$

Write in standard form.

From the standard form, you can see that the graph of  $f$  is a parabola that opens upward with vertex  $(-2, -1)$ , as shown in Figure 3.5. This corresponds to a left shift of two units and a downward shift of one unit relative to the graph of  $y = 2x^2$ .

 **CHECKPOINT** Now try Exercise 13.

To find the  $x$ -intercepts of the graph of  $f(x) = ax^2 + bx + c$ , solve the equation  $ax^2 + bx + c = 0$ . If  $ax^2 + bx + c$  does not factor, you can use the Quadratic Formula to find the  $x$ -intercepts, or a graphing utility to approximate the  $x$ -intercepts. Remember, however, that a parabola may not have  $x$ -intercepts.

## Exploration

Use a graphing utility to graph  $y = ax^2$  with  $a = -2, -1, -0.5, 0.5, 1,$  and  $2$ . How does changing the value of  $a$  affect the graph?

Use a graphing utility to graph  $y = (x - h)^2$  with  $h = -4, -2, 2,$  and  $4$ . How does changing the value of  $h$  affect the graph?

Use a graphing utility to graph  $y = x^2 + k$  with  $k = -4, -2, 2,$  and  $4$ . How does changing the value of  $k$  affect the graph?

## Prerequisite Skills

If you have difficulty with this example, review the process of completing the square for an algebraic expression in Section 2.4, paying special attention to problems in which  $a \neq 1$ .

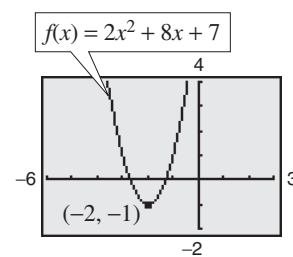


Figure 3.5

**Example 3** Identifying  $x$ -Intercepts of a Quadratic Function

Describe the graph of  $f(x) = -x^2 + 6x - 8$  and identify any  $x$ -intercepts.

**Solution**

$$\begin{aligned}
 f(x) &= -x^2 + 6x - 8 && \text{Write original function.} \\
 &= -(x^2 - 6x) - 8 && \text{Factor } -1 \text{ out of } x\text{-terms.} \\
 &= -(x^2 - 6x + 9 - 9) - 8 && \text{Because } b = 6, \text{ add and subtract } (6/2)^2 = 9 \text{ within parentheses.} \\
 &\quad \quad \quad \underbrace{\hspace{2cm}}_{\left(\frac{6}{2}\right)^2} \\
 &= -(x^2 - 6x + 9) - (-9) - 8 && \text{Regroup terms.} \\
 &= -(x - 3)^2 + 1 && \text{Write in standard form.}
 \end{aligned}$$

The graph of  $f$  is a parabola that opens downward with vertex  $(3, 1)$ , as shown in Figure 3.6. The  $x$ -intercepts are determined as follows.

$$\begin{aligned}
 -(x^2 - 6x + 8) &= 0 && \text{Factor out } -1. \\
 -(x - 2)(x - 4) &= 0 && \text{Factor.} \\
 x - 2 &= 0 && \Rightarrow x = 2 && \text{Set 1st factor equal to 0.} \\
 x - 4 &= 0 && \Rightarrow x = 4 && \text{Set 2nd factor equal to 0.}
 \end{aligned}$$

So, the  $x$ -intercepts are  $(2, 0)$  and  $(4, 0)$ , as shown in Figure 3.6.

**CHECKPOINT** Now try Exercise 17.

**Example 4** Writing the Equation of a Parabola in Standard Form

Write the standard form of the equation of the parabola whose vertex is  $(1, 2)$  and that passes through the point  $(3, -6)$ .

**Solution**

Because the vertex of the parabola is  $(h, k) = (1, 2)$ , the equation has the form

$$f(x) = a(x - 1)^2 + 2. \quad \text{Substitute for } h \text{ and } k \text{ in standard form.}$$

Because the parabola passes through the point  $(3, -6)$ , it follows that  $f(3) = -6$ . So, you obtain

$$\begin{aligned}
 -6 &= a(3 - 1)^2 + 2 \\
 -6 &= 4a + 2 \\
 -2 &= a.
 \end{aligned}$$

The equation in standard form is  $f(x) = -2(x - 1)^2 + 2$ . You can confirm this answer by graphing  $f(x) = -2(x - 1)^2 + 2$  with a graphing utility, as shown in Figure 3.7 Use the *zoom* and *trace* features or the *maximum* and *value* features to confirm that its vertex is  $(1, 2)$  and that it passes through the point  $(3, -6)$ .

**CHECKPOINT** Now try Exercise 29.

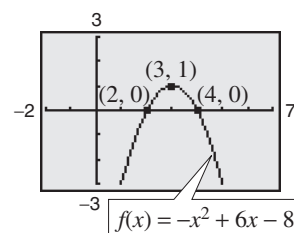


Figure 3.6

**STUDY TIP**

In Example 4, there are infinitely many different parabolas that have a vertex at  $(1, 2)$ . Of these, however, the only one that passes through the point  $(3, -6)$  is the one given by

$$f(x) = -2(x - 1)^2 + 2.$$

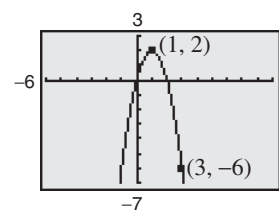


Figure 3.7

## Finding Minimum and Maximum Values

Many applications involve finding the maximum or minimum value of a quadratic function. By completing the square of the quadratic function  $f(x) = ax^2 + bx + c$ , you can rewrite the function in standard form.

$$f(x) = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right) \quad \text{Standard form}$$

You can see that the vertex occurs at  $x = -b/(2a)$ , which implies the following.

### Minimum and Maximum Values of Quadratic Functions

1. If  $a > 0$ ,  $f$  has a *minimum* value at  $x = -\frac{b}{2a}$ .
2. If  $a < 0$ ,  $f$  has a *maximum* value at  $x = -\frac{b}{2a}$ .

### Example 5 The Maximum Height of a Baseball



A baseball is hit at a point 3 feet above the ground at a velocity of 100 feet per second and at an angle of  $45^\circ$  with respect to the ground. The path of the baseball is given by the function  $f(x) = -0.0032x^2 + x + 3$ , where  $f(x)$  is the height of the baseball (in feet) and  $x$  is the horizontal distance from home plate (in feet). What is the maximum height reached by the baseball?

#### Algebraic Solution

For this quadratic function, you have

$$f(x) = ax^2 + bx + c = -0.0032x^2 + x + 3$$

which implies that  $a = -0.0032$  and  $b = 1$ . Because the function has a maximum when  $x = -b/(2a)$ , you can conclude that the baseball reaches its maximum height when it is  $x$  feet from home plate, where  $x$  is

$$\begin{aligned} x &= -\frac{b}{2a} = -\frac{1}{2(-0.0032)} \\ &= 156.25 \text{ feet.} \end{aligned}$$

At this distance, the maximum height is

$$\begin{aligned} f(156.25) &= -0.0032(156.25)^2 + 156.25 + 3 \\ &= 81.125 \text{ feet.} \end{aligned}$$

**CHECKPOINT** Now try Exercise 55.

#### TECHNOLOGY TIP

Note in the graphical solution for Example 5, that when using the *zoom* and *trace* features, you might have to change the  $y$ -scale in order to avoid a graph that is “too flat.”

#### Graphical Solution

Use a graphing utility to graph  $y = -0.0032x^2 + x + 3$  so that you can see the important features of the parabola. Use the *maximum* feature (see Figure 3.8) or the *zoom* and *trace* features (see Figure 3.9) of the graphing utility to approximate the maximum height on the graph to be  $y \approx 81.125$  feet at  $x \approx 156.25$ .

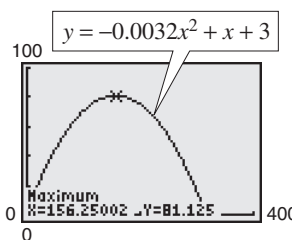


Figure 3.8

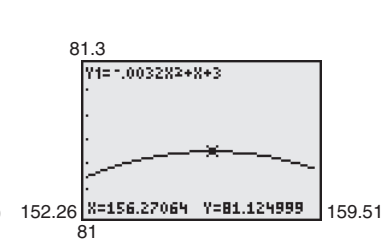


Figure 3.9

**TECHNOLOGY SUPPORT** For instructions on how to use the *maximum*, the *minimum*, the *table*, and the *zoom* and *trace* features, see Appendix A; for specific keystrokes, go to this textbook's *Online Study Center*.

**Example 6** Cost

A soft drink manufacturer has daily production costs of

$$C(x) = 70,000 - 120x + 0.055x^2$$

where  $C$  is the total cost (in dollars) and  $x$  is the number of units produced. Estimate numerically the number of units that should be produced each day to yield a minimum cost.

**Solution**

Enter the function  $y = 70,000 - 120x + 0.055x^2$  into your graphing utility. Then use the *table* feature of the graphing utility to create a table. Set the table to start at  $x = 0$  and set the table step to 100. By scrolling through the table you can see that the minimum cost is between 1000 units and 1200 units, as shown in Figure 3.10. You can improve this estimate by starting the table at  $x = 1000$  and setting the table step to 10. From the table in Figure 3.11, you can see that approximately 1090 units should be produced to yield a minimum cost of \$4545.50.

**CHECKPOINT** Now try Exercise 57.

X	Y1
800	5200
900	6550
1000	5000
1100	4550
1200	5200
1300	6950
1400	9800

X=1100

Figure 3.10

X	Y1
1060	4598
1070	4569.5
1080	4552
1090	4545.5
1100	4550
1110	4565.5
1120	4592

X=1090

Figure 3.11

**Example 7** Grants

The numbers  $g$  of grants awarded from the National Endowment for the Humanities fund from 1999 to 2003 can be approximated by the model

$$g(t) = -99.14t^2 + 2,201.1t - 10,896, \quad 9 \leq t \leq 13$$

where  $t$  represents the year, with  $t = 9$  corresponding to 1999. Using this model, determine the year in which the number of grants awarded was greatest.

(Source: U.S. National Endowment for the Arts)

**Algebraic Solution**

Use the fact that the maximum point of the parabola occurs when  $t = -b/(2a)$ . For this function, you have  $a = -99.14$  and  $b = 2201.1$ . So,

$$\begin{aligned} t &= -\frac{b}{2a} \\ &= -\frac{2201.1}{2(-99.14)} \\ &\approx 11.1 \end{aligned}$$

From this  $t$ -value and the fact that  $t = 9$  represents 1999, you can conclude that the greatest number of grants were awarded during 2001.

**CHECKPOINT** Now try Exercise 61.

**Graphical Solution**

Use a graphing utility to graph

$$y = -99.14x^2 + 2,201.1x - 10,896$$

for  $9 \leq x \leq 13$ , as shown in Figure 3.12. Use the *maximum* feature (see Figure 3.12) or the *zoom* and *trace* features (see Figure 3.13) of the graphing utility to approximate the maximum point of the parabola to be  $x \approx 11.1$ . So, you can conclude that the greatest number of grants were awarded during 2001.

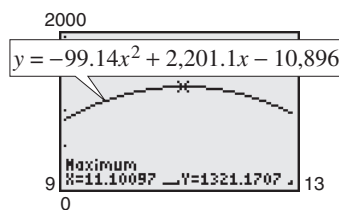


Figure 3.12

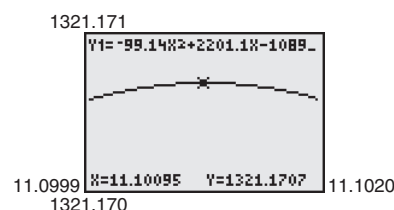


Figure 3.13

## 3.1 Exercises

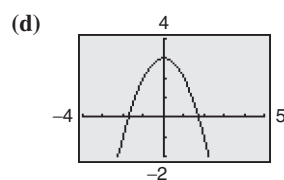
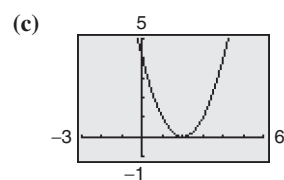
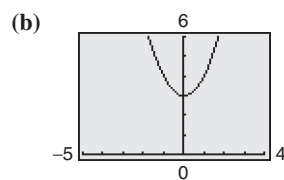
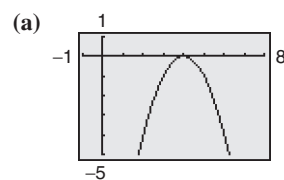
See [www.CalcChat.com](http://www.CalcChat.com) for worked-out solutions to odd-numbered exercises.

## Vocabulary Check

Fill in the blanks.

- A polynomial function of degree  $n$  and leading coefficient  $a_n$  is a function of the form  $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ ,  $a_n \neq 0$  where  $n$  is a \_\_\_\_\_ and  $a_n, a_{n-1}, \dots, a_2, a_1, a_0$  are \_\_\_\_\_ numbers.
- A \_\_\_\_\_ function is a second-degree polynomial function, and its graph is called a \_\_\_\_\_.
- The graph of a quadratic function is symmetric about its \_\_\_\_\_.
- If the graph of a quadratic function opens upward, then its leading coefficient is \_\_\_\_\_ and the vertex of the graph is a \_\_\_\_\_.
- If the graph of a quadratic function opens downward, then its leading coefficient is \_\_\_\_\_ and the vertex of the graph is a \_\_\_\_\_.

In Exercises 1–4, match the quadratic function with its graph. [The graphs are labeled (a), (b), (c), and (d).]



- $f(x) = (x - 2)^2$
- $f(x) = 3 - x^2$
- $f(x) = x^2 + 3$
- $f(x) = -(x - 4)^2$

In Exercises 5 and 6, use a graphing utility to graph each function in the same viewing window. Describe how the graph of each function is related to the graph of  $y = x^2$ .

- (a)  $y = \frac{1}{2}x^2$  (b)  $y = \frac{1}{2}x^2 - 1$   
(c)  $y = \frac{1}{2}(x + 3)^2$  (d)  $y = -\frac{1}{2}(x + 3)^2 - 1$
- (a)  $y = \frac{3}{2}x^2$  (b)  $y = \frac{3}{2}x^2 + 1$   
(c)  $y = \frac{3}{2}(x - 3)^2$  (d)  $y = -\frac{3}{2}(x - 3)^2 + 1$

In Exercises 7–20, sketch the graph of the quadratic function. Identify the vertex and  $x$ -intercept(s). Use a graphing utility to verify your results.

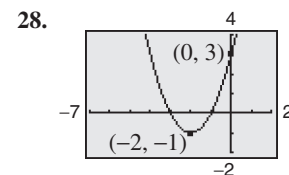
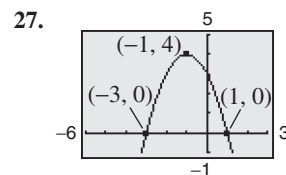
- $f(x) = 25 - x^2$
- $f(x) = x^2 - 7$
- $f(x) = \frac{1}{2}x^2 - 4$
- $f(x) = 16 - \frac{1}{4}x^2$

- $f(x) = (x + 4)^2 - 3$
- $f(x) = (x - 6)^2 + 3$
- $h(x) = x^2 - 8x + 16$
- $g(x) = x^2 + 2x + 1$
- $f(x) = x^2 - x + \frac{5}{4}$
- $f(x) = x^2 + 3x + \frac{1}{4}$
- $f(x) = -x^2 + 2x + 5$
- $f(x) = -x^2 - 4x + 1$
- $h(x) = 4x^2 - 4x + 21$
- $f(x) = 2x^2 - x + 1$

In Exercises 21–26, use a graphing utility to graph the quadratic function. Identify the vertex and  $x$ -intercept(s). Then check your results algebraically by writing the quadratic function in standard form.

- $f(x) = -(x^2 + 2x - 3)$
- $f(x) = -(x^2 + x - 30)$
- $g(x) = x^2 + 8x + 11$
- $f(x) = x^2 + 10x + 14$
- $f(x) = -2x^2 + 16x - 31$
- $f(x) = -4x^2 + 24x - 41$

In Exercises 27 and 28, write an equation for the parabola in standard form. Use a graphing utility to graph the equation and verify your result.





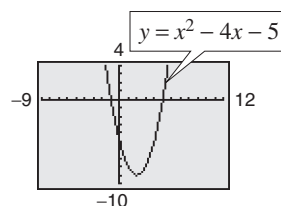
## 260 Chapter 3 Polynomial and Rational Functions

In Exercises 29–34, write the standard form of the quadratic function that has the indicated vertex and whose graph passes through the given point. Verify your result with a graphing utility.

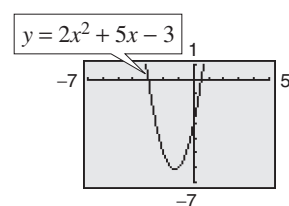
29. Vertex:  $(-2, 5)$ ; Point:  $(0, 9)$   
 30. Vertex:  $(4, 1)$ ; Point:  $(6, -7)$   
 31. Vertex:  $(1, -2)$ ; Point:  $(-1, 14)$   
 32. Vertex:  $(-4, -1)$ ; Point:  $(-2, 4)$   
 33. Vertex:  $(\frac{1}{2}, 1)$ ; Point:  $(-2, -\frac{21}{5})$   
 34. Vertex:  $(-\frac{1}{4}, -1)$ ; Point:  $(0, -\frac{17}{16})$

**Graphical Reasoning** In Exercises 35–38, determine the  $x$ -intercept(s) of the graph visually. How do the  $x$ -intercepts correspond to the solutions of the quadratic equation when  $y = 0$ ?

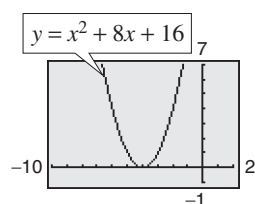
35.



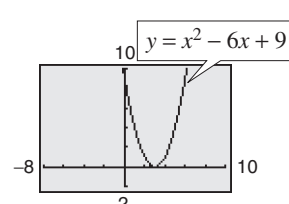
36.



37.



38.



In Exercises 39–44, use a graphing utility to graph the quadratic function. Find the  $x$ -intercepts of the graph and compare them with the solutions of the corresponding quadratic equation when  $y = 0$ .

39.  $y = x^2 - 4x$       40.  $y = -2x^2 + 10x$   
 41.  $y = 2x^2 - 7x - 30$       42.  $y = 4x^2 + 25x - 21$   
 43.  $y = -\frac{1}{2}(x^2 - 6x - 7)$       44.  $y = \frac{7}{10}(x^2 + 12x - 45)$

In Exercises 45–48, find two quadratic functions, one that opens upward and one that opens downward, whose graphs have the given  $x$ -intercepts. (There are many correct answers.)

45.  $(-1, 0), (3, 0)$       46.  $(0, 0), (10, 0)$   
 47.  $(-3, 0), (-\frac{1}{2}, 0)$       48.  $(-\frac{5}{2}, 0), (2, 0)$

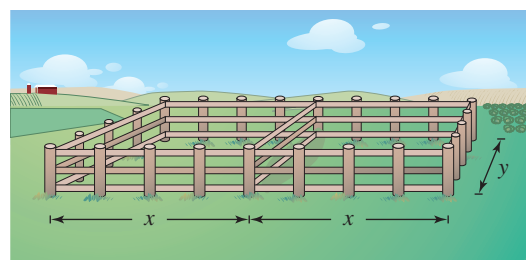
In Exercises 49–52, find two positive real numbers whose product is a maximum.

49. The sum is 110.  
 50. The sum is  $S$ .  
 51. The sum of the first and twice the second is 24.  
 52. The sum of the first and three times the second is 42.

53. **Geometry** An indoor physical fitness room consists of a rectangular region with a semicircle on each end. The perimeter of the room is to be a 200-meter single-lane running track.

- (a) Draw a diagram that illustrates the problem. Let  $x$  and  $y$  represent the length and width of the rectangular region, respectively.  
 (b) Determine the radius of the semicircular ends of the track. Determine the distance, in terms of  $y$ , around the inside edge of the two semicircular parts of the track.  
 (c) Use the result of part (b) to write an equation, in terms of  $x$  and  $y$ , for the distance traveled in one lap around the track. Solve for  $y$ .  
 (d) Use the result of part (c) to write the area  $A$  of the rectangular region as a function of  $x$ .  
 (e) Use a graphing utility to graph the area function from part (d). Use the graph to approximate the dimensions that will produce a rectangle of maximum area.

54. **Numerical, Graphical, and Analytical Analysis** A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals (see figure). Use the following methods to determine the dimensions that will produce a maximum enclosed area.



- (a) Write the area  $A$  of the corral as a function of  $x$ .  
 (b) Use the *table* feature of a graphing utility to create a table showing possible values of  $x$  and the corresponding areas  $A$  of the corral. Use the table to estimate the dimensions that will produce the maximum enclosed area.  
 (c) Use a graphing utility to graph the area function. Use the graph to approximate the dimensions that will produce the maximum enclosed area.

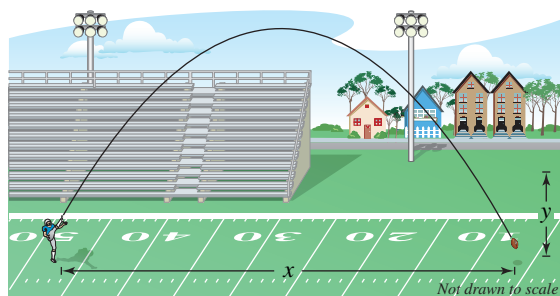
(d) Write the area function in standard form to find algebraically the dimensions that will produce the maximum area.

(e) Compare your results from parts (b), (c), and (d).

- 55. Height of a Ball** The height  $y$  (in feet) of a punted football is approximated by

$$y = -\frac{16}{2025}x^2 + \frac{9}{5}x + \frac{3}{2}$$

where  $x$  is the horizontal distance (in feet) from where the football is punted.

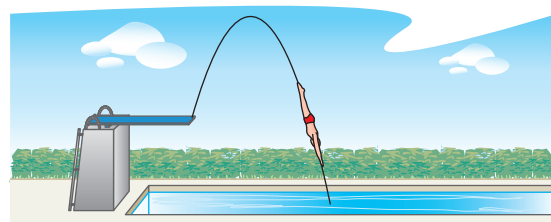


- (a) Use a graphing utility to graph the path of the football.  
 (b) How high is the football when it is punted? (*Hint*: Find  $y$  when  $x = 0$ .)  
 (c) What is the maximum height of the football?  
 (d) How far from the punter does the football strike the ground?

- 56. Path of a Diver** The path of a diver is approximated by

$$y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$$

where  $y$  is the height (in feet) and  $x$  is the horizontal distance (in feet) from the end of the diving board (see figure). What is the maximum height of the diver? Verify your answer using a graphing utility.



- 57. Cost** A manufacturer of lighting fixtures has daily production costs of

$$C(x) = 800 - 10x + 0.25x^2$$

where  $C$  is the total cost (in dollars) and  $x$  is the number of units produced. Use the *table* feature of a graphing utility to determine how many fixtures should be produced each day to yield a minimum cost.

- 58. Automobile Aerodynamics** The number of horsepower  $H$  required to overcome wind drag on a certain automobile is approximated by

$$H(s) = 0.002s^2 + 0.05s - 0.029, \quad 0 \leq s \leq 100$$

where  $s$  is the speed of the car (in miles per hour).

- (a) Use a graphing utility to graph the function.  
 (b) Graphically estimate the maximum speed of the car if the power required to overcome wind drag is not to exceed 10 horsepower. Verify your result algebraically.

- 59. Revenue** The total revenue  $R$  (in thousands of dollars) earned from manufacturing and selling hand-held video games is given by

$$R(p) = -25p^2 + 1200p$$

where  $p$  is the price per unit (in dollars).

- (a) Find the revenue when the price per unit is \$20, \$25, and \$30.  
 (b) Find the unit price that will yield a maximum revenue.  
 (c) What is the maximum revenue?  
 (d) Explain your results.

- 60. Revenue** The total revenue  $R$  (in dollars) earned by a dog walking service is given by

$$R(p) = -12p^2 + 150p$$

where  $p$  is the price charged per dog (in dollars).

- (a) Find the revenue when the price per dog is \$4, \$6, and \$8.  
 (b) Find the price that will yield a maximum revenue.  
 (c) What is the maximum revenue?  
 (d) Explain your results.

- 61. Graphical Analysis** From 1960 to 2004, the annual per capita consumption  $C$  of cigarettes by Americans (age 18 and older) can be modeled by

$$C(t) = 4306 - 3.4t - 1.32t^2, \quad 0 \leq t \leq 44$$

where  $t$  is the year, with  $t = 0$  corresponding to 1960.

(Source: U.S. Department of Agriculture)

- (a) Use a graphing utility to graph the model.  
 (b) Use the graph of the model to approximate the year when the maximum annual consumption of cigarettes occurred. Approximate the maximum average annual consumption. Beginning in 1966, all cigarette packages were required by law to carry a health warning. Do you think the warning had any effect? Explain.  
 (c) In 2000, the U.S. population (age 18 and older) was 209,117,000. Of those, about 48,306,000 were smokers. What was the average annual cigarette consumption *per smoker* in 2000? What was the average daily cigarette consumption *per smoker*?

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**62. Data Analysis** The factory sales  $S$  of VCRs (in millions of dollars) in the United States from 1990 to 2004 can be modeled by  $S = -28.40t^2 + 218.1t + 2435$ , for  $0 \leq t \leq 14$ , where  $t$  is the year, with  $t = 0$  corresponding to 1990. (Source: Consumer Electronics Association)

- According to the model, when did the maximum value of factory sales of VCRs occur?
- According to the model, what was the value of the factory sales in 2004? Explain your result.
- Would you use the model to predict the value of the factory sales for years beyond 2004? Explain.

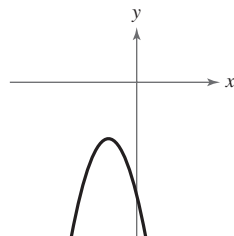
**Synthesis**

**True or False?** In Exercises 63 and 64, determine whether the statement is true or false. Justify your answer.

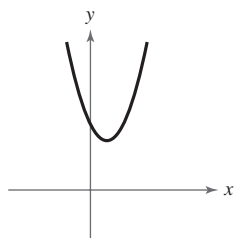
- 63.** The function  $f(x) = -12x^2 - 1$  has no  $x$ -intercepts.
- 64.** The graphs of  $f(x) = -4x^2 - 10x + 7$  and  $g(x) = 12x^2 + 30x + 1$  have the same axis of symmetry.

**Library of Parent Functions** In Exercises 65 and 66, determine which equation(s) may be represented by the graph shown. (There may be more than one correct answer.)

- 65.** (a)  $f(x) = -(x - 4)^2 + 2$   
 (b)  $f(x) = -(x + 2)^2 + 4$   
 (c)  $f(x) = -(x + 2)^2 - 4$   
 (d)  $f(x) = -x^2 - 4x - 8$   
 (e)  $f(x) = -(x - 2)^2 - 4$   
 (f)  $f(x) = -x^2 + 4x - 8$



- 66.** (a)  $f(x) = (x - 1)^2 + 3$   
 (b)  $f(x) = (x + 1)^2 + 3$   
 (c)  $f(x) = (x - 3)^2 + 1$   
 (d)  $f(x) = x^2 + 2x + 4$   
 (e)  $f(x) = (x + 3)^2 + 1$   
 (f)  $f(x) = x^2 + 6x + 10$



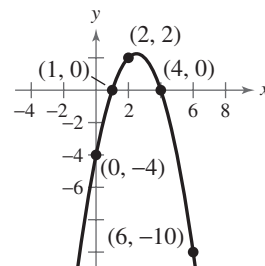
**Think About It** In Exercises 67–70, find the value of  $b$  such that the function has the given maximum or minimum value.

- 67.**  $f(x) = -x^2 + bx - 75$ ; Maximum value: 25  
**68.**  $f(x) = -x^2 + bx - 16$ ; Maximum value: 48  
**69.**  $f(x) = x^2 + bx + 26$ ; Minimum value: 10  
**70.**  $f(x) = x^2 + bx - 25$ ; Minimum value: -50

**71. Profit** The profit  $P$  (in millions of dollars) for a recreational vehicle retailer is modeled by a quadratic function of the form  $P = at^2 + bt + c$ , where  $t$  represents the year. If you were president of the company, which of the following models would you prefer? Explain your reasoning.

- $a$  is positive and  $t \geq -b/(2a)$ .
- $a$  is positive and  $t \leq -b/(2a)$ .
- $a$  is negative and  $t \geq -b/(2a)$ .
- $a$  is negative and  $t \leq -b/(2a)$ .

**72. Writing** The parabola in the figure below has an equation of the form  $y = ax^2 + bx - 4$ . Find the equation of this parabola in two different ways, by hand and with technology (graphing utility or computer software). Write a paragraph describing the methods you used and comparing the results of the two methods.

**Skills Review**

In Exercises 73–76, determine algebraically any point(s) of intersection of the graphs of the equations. Verify your results using the *intersect* feature of a graphing utility.

- 73.**  $x + y = 8$   
 $-\frac{2}{3}x + y = 6$
- 74.**  $y = 3x - 10$   
 $y = \frac{1}{4}x + 1$
- 75.**  $y = 9 - x^2$   
 $y = x + 3$
- 76.**  $y = x^3 + 2x - 1$   
 $y = -2x + 15$

In Exercises 77–80, perform the operation and write the result in standard form.

- 77.**  $(6 - i) - (2i + 11)$   
**78.**  $(2i + 5)^2 - 21$   
**79.**  $(3i + 7)(-4i + 1)$   
**80.**  $(4 - i)^3$

**81. Make a Decision** To work an extended application analyzing the height of a basketball after it has been dropped, visit this textbook's *Online Study Center*.