1.2 THE REAL NUMBERS

Objectives

a. State the integer that corresponds to a real-world situation.

b. Graph rational numbers on a number line.

c. Convert from fraction notation to decimal notation for a rational number.

d. Determine which of two real numbers is greater and indicate which, using < or >; given an inequality like a > b, write another inequality with the same meaning. Determine whether an inequality like −3 </= 5 is true or false.

e. Find the absolute value of a real number.

A set is a collection of objects. For our purposes, we will most often be considering sets of numbers. One way to name a set uses what is called roster notation. For example, roster notation for the set containing the numbers 0, 2, and 5 is {0, 2, 5}.

Sets that are part of other sets are called subsets. In this section, we become acquainted with the set of real numbers and its various subsets.

Two important subsets of the real numbers are listed below using roster notation:

Natural Numbers.

The set of natural numbers = {1, 2, 3, and so on}. These are the numbers used for counting.

Whole Numbers
The set of whole numbers = {0, 1, 2, 3, and so on}. This is the set of natural numbers with 0 included.

We can represent these sets on a number line. The natural numbers are those to the right of zero. The whole numbers are the natural numbers and zero.

We create a new set, called the integers, by starting with the whole numbers, 0, 1, 2, 3, and so on. For each natural number 1, 2, 3, and so on, we obtain a new number to the left of zero on the number line:
For the number 1, there will be an opposite number −1 (negative 1).
For the number 2, there will be an opposite number −2 (negative 2).
For the number 3, there will be an opposite number −3 (negative 3), and so on.

The integers consist of the whole numbers and these new numbers.

Integers

The set of integers = {from negative infinity, −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, and so on to positive infinity}.

We call these new numbers to the left of 0 negative integers. The natural numbers are also called positive integers. Zero is neither positive nor negative. We call −1 and 1 opposites of each other. Similarly, −2 and 2 are opposites, −3 and 3 are opposites, −100 and 100 are opposites, and 0 is its own opposite. Pairs of opposite numbers like −3 and 3 are the same distance from 0. The integers extend infinitely on the number line to the left and right of zero.

a. Integers and the Real World

Integers correspond to many real world problems and situations. The following examples will help you get ready to translate problem situations that involve integers to mathematical language.

EXAMPLE 1. Tell which integer corresponds to this situation:
The temperature is 4 degrees below zero. The integer −4 corresponds to the situation. The temperature is −4 degrees.

EXAMPLE 2. Jeopardy. Tell which integer corresponds to this situation:
A contestant missed a $600 question on the television game show Jeopardy. Missing a $600 question means -600. Missing a $600 question causes a $600 loss on the score; that is, the contestant earns negative 600 dollars.

EXAMPLE 3. Elevation. Tell which integer corresponds to this situation:
The shores of California’s largest lake, the Salton Sea, are 227 feet below sea level. The integer −227 corresponds to the situation. The elevation is −227 ft.

EXAMPLE 4. Stock Price Change. Tell which integers correspond to this situation:
The price of Pearson Education stock decreased from $27 per share to $11 per share over a recent time period. The price of Safeway stock increased from $20 per share to $22 per share over a recent time period. The integer −16 corresponds to the decrease in the Pearson Education stock value. The integer 2 represents the increase in the Safeway stock value.

Do Exercises 1–5.

1. The halfback gained 8 yards on the first down. The quarterback was sacked for a 5 yard loss on the second down.

2. Temperature High and Low. The highest recorded temperature in Nevada is 125 degrees F on June 29, 1994, in Laughlin. The lowest recorded temperature in Nevada is 50 degrees F below zero on June 8, 1937, in San Jacinto.

3. Stock Decrease. The price of Wendy’s stock decreased from $41 per share to $38 per share over a recent time period.

4. At 10 seconds before liftoff, ignition occurs. At 156 sec after liftoff, the first stage is detached from the rocket.

5. A submarine dove 120 feet, rose 50 ft, and then dove 80 ft.

Answers to Exercises 1 – 5.

1. 8; –5
2. 125; –50
3. –3
4. –10; 156
5. –120; 50; –80

b. The Rational Numbers.

We created the set of integers by obtaining a negative number for each natural number and also including 0. To create a larger number system, called the set of rational numbers, we consider quotients of integers with nonzero divisors. The following are some examples of rational numbers:

Fraction 2/3, fraction –2/3, fraction 7/1, 4, –3, 0, fraction 23/(–8), 2.4, –0.17, mixed number 10 and one half.

The number –2/3 (read negative two-thirds) can also be named fraction –2/3 or fraction 2/(–3); that is,
Fraction –a/b = fraction –a/b = fraction a/(–b).

The number 2.4 can be named fraction 24/10 or, fraction 12/5 and decimal –0.17 can be named fraction –17/100. We can describe the set of rational numbers as follows.

Rational Numbers.

The set of rational numbers = the set of numbers fraction a/b, where a and b are integers and b is not equal to 0. Note that this new set of numbers, the rational numbers, contains whole numbers, the integers, the arithmetic numbers (also called the non negative rational numbers), and the negative rational numbers

To graph a number means to find and mark its point on the number line. Some rational numbers are graphed in the preceding figure.

c. Notation for Rational Numbers.
Each rational number can be named using fraction or decimal notation.

EXAMPLE 8. Convert to decimal notation: fraction -5/8.
We first find decimal notation for fraction 5/8. Since 5/8 means 5 division symbol 8, we divide. Thus, fraction 5/8 = decimal 0.625, so fractions –5/8 = decimal –0.625.
Decimal notation for fraction –5/8 is decimal –0.625. We consider decimal –0.625 to be a terminating decimal. Decimal notation for some numbers repeats.

EXAMPLE 9. Convert to decimal notation: fraction 7/11.

We can abbreviate repeating decimal notation by writing a bar over the repeating part, in this case, 0.63, with a bar over the 3. Thus, fraction 7/11 = decimal 0.63, with a bar over the 3.

Each rational number can be expressed in either terminating or repeating decimal notation.

The following are other examples to show how each rational number can be named using fraction or decimal notation:

0 = fraction 0/8
fraction 27/100 = decimal 0.27
mixed number -8 and three quareters = decimal -8.75
-13 / 6 = -2.16

Do Exercises 9-11.
Convert to decimal notation.
9. fraction -3/8
10. fraction -6/11
11. 4 / 3

Answers to Exercises 9-11.
9. decimal -0.375
10. decimal -0.54, repeating decimal line over the 54
11. decimal 1.3,repeating decimal line over the 3

d. The Real Numbers and Order.
Every rational number has a point on the number line. However, there are some points on the line for which there is no rational number. These points correspond to what are called irrational numbers.

What kinds of numbers are irrational? One example is the number pie, which is used in finding the area and the circumference of a circle: A = pie times r squared and See = 2 times pi times r.

Another example of an irrational number is the square root of 2, named radical 2. It is the length of the diagonal of a square with sides of length 1. It is also the number that when multiplied by itself gives 2; that is, (radical 2) times (radical 2) = 2. There is no rational number that can be multiplied by itself to get 2, but the following are rational approximations:

1.4 is an approximation of radical 2 because (1.4)(1.4) = 1.96
1.41 is a better approximation because (1.41)(1.4) = 1.9881
1.4142 is an even better approximation because (1.4142)(1.4142) = 1.99996164.

Decimal notation for rational numbers either terminates or repeats.
Decimal notation for irrational numbers neither terminates nor repeats.

Some other examples of irrational numbers are radical 3, negative radical 8, radical 11, and 0.121221222122221 repeating. Whenever we take the square root of a number that is not a perfect square, we will get an irrational number.

The rational numbers and the irrational numbers together correspond to all the points on a number line and make up what is called the real number system.

Following is a diagram of a number line showing the integers from −4 to 4 and also showing the rational numbers −2.5, fraction -1/2, and fraction 1/2, as well as the irrational numbers radical 2 and pie.

A number line is a horizontal line with arrows at each end indicating that the line continues to infinity at both ends. The line is divided into equal segments. In the middle of the line is the value of 0. Negative numbers fall to the left of zero and positive numbers fall to the right of zero. The integer negative 4 is located four divisions to the left of zero. The integer positive four is found four divisions to the right of zero. The rational number negative 2 point 5 is found half way between the values of negative two and negative three. The fraction negative one half is found half way between zero and negative one. The value one half is found half way between zero and one. The irrational number radical 2 is found between one and two at about 1.4 and the irrational number pi is found between the three and four at 3.14.

REAL NUMBERS.

The set of real numbers is the set of all numbers corresponding to points on the number line.

The real numbers consist of the rational numbers and the irrational numbers.

ORDER.

Real numbers are named in order on the number line, with larger numbers named farther to the right. For any two numbers on the line, the one to the left is less than the one to the right.

We use the symbol < to mean “is less than.” The sentence −8 < 6 means “−8 is less than 6.” The symbol > means “is greater than.” The sentence −3 > −7 means “3 is greater than 7.” The sentences −8 < 6 and −3 > −7 are inequalities.

EXAMPLES. Use either < or > to write a true sentence.
10. 2 blank 9
Since 2 is to the left of 9, 2 is less than 9, so 2 < 9.

11. −7 blank 3
Since 7 is to the left of 3, we have −7 < 3.

12. 6 blank −12
Since 6 is to the right of −12, then 6 > −12.

13. −18 blank − 5

Since −18 is to the left of 5, we have 18 < 5.

14.   −2.7 blank −3/2
The answer is −2.7 < −3/2.

15.   1.5 blank −2.7
The answer is 1.5 > 2.7.

16.   1.38 blank 1.83
The answer is 1.38 < 1.83.

17.   −3.45 blank 1.32
The answer is 3.45 < 1.32.

18.   −4 blank 0
The answer is 4 < 0.

19.   5.8 blank 0
The answer is 5.8 > 0.

20.   5 / 8 blank 7 / 11
We convert to decimal notation: fraction 5/8 = decimal 0.625 and fraction 7/11 = decimal 0.6363 repeating. Thus, fraction 5/8 < fraction 7/11.

21.   −1/ blank 1/3
The answer is −1/2 < −1/3

22.   −2 3/5 blank −11/4
The answer is −2 3/5 > −11/4

Do Exercises 12–19.
12.   -3 blank 7
13.   -8 blank -5
14.   7 blank −10
15.   3.1 blank −9.5
16.   −2/3 blank −1
17.   −11/8 blank 23/15
18.   −2/3 blank −5/9
19.   −4.78 blank −5.01

Answers to Exercises 12–19.
12.   <
13.   <
14.   >
15.   >
16.   >
17.   <
18.   <
19.   >

Note that both −8 < 6 and 6 > -8 are true. Every true inequality yields another true inequality when we interchange the numbers or variables and reverse the direction of the inequality sign.

Order; >, <
a < b also has the meaning b > a.

EXAMPLES. Write another inequality with the same meaning.
23. -3 > -8
The inequality -8 < -3 has the same meaning.

24. a < -5
The inequality -5 > a has the same meaning.
A helpful mental device is to think of an inequality sign as an "arrow" with the arrow pointing to the smaller number.

Do Exercises 20 and 21.
Write another inequality with the same meaning.
20. -5 < 7
21. x > 4

Answers to Exercise 20-21.
20. 7 > -5
21. 4 < x
Note that all positive real numbers are greater than zero and all negative real numbers are less than zero.

If b is a positive real number, then b > 0.
If a is a negative real number, then a < 0.

Expressions like a symbol </= b and b symbol >/= a are also inequalities. We read a symbol </= b as "a is less than or equal to b." We read a symbol >/= b as "a is greater than or equal to b."
EXAMPLES. Write true or false for the statement.
25. -3 symbol </= 5.4
True since -3 < 5.4 is true.

26. -3 symbol </= -3
True since -3 = -3 is true.

27. -5 symbol >/= 1 2/3
False since neither -5 > 1 2/3 nor -5 = 1 2/3 is true.

Do Exercises 22-24.
Write true or false.
22. -4 symbol </= -6
23. 7.8 symbol >/= 7.8
24. -2 symbol </= 3/8

Answers to Exercise 22-24.
22. False
23. True
24. True

e. Absolute Value.

Distance is always a nonnegative number. We call the distance of a number from zero on a number line the absolute value of the number.

Absolute Value.

The absolute value of a number is its distance from zero on a number line. We use | x | (x enclosed in two vertical lines) to represent the absolute value of a number x.

Finding Absolute Value

a) If a number is negative, its absolute value is positive.

b) If a number is positive or zero, its absolute value is the same as the number.

EXAMPLES Find the absolute value.

28. | -7 |

The distance of -7 from 0 is 7, so | 7 | = 7.

29. | 12 |

The distance of 12 from 0 is | 12 |, so | 12 | = 12.

30. | 0 |

The distance of 0 from 0 is 0, so | 0 | = 0.

31. | 3 / 2 | = 3 / 2

32. | -2.73 | = 2.73

Do Exercises 25-28

25. | 8 |
26. | -9 |
27. | -2 / 3 |
28. | 5.6 |

Answers to Exercises 25-28

25. 8
26. 9
27. 2/3
28. 5.6