

## Chapter 1: Introduction to Real Numbers and Algebraic Expressions

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#### 1.1 INTRODUCTION TO ALGEBRA

### CHAPTER 1: Introduction to Real Numbers and Algebraic Expressions

The study of algebra involves the use of equations to solve problems. Equations are constructed from algebraic expressions. The purpose of this section is to introduce you to the types of expressions encountered in algebra.

#### Evaluating Algebraic Expressions

In arithmetic, you have worked with expressions such as

$49 + 75$ ,  $8$  times  $6.07$ ,  $29 - 14$ , and  $5/6$

In algebra, we use certain letters for numbers and work with algebraic expressions such as

$x + 75$ ,  $8$  times  $y$ ,  $29 - t$ , and  $a/b$ .

Sometimes a letter can represent various numbers. In that case, we call the letter a variable. Let  $a =$  your age. Then  $a$  is a variable since  $a$  changes from year to year. Sometimes a letter can stand for just one number. In that case, we call the letter a constant. Let  $b =$  your date of birth. Then  $b$  is a constant.

An algebraic expression consists of variables, constants, numerals, and operation signs. When we replace a variable with a number, we say that we are substituting for the variable. This process is called evaluating the expression.

**EXAMPLE 1** Evaluate  $x + y$  when  $x = 37$  and  $y = 29$ .

We substitute  $37$  for  $x$  and  $29$  for  $y$  and carry out the addition:

$$x + y = 37 + 29 = 66$$

The number  $66$  is called the value of the expression.

Algebraic expressions involving multiplication can be written in several ways. For example, "8 times a" can be written as

8 cross a, 8 dot a, (8a), or simply 8a.

Two letters written together without an operation symbol, such as ab, also indicate a multiplication.

EXAMPLE 2 Evaluate  $3y$  when  $y = 14$ .

$$3y = 3(14) = 42$$

Do Exercises 2-4.

2. Evaluate  $a + b$  when  $a = 38$  and  $b = 26$ .

3. Evaluate  $x - y$  when  $x = 57$  and  $y = 29$ .

4. Evaluate  $4t$  when  $t = 15$ .

Answers to Exercises 2-4:

2. 64

3. 28

4. 60

EXAMPLE 3 Area of a Rectangle. The area  $A$  of a rectangle of length  $l$  and width  $w$  is given by the formula  $A = lw$ . Find the area when  $l$  is 24.5 in. and  $w$  is 16 in.

We substitute 24.5 in. for  $l$  and 16 in. for  $w$  and carry out the multiplication:

$$\begin{aligned} A = lw &= (24.5 \text{ in.}) (16 \text{ in.}) \\ &= (24.5)(16) (\text{in.})(\text{in.}) \\ &= 392 \text{ in}^2, \text{ or } 392 \text{ square inches.} \end{aligned}$$

Do Exercise 5.

5. Find the area of a rectangle when  $l$  is 24 ft and  $w$  is 8 ft.

Answer to Exercise 5:

5.  $192 \text{ ft}^2$

Algebraic expressions involving division can also be written in several ways. For example, "8 divided by  $t$ " can be written as

8 division symbol  $t$ ,  $8/t$ , or 8 times  $1/t$

where the fraction bar (slash mark) is a division symbol.

EXAMPLE 4 Evaluate  $a/b$  when  $a = 63$  and  $b = 9$ .

We substitute 63 for  $a$  and 9 for  $b$  and carry out the division:

$$a/b = 63/9 = 7$$

EXAMPLE 5

Evaluate  $(12m)/n$  when  $m = 8$  and  $n = 16$ .

$$(12m)/n = (12 \text{ times } 8)/16 = 96/16 = 6$$

Do Exercises 6 and 7.

6. Evaluate  $a/b$  when  $a = 200$  and  $b = 8$ .

7. Evaluate  $10p/q$  when  $p = 40$  and  $q = 25$ .

Answers to Exercise 6-7:

6. 25

7. 16

EXAMPLE 6 Ed takes a trip on his motorcycle. He wants to travel 660 mi on a particular day. The time  $t$ , in hours, that it takes to travel 660 mi. is given by

$$t = 660/t,$$

where  $r$  is the speed of Ed's motorcycle. Find the time of travel if the speed  $r$  is 60 mph. We substitute 60 for  $r$  and carry out the division:

$$t = 660/t = 660 = 11 \text{ hr}$$

Do Exercise 8.

8. Motorcycle Travel. Find the time it takes to travel 660 mi. if the speed is 55 mph.

Answers to Exercise 8:

8. 12 hr

### Translating to Algebraic Expressions

In algebra, we translate problems to equations. The different parts of an equation are translations of word phrases to algebraic expressions. It is easier to translate if we know that certain words often translate to certain operation symbols.

#### KEY WORDS, PHRASES, AND CONCEPTS

ADDITION ( + )

add

added to

sum

total

plus  
more than  
increased by

SUBTRACTION ( - )  
subtract  
subtracted from  
difference  
minus  
less than  
decreased by  
take away

MULTIPLICATION  
multiply  
multiplied by  
product  
times  
of

DIVISION  
divide  
divided by  
quotient

EXAMPLE 7 Translate to an algebraic expression:

Twice (or two times) some number.

Think of some number, say, 8. We can write 2 times 8 as 2 times 8. We multiplied by 2. Do the same thing using a variable. We can use any variable we wish, such as  $x$ ,  $y$ ,  $m$ , or  $n$ . Let's use  $y$  to stand for some number. If we multiply by 2, we get an expression

$y$  times 2,      2 times  $y$ ,      2 times  $y$ , or       $2y$ .

In algebra,  $2y$  is the expression generally used.

EXAMPLE 8 Translate to an algebraic expression:

Thirty-eight percent of some number.

Let  $n$  = the number. The word "of" translates to a multiplication symbol, so we get the following expressions as a translation:

38% times  $n$ , 0.38 times  $n$ , or  $0.38n$ .

EXAMPLE 9 Translate to an algebraic expression:

Seven less than some number.

We let  $x$  represent the number.

Now if the number were 23, then 7 less than 23 is 16, that is,  $(23 - 7)$ , not  $(7 - 23)$ . If we knew the number to be 345, then the translation would be  $345 - 7$ . If the number is  $x$ , then the translation is  $x - 7$ .

Caution!

Note that  $7 - x$  is not a correct translation of the expression in Example 9. The expression  $7 - x$  is a translation of "seven minus some number" or "some number less than seven."

EXAMPLE 10 Translate to an algebraic expression:

Eighteen more than a number.

We let  $t$  = the number.

Now if the number were 26, then the translation would be  $26 + 18$ , or  $18 + 26$ . If we knew the number to be 174, then the translation would be  $174 + 18$ , or  $18 + 174$ . If the number is  $t$ , then the translation is  $t + 18$ , or  $18 + t$ .

EXAMPLE 11 Translate to an algebraic expression:

A number divided by 5.

We let  $m$  = the number.

Now if the number were 76, then the translation would be 76 division symbol 5, or  $76/5$ .

If the number were 213, then the translation would be 213 division symbol 5 or  $213/5$ .

If the number is  $m$ , then the translation is

$m$  division symbol 5 or  $m / 5$ .

EXAMPLE 12 Translate each phrase to an algebraic expression.

PHRASE

Five more than some number

EXPRESSION

$n + 5$ , or  $5 + n$

PHRASE

Half of a number

EXPRESSION

$(1/2)t$  or  $t/2$

PHRASE

Five more than three times some number

EXPRESSION

$3p + 5$ , or  $5 + 3p$

PHRASE

The difference of two numbers

EXPRESSION

$$x - y$$

PHRASE

Six less than the product of two numbers

EXPRESSION

$$mn - 6$$

PHRASE

Seventy-six percent of some number

EXPRESSION

$$76\%z, \text{ or } 0.76z$$

PHRASE

Four less than twice some number

EXPRESSION

$$2x - 4$$

Do Exercises 9-17.

Translate to an algebraic expression.

9. Eight less than some number
10. Eight more than some number
11. Four less than some number
12. Half of a number
13. Six more than eight times some number
14. The difference of two numbers
15. Fifty-nine percent of some number
16. Two hundred less than the product of two numbers
17. The sum of two numbers

Answers to Exercise 9-17:

9.  $x - 8$
10.  $y + 8$  or  $8 + y$
11.  $m - 4$
12.  $\frac{1}{2}p$
13.  $6 + 8x$  or  $8x + 6$
14.  $a - b$

15.  $59\%x$  or  $0.59x$

16.  $xy - 200$

17.  $p + q$