1.2 Lines in the Plane

The Slope of a Line

In this section, you will study lines and their equations. The **slope** of a nonvertical line represents the number of units the line rises or falls vertically for each unit of horizontal change from left to right. For instance, consider the two points (x_1, y_1) and (x_2, y_2) on the line shown in Figure 1.16. As you move from left to right along this line, a change of $(y_2 - y_1)$ units in the vertical direction corresponds to a change of $(x_2 - x_1)$ units in the horizontal direction. That is,

$$y_2 - y_1 = \text{the change in } y$$

and

$$x_2 - x_1 = \text{the change in } x.$$

The slope of the line is given by the ratio of these two changes.

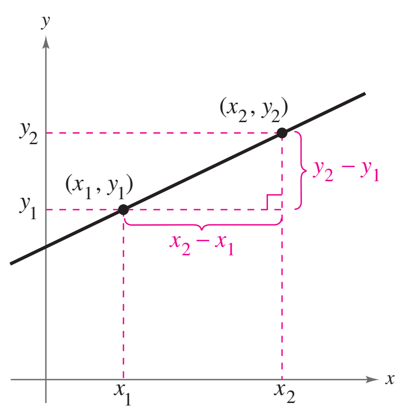


Figure 1.16

What you should learn

- Find the slopes of lines.
- Write linear equations given points on lines and their slopes.
- Use slope-intercept forms of linear equations to sketch lines.
- Use slope to identify parallel and perpendicular lines.

Why you should learn it

The slope of a line can be used to solve real-life problems. For instance, in Exercise 87 on page 99. you will use a linear equation to model student enrollment at Penn State University.

Sky Bonillo/PhotoEdit

Definition of the Slope of a Line

The **slope** m of the nonvertical line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$$

where $x_1 \neq x_2$.

When this formula for slope is used, the *order of subtraction* is important. Given two points on a line, you are free to label either one of them as (x_1, y_1) and the other as (x_2, y_2). However, once you have done this, you must form the numerator and denominator using the same order of subtraction.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \frac{y_1 - y_2}{x_1 - x_2} \qquad m = \frac{y_2 - y_1}{x_1 - x_2}$$

Correct   Correct   Incorrect

Throughout this text, the term *line* always means a *straight* line.

Example 1 Finding the Slope of a Line

Find the slope of the line passing through each pair of points.

a. $(-2, 0)$ and $(3, 1)$ **b.** $(-1, 2)$ and $(2, 2)$ **c.** $(0, 4)$ and $(1, -1)$

Solution

Difference in y-values

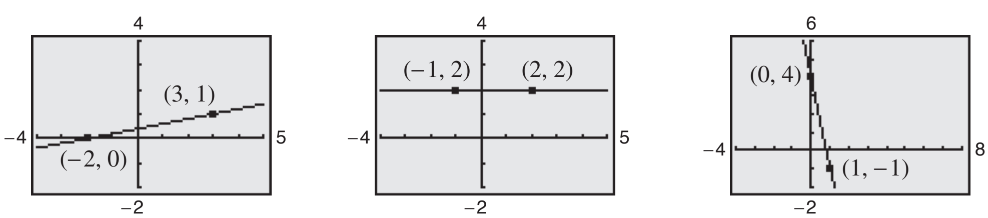
$$\textbf{a.}\ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{3 - (-2)} = \frac{1}{3 + 2} = \frac{1}{5}$$

Difference in x-values

$$\textbf{b.}\ m = \frac{2 - 2}{2 - (-1)} = \frac{0}{3} = 0$$

$$\textbf{c.}\ m = \frac{-1 - 4}{1 - 0} = \frac{-5}{1} = -5$$

The graphs of the three lines are shown in Figure 1.17. Note that the *square setting* gives the correct "steepness" of the lines.



(a) (b) (c)

Figure 1.17

CHECKPOINT Now try Exercise 9.

The definition of slope does not apply to vertical lines. For instance, consider the points $(3, 4)$ and $(3, 1)$ on the vertical line shown in Figure 1.18. Applying the formula for slope, you obtain

$$m = \frac{4 - 1}{3 - 3} = \frac{3}{0}. \quad \text{Undefined}$$

Because division by zero is undefined, the slope of a vertical line is undefined.

From the slopes of the lines shown in Figures 1.17 and 1.18, you can make the following generalizations about the slope of a line.

The Slope of a Line

1. A line with positive slope $(m > 0)$ *rises* from left to right.
2. A line with negative slope $(m < 0)$ *falls* from left to right.
3. A line with zero slope $(m = 0)$ is *horizontal*.
4. A line with undefined slope is *vertical*.

Exploration

Use a graphing utility to compare the slopes of the lines $y = 0.5x$, $y = x$, $y = 2x$, and $y = 4x$. What do you observe about these lines? Compare the slopes of the lines $y = -0.5x$, $y = -x$, $y = -2x$, and $y = -4x$. What do you observe about these lines? (*Hint:* Use a *square setting* to guarantee a true geometric perspective.)

Common Error

A common error when finding the slope of a line is combining x- and y-coordinates in either the numerator or denominator, or both, as in

$$m = \frac{y_2 - x_1}{x_2 - y_1}.$$

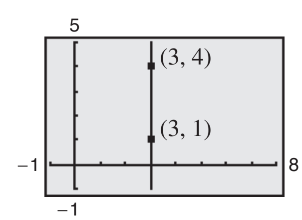


Figure 1.18

Point out to your students that the vertical line shown in Figure 1.18 must be drawn on a graphing utility with a special command because there is no way to express the line's equation in the "$y =$" format.

The Point-Slope Form of the Equation of a Line

If you know the slope of a line *and* you also know the coordinates of one point on the line, you can find an equation for the line. For instance, in Figure 1.19, let (x_1, y_1) be a point on the line whose slope is m. If (x, y) is any *other* point on the line, it follows that

$$\frac{y - y_1}{x - x_1} = m.$$

This equation in the variables x and y can be rewritten in the **point-slope form** of the equation of a line.

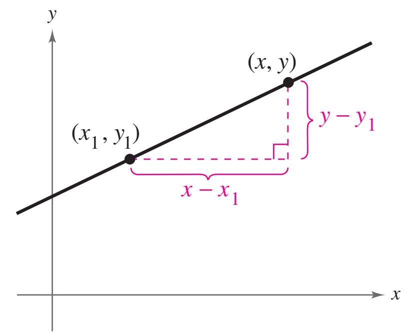


Figure 1.19

Point-Slope Form of the Equation of a Line

The **point-slope form** of the equation of the line that passes through the point (x_1, y_1) and has a slope of m is

$$y - y_1 = m(x - x_1).$$

The point-slope form is most useful for finding the equation of a line if you know at least one point that the line passes through and the slope of the line. You should remember this form of the equation of a line.

Example 2 The Point-Slope Form of the Equation of a Line

Find an equation of the line that passes through the point $(1, -2)$ and has a slope of 3.

Solution

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-2) = 3(x - 1)$	Substitute for y_1, m, and x_1.
$y + 2 = 3x - 3$	Simplify.
$y = 3x - 5$	Solve for y.

The line is shown in Figure 1.20.

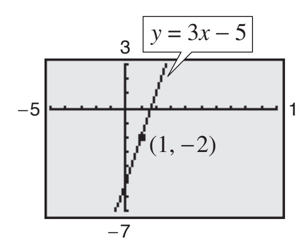


Figure 1.20

CHECKPOINT Now try Exercise 25.

The point-slope form can be used to find an equation of a nonvertical line passing through two points (x_1, y_1) and (x_2, y_2). First, find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2$$

Then use the point-slope form to obtain the equation

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

This is sometimes called the **two-point form** of the equation of a line.

STUDY TIP

When you find an equation of the line that passes through two given points, you need to substitute the coordinates of only one of the points into the point-slope form. It does not matter which point you choose because both points will yield the same result.

Example 3 A Linear Model for Sales Prediction

During 2004, Nike's net sales were \$12.25 billion, and in 2005 net sales were \$13.74 billion. Write a linear equation giving the net sales y in terms of the year x. Then use the equation to predict the net sales for 2006. (Source: Nike, Inc.)

Solution

Let $x = 0$ represent 2000. In Figure 1.21, let (4, 12.25) and (5, 13.74) be two points on the line representing the net sales. The slope of this line is

$$m = \frac{13.74 - 12.25}{5 - 4} = 1.49. \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

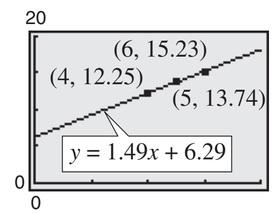


Figure 1.21

By the point-slope form, the equation of the line is as follows.

$$y - 12.25 = 1.49(x - 4) \qquad \text{Write in point-slope form.}$$

$$y = 1.49x + 6.29 \qquad \text{Simplify.}$$

Now, using this equation, you can predict the 2006 net sales ($x = 6$) to be

$$y = 1.49(6) + 6.29 = 8.94 + 6.29 = \$15.23 \text{ billion.}$$

CHECKPOINT Now try Exercise 45.

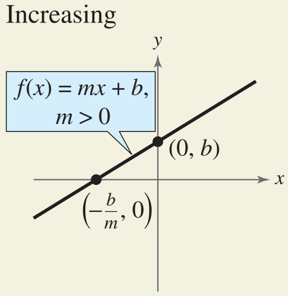
Library of Parent Functions: Linear Function

In the next section, you will be introduced to the precise meaning of the term *function*. The simplest type of function is a *linear function* of the form

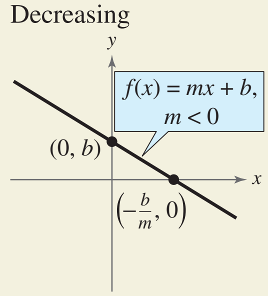
$$f(x) = mx + b.$$

As its name implies, the graph of a linear function is a line that has a slope of m and a y-intercept at $(0, b)$. The basic characteristics of a linear function are summarized below. (Note that some of the terms below will be defined later in the text.) A review of linear functions can be found in the *Study Capsules.*

Graph of $f(x) = mx + b, m > 0$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
x-intercept: $(-b/m, 0)$
y-intercept: $(0, b)$
Increasing



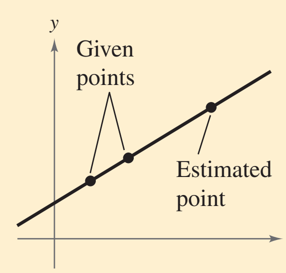
Graph of $f(x) = mx + b, m < 0$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
x-intercept: $(-b/m, 0)$
y-intercept: $(0, b)$
Decreasing



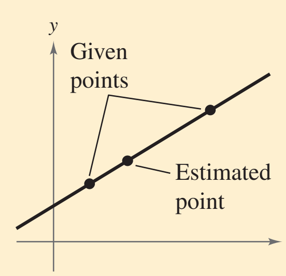
When $m = 0$, the function $f(x) = b$ is called a *constant function* and its graph is a horizontal line.

STUDY TIP

The prediction method illustrated in Example 3 is called **linear extrapolation.** Note in the top figure below that an extrapolated point does not lie between the given points. When the estimated point lies between two given points, as shown in the bottom figure, the procedure used to predict the point is called **linear interpolation.**



Linear Extrapolation



Linear Interpolation

Sketching Graphs of Lines

Many problems in coordinate geometry can be classified as follows.

1. Given a graph (or parts of it), find its equation.
2. Given an equation, sketch its graph.

For lines, the first problem is solved easily by using the point-slope form. This formula, however, is not particularly useful for solving the second type of problem. The form that is better suited to graphing linear equations is the **slope-intercept form** of the equation of a line, $y = mx + b$.

Slope-Intercept Form of the Equation of a Line

The graph of the equation

$$y = mx + b$$

is a line whose slope is m and whose y-intercept is $(0, b)$.

Example 4 Using the Slope-Intercept Form

Determine the slope and y-intercept of each linear equation. Then describe its graph.

a. $x + y = 2$ **b.** $y = 2$

Algebraic Solution

a. Begin by writing the equation in slope-intercept form.

$$x + y = 2 \quad \text{Write original equation.}$$
$$y = 2 - x \quad \text{Subtract } x \text{ from each side.}$$
$$y = -x + 2 \quad \text{Write in slope-intercept form.}$$

From the slope-intercept form of the equation, the slope is -1 and the y-intercept is $(0, 2)$. Because the slope is negative, you know that the graph of the equation is a line that falls one unit for every unit it moves to the right.

b. By writing the equation $y = 2$ in slope-intercept form

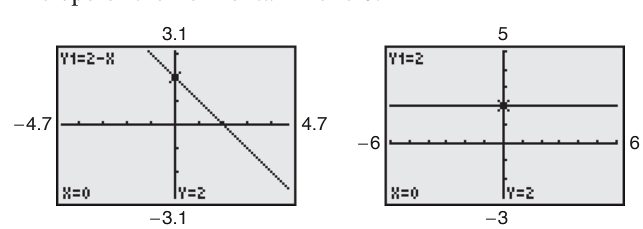
$$y = (0)x + 2$$

you can see that the slope is 0 and the y-intercept is $(0, 2)$. A zero slope implies that the line is horizontal.

Graphical Solution

a. Solve the equation for y to obtain $y = 2 - x$. Enter this equation in your graphing utility. Use a decimal viewing window to graph the equation. To find the y-intercept, use the *value* or *trace* feature. When $x = 0$, $y = 2$, as shown in Figure 1.22(a). So, the y-intercept is $(0, 2)$. To find the slope, continue to use the *trace* feature. Move the cursor along the line until $x = 1$. At this point, $y = 1$. So the graph falls 1 unit for every unit it moves to the right, and the slope is -1.

b. Enter the equation $y = 2$ in your graphing utility and graph the equation. Use the *trace* feature to verify the y-intercept $(0, 2)$, as shown in Figure 1.22(b), and to see that the value of y is the same for all values of x. So, the slope of the horizontal line is 0.



(a) (b)

Figure 1.22

CHECKPOINT Now try Exercise 47.

From the slope-intercept form of the equation of a line, you can see that a horizontal line ($m = 0$) has an equation of the form $y = b$. This is consistent with the fact that each point on a horizontal line through $(0, b)$ has a y-coordinate of b. Similarly, each point on a vertical line through $(a, 0)$ has an x-coordinate of a. So, a vertical line has an equation of the form $x = a$. This equation cannot be written in slope-intercept form because the slope of a vertical line is undefined. However, *every* line has an equation that can be written in the **general form**

$$Ax + By + C = 0 \qquad \text{General form of the equation of a line}$$

where A and B are not *both* zero.

Exploration

Graph the lines $y_1 = 2x + 1$, $y_2 = \frac{1}{2}x + 1$, and $y_3 = -2x + 1$ in the same viewing window. What do you observe?

Graph the lines $y_1 = 2x + 1$, $y_2 = 2x$, and $y_3 = 2x - 1$ in the same viewing window. What do you observe?

Summary of Equations of Lines

1. General form: $Ax + By + C = 0$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Slope-intercept form: $y = mx + b$
5. Point-slope form: $y - y_1 = m(x - x_1)$

Example 5 Different Viewing Windows

The graphs of the two lines

$$y = -x - 1 \qquad \text{and} \qquad y = -10x - 1$$

are shown in Figure 1.23. Even though the slopes of these lines are quite different (-1 and -10, respectively), the graphs seem misleadingly similar because the viewing windows are different.

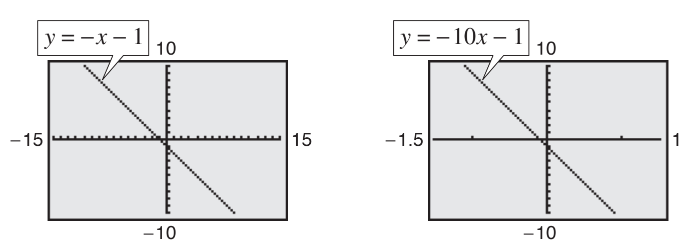
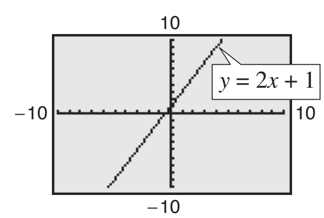


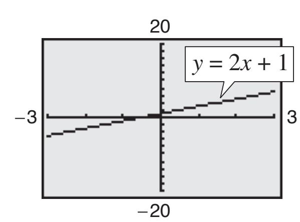
Figure 1.23

✓CHECKPOINT Now try Exercise 51.

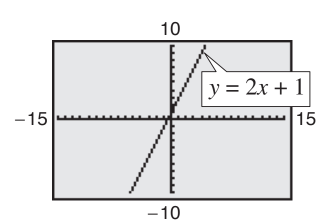
TECHNOLOGY TIP When a graphing utility is used to graph a line, it is important to realize that the graph of the line may not visually appear to have the slope indicated by its equation. This occurs because of the viewing window used for the graph. For instance, Figure 1.24 shows graphs of $y = 2x + 1$ produced on a graphing utility using three different viewing windows. Notice that the slopes in Figures 1.24(a) and (b) do not visually appear to be equal to 2. However, if you use a *square setting*, as in Figure 1.24(c), the slope visually appears to be 2.



(a)



(b)



(c)

Figure 1.24

Parallel and Perpendicular Lines

The slope of a line is a convenient tool for determining whether two lines are parallel or perpendicular.

Parallel Lines

Two distinct nonvertical lines are **parallel** if and only if their slopes are equal. That is,

$$m_1 = m_2.$$

Example 6 Equations of Parallel Lines

Find the slope-intercept form of the equation of the line that passes through the point $(2, -1)$ and is parallel to the line $2x - 3y = 5$.

Solution

Begin by writing the equation of the given line in slope-intercept form.

$$2x - 3y = 5 \quad \text{Write original equation.}$$

$$-2x + 3y = -5 \quad \text{Multiply by } -1.$$

$$3y = 2x - 5 \quad \text{Add } 2x \text{ to each side.}$$

$$y = \frac{2}{3}x - \frac{5}{3} \quad \text{Write in slope-intercept form.}$$

Therefore, the given line has a slope of $m = \frac{2}{3}$. Any line parallel to the given line must also have a slope of $\frac{2}{3}$. So, the line through $(2, -1)$ has the following equation.

$$y - (-1) = \frac{2}{3}(x - 2) \quad \text{Write in point-slope form.}$$

$$y + 1 = \frac{2}{3}x - \frac{4}{3} \quad \text{Simplify.}$$

$$y = \frac{2}{3}x - \frac{7}{3} \quad \text{Write in slope-intercept form.}$$

Notice the similarity between the slope-intercept form of the original equation and the slope-intercept form of the parallel equation. The graphs of both equations are shown in Figure 1.25.

CHECKPOINT Now try Exercise 57(a).

Perpendicular Lines

Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other. That is,

$$m_1 = -\frac{1}{m_2}.$$

TECHNOLOGY TIP

Be careful when you graph equations such as $y = \frac{2}{3}x - \frac{7}{3}$ with your graphing utility. A common mistake is to type in the equation as

Y1 = 2/3X − 7/3

which may not be interpreted by your graphing utility as the original equation. You should use one of the following formulas.

Y1 = 2X/3 − 7/3

Y1 = (2/3)X − 7/3

Do you see why?

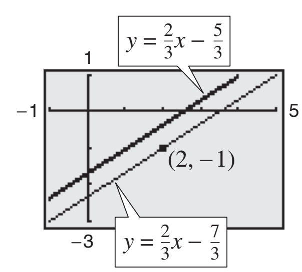


Figure 1.25

Example 7 Equations of Perpendicular Lines

Find the slope-intercept form of the equation of the line that passes through the point $(2, -1)$ and is perpendicular to the line

$$2x - 3y = 5.$$

Solution

From Example 6, you know that the equation can be written in the slope-intercept form $y = \frac{2}{3}x - \frac{5}{3}$. You can see that the line has a slope of $\frac{2}{3}$. So, any line perpendicular to this line must have a slope of $-\frac{3}{2}$ (because $-\frac{3}{2}$ is the negative reciprocal of $\frac{2}{3}$). So, the line through the point $(2, -1)$ has the following equation.

$$y - (-1) = -\frac{3}{2}(x - 2)$$ Write in point-slope form.

$$y + 1 = -\frac{3}{2}x + 3$$ Simplify.

$$y = -\frac{3}{2}x + 2$$ Write in slope-intercept form.

The graphs of both equations are shown in Figure 1.26.

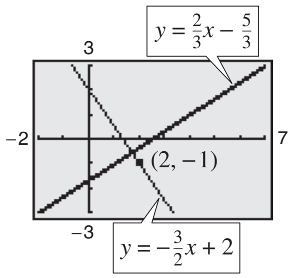


Figure 1.26

CHECKPOINT Now try Exercise 57(b).

Example 8 Graphs of Perpendicular Lines

Use a graphing utility to graph the lines

$$y = x + 1$$

and

$$y = -x + 3$$

in the same viewing window. The lines are supposed to be perpendicular (they have slopes of $m_1 = 1$ and $m_2 = -1$). Do they appear to be perpendicular on the display?

Solution

If the viewing window is nonsquare, as in Figure 1.27, the two lines will not appear perpendicular. If, however, the viewing window is square, as in Figure 1.28, the lines will appear perpendicular.

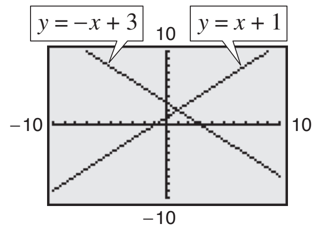


Figure 1.27

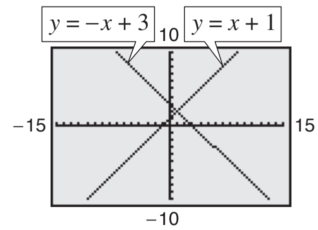


Figure 1.28

CHECKPOINT Now try Exercise 67.

Activities

1. Write an equation of the line that passes through the points $(-2, 1)$ and $(3, 2)$.
   Answer: $x - 5y + 7 = 0$
2. Find the slope of the line that is perpendicular to the line $4x - 7y = 12$.
   Answer: $m = -\frac{7}{4}$
3. Write the equation of the vertical line that passes through the point $(3, 2)$.
   Answer: $x = 3$

1.2 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

1. Match each equation with its form.

(a) $Ax + By + C = 0$	(i) vertical line
(b) $x = a$	(ii) slope-intercept form
(c) $y = b$	(iii) general form
(d) $y = mx + b$	(iv) point-slope form
(e) $y - y_1 = m(x - x_1)$	(v) horizontal line

In Exercises 2–5, fill in the blanks.

2. For a line, the ratio of the change in y to the change in x is called the _______ of the line.

3. Two lines are _______ if and only if their slopes are equal.

4. Two lines are _______ if and only if their slopes are negative reciprocals of each other.

5. The prediction method _______ is the method used to estimate a point on a line that does not lie between the given points.

In Exercises 1 and 2, identify the line that has the indicated slope.

1. (a) $m = \frac{2}{3}$ (b) m is undefined. (c) $m = -2$

2. (a) $m = 0$ (b) $m = -\frac{3}{4}$ (c) $m = 1$

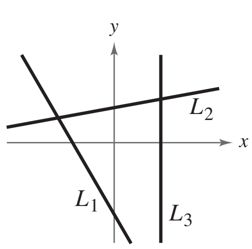


Figure for 1

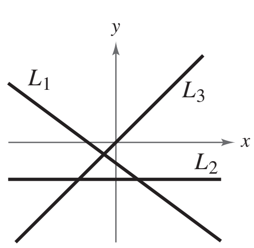


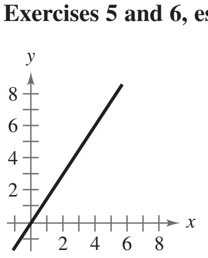
Figure for 2

In Exercises 3 and 4, sketch the lines through the point with the indicated slopes on the same set of coordinate axes.

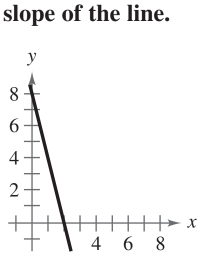
	Point	*Slopes*			
3.	(2, 3)	(a) 0	(b) 1	(c) 2	(d) -3
4.	$(-4, 1)$	(a) 3	(b) -3	(c) $\frac{1}{2}$	(d) Undefined

In Exercises 5 and 6, estimate the slope of the line.

5.



6.



In Exercises 7–10, find the slope of the line passing through the pair of points. Then use a graphing utility to plot the points and use the *draw* feature to graph the line segment connecting the two points. (Use a *square setting*.)

7. $(0, -10), (-4, 0)$

8. $(2, 4), (4, -4)$

9. $(-6, -1), (-6, 4)$

10. $(-3, -2), (1, 6)$

In Exercises 11–18, use the point on the line and the slope of the line to find three additional points through which the line passes. (There are many correct answers.)

	Point	*Slope*
11.	(2, 1)	$m = 0$
12.	$(3, -2)$	$m = 0$
13.	(1, 5)	m is undefined.
14.	$(-4, 1)$	m is undefined.
15.	$(0, -9)$	$m = -2$
16.	$(-5, 4)$	$m = 2$
17.	$(7, -2)$	$m = \frac{1}{2}$
18.	$(-1, -6)$	$m = -\frac{1}{2}$

In Exercises 19–24, (a) find the slope and y-intercept (if possible) of the equation of the line algebraically, and (b) sketch the line by hand. Use a graphing utility to verify your answers to parts (a) and (b).

19. $5x - y + 3 = 0$

20. $2x + 3y - 9 = 0$

21. $5x - 2 = 0$

22. $3x + 7 = 0$

23. $3y + 5 = 0$

24. $-11 - 8y = 0$

In Exercises 25–32, find the general form of the equation of the line that passes through the given point and has the indicated slope. Sketch the line by hand. Use a graphing utility to verify your sketch, if possible.

	Point	*Slope*
25.	$(0, -2)$	$m = 3$
26.	$(-3, 6)$	$m = -2$
27.	$(2, -3)$	$m = -\frac{1}{2}$
28.	$(-2, -5)$	$m = \frac{3}{4}$
29.	$(6, -1)$	m is undefined.
30.	$(-10, 4)$	m is undefined.
31.	$\left(-\frac{1}{2}, \frac{3}{2}\right)$	$m = 0$
32.	$(2.3, -8.5)$	$m = 0$

In Exercises 33–42, find the slope-intercept form of the equation of the line that passes through the points. Use a graphing utility to graph the line.

33. $(5, -1), (-5, 5)$

34. $(4, 3), (-4, -4)$

35. $(-8, 1), (-8, 7)$

36. $(-1, 4), (6, 4)$

37. $\left(2, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{5}{4}\right)$

38. $(1, 1), \left(6, -\frac{2}{3}\right)$

39. $\left(-\frac{1}{10}, -\frac{3}{5}\right), \left(\frac{9}{10}, -\frac{9}{5}\right)$

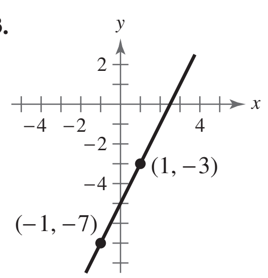
40. $\left(\frac{3}{4}, \frac{3}{2}\right), \left(-\frac{4}{3}, \frac{7}{4}\right)$

41. $(1, 0.6), (-2, -0.6)$

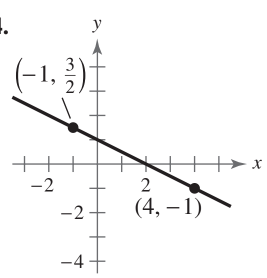
42. $(-8, 0.6), (2, -2.4)$

In Exercises 43 and 44, find the slope-intercept form of the equation of the line shown.

43.



44.



45. ***Annual Salary*** A jeweler's salary was $28,500 in 2004 and $32,900 in 2006. The jeweler's salary follows a linear growth pattern. What will the jeweler's salary be in 2008?

46. ***Annual Salary*** A librarian's salary was $25,000 in 2004 and $27,500 in 2006. The librarian's salary follows a linear growth pattern. What will the librarian's salary be in 2008?

In Exercises 47–50, determine the slope and *y*-intercept of the linear equation. Then describe its graph.

47. $x - 2y = 4$

48. $3x + 4y = 1$

49. $x = -6$

50. $y = 12$

In Exercises 51 and 52, use a graphing utility to graph the equation using each of the suggested viewing windows. Describe the difference between the two graphs.

51. $y = 0.5x - 3$

Xmin = -5	Xmin = -2
Xmax = 10	Xmax = 10
Xscl = 1	Xscl = 1
Ymin = -1	Ymin = -4
Ymax = 10	Ymax = 1
Yscl = 1	Yscl = 1

52. $y = -8x + 5$

Xmin = -5	Xmin = -5
Xmax = 5	Xmax = 10
Xscl = 1	Xscl = 1
Ymin = -10	Ymin = -80
Ymax = 10	Ymax = 80
Yscl = 1	Yscl = 20

In Exercises 53–56, determine whether the lines L_1 and L_2 passing through the pairs of points are parallel, perpendicular, or neither.

53. L_1: $(0, -1), (5, 9)$
L_2: $(0, 3), (4, 1)$

54. L_1: $(-2, -1), (1, 5)$
L_2: $(1, 3), (5, -5)$

55. L_1: $(3, 6), (-6, 0)$
L_2: $(0, -1), \left(5, \frac{7}{3}\right)$

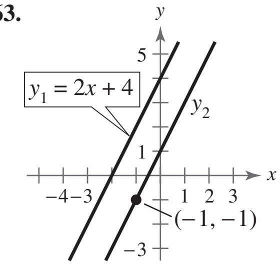
56. L_1: $(4, 8), (-4, 2)$
L_2: $(3, -5), \left(-1, \frac{1}{3}\right)$

In Exercises 57–62, write the slope-intercept forms of the equations of the lines through the given point (a) parallel to the given line and (b) perpendicular to the given line.

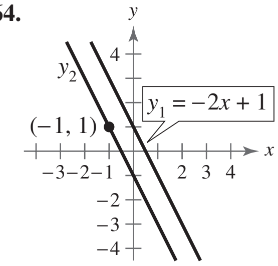
	Point	*Line*
57.	$(2, 1)$	$4x - 2y = 3$
58.	$(-3, 2)$	$x + y = 7$
59.	$\left(-\frac{2}{3}, \frac{7}{8}\right)$	$3x + 4y = 7$
60.	$(-3.9, -1.4)$	$6x + 2y = 9$
61.	$(3, -2)$	$x - 4 = 0$
62.	$(-4, 1)$	$y + 2 = 0$

In Exercises 63 and 64, the lines are parallel. Find the slope-intercept form of the equation of line y_2.

63.

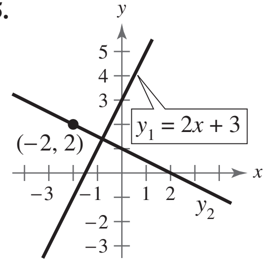


64.

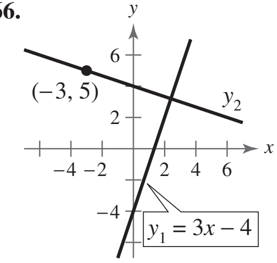


In Exercises 65 and 66, the lines are perpendicular. Find the slope-intercept form of the equation of line y_2.

65.



66.



Graphical Analysis **In Exercises 67–70, identify any relationships that exist among the lines, and then use a graphing utility to graph the three equations in the same viewing window. Adjust the viewing window so that each slope appears visually correct. Use the slopes of the lines to verify your results.**

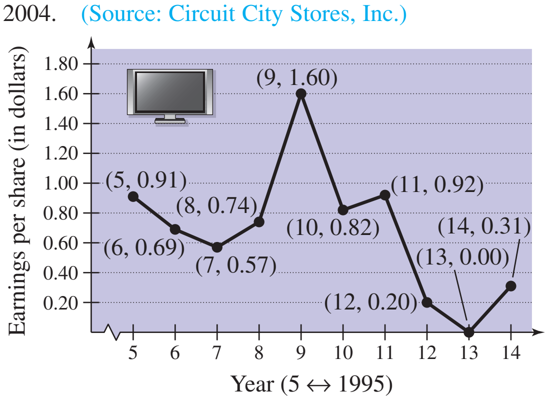
67. (a) $y = 2x$ (b) $y = -2x$ (c) $y = \frac{1}{2}x$

68. (a) $y = \frac{2}{3}x$ (b) $y = -\frac{3}{2}x$ (c) $y = \frac{2}{3}x + 2$

69. (a) $y = -\frac{1}{2}x$ (b) $y = -\frac{1}{2}x + 3$ (c) $y = 2x - 4$

70. (a) $y = x - 8$ (b) $y = x + 1$ (c) $y = -x + 3$

71. ***Earnings per Share*** The graph shows the earnings per share of stock for Circuit City for the years 1995 through 2004. (Source: Circuit City Stores, Inc.)



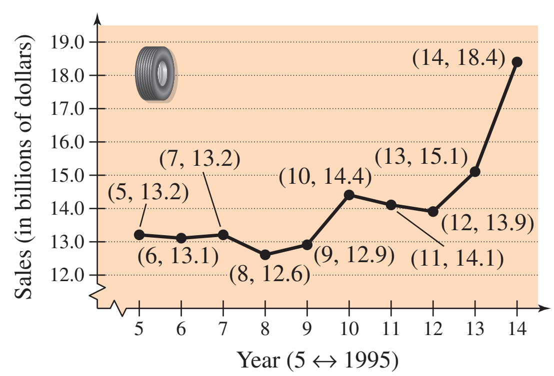
(a) Use the slopes to determine the years in which the earnings per share of stock showed the greatest increase and greatest decrease.

(b) Find the equation of the line between the years 1995 and 2004.

(c) Interpret the meaning of the slope of the equation from part (b) in the context of the problem.

(d) Use the equation from part (b) to estimate the earnings per share of stock in the year 2010. Do you think this is an accurate estimation? Explain.

72. ***Sales*** The graph shows the sales (in billions of dollars) for Goodyear Tire for the years 1995 through 2004, where $t = 5$ represents 1995. (Source: Goodyear Tire)



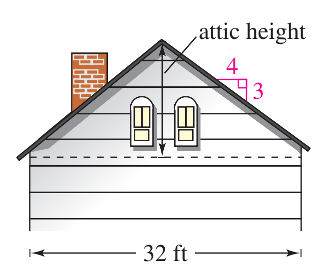
(a) Use the slopes to determine the years in which the sales for Goodyear Tire showed the greatest increase and the smallest increase.

(b) Find the equation of the line between the years 1995 and 2004.

(c) Interpret the meaning of the slope of the equation from part (b) in the context of the problem.

(d) Use the equation from part (b) to estimate the sales for Goodyear Tire in the year 2010. Do you think this is an accurate estimation? Explain.

73. ***Height*** The "rise to run" ratio of the roof of a house determines the steepness of the roof. The rise to run ratio of the roof in the figure is 3 to 4. Determine the maximum height in the attic of the house if the house is 32 feet wide.

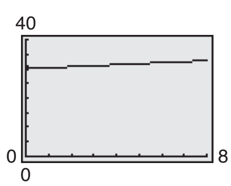


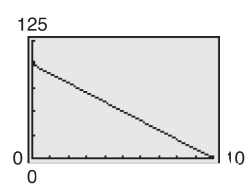
74. ***Road Grade*** When driving down a mountain road, you notice warning signs indicating that it is a "12% grade." This means that the slope of the road is $-\frac{12}{100}$. Approximate the amount of horizontal change in your position if you note from elevation markers that you have descended 2000 feet vertically.

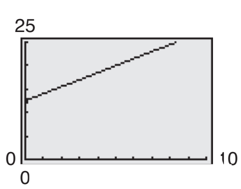
Rate of Change **In Exercises 75–78, you are given the dollar value of a product in 2006 and the rate at which the value of the product is expected to change during the next 5 years. Write a linear equation that gives the dollar value V of the product in terms of the year t. (Let $t = 6$ represent 2006.)**

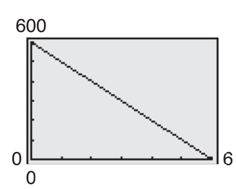
	2006 Value	*Rate*
75.	\$2540	\$125 increase per year
76.	\$156	\$4.50 increase per year
77.	\$20,400	\$2000 decrease per year
78.	\$245,000	\$5600 decrease per year

Graphical Interpretation **In Exercises 79–82, match the description with its graph. Determine the slope of each graph and how it is interpreted in the given context. [The graphs are labeled (a), (b), (c), and (d).]**

(a)


(b)


(c)


(d)


79. You are paying \$10 per week to repay a \$100 loan.

80. An employee is paid \$12.50 per hour plus \$1.50 for each unit produced per hour.

81. A sales representative receives \$30 per day for food plus \$.35 for each mile traveled.

82. A computer that was purchased for \$600 depreciates \$100 per year.

83. ***Depreciation*** A school district purchases a high-volume printer, copier, and scanner for \$25,000. After 10 years, the equipment will have to be replaced. Its value at that time is expected to be \$2000.

(a) Write a linear equation giving the value V of the equipment during the 10 years it will be used.

(b) Use a graphing utility to graph the linear equation representing the depreciation of the equipment, and use the *value* or *trace* feature to complete the table.

t	0	1	2	3	4	5	6	7	8	9	10
V											

(c) Verify your answers in part (b) algebraically by using the equation you found in part (a).

84. ***Meteorology*** Recall that water freezes at 0°C (32°F) and boils at 100°C (212°F).

(a) Find an equation of the line that shows the relationship between the temperature in degrees Celsius C and degrees Fahrenheit F.

(b) Use the result of part (a) to complete the table.

C		$-10°$	$10°$			$177°$
F	$0°$			$68°$	$90°$	

85. ***Cost, Revenue, and Profit*** A contractor purchases a bulldozer for \$36,500. The bulldozer requires an average expenditure of \$5.25 per hour for fuel and maintenance, and the operator is paid \$11.50 per hour.

(a) Write a linear equation giving the total cost C of operating the bulldozer for t hours. (Include the purchase cost of the bulldozer.)

(b) Assuming that customers are charged \$27 per hour of bulldozer use, write an equation for the revenue R derived from t hours of use.

(c) Use the profit formula $(P = R - C)$ to write an equation for the profit derived from t hours of use.

(d) Use the result of part (c) to find the break-even point (the number of hours the bulldozer must be used to yield a profit of 0 dollars).

86. ***Rental Demand*** A real estate office handles an apartment complex with 50 units. When the rent per unit is \$580 per month, all 50 units are occupied. However, when the rent is \$625 per month, the average number of occupied units drops to 47. Assume that the relationship between the monthly rent p and the demand x is linear.

(a) Write the equation of the line giving the demand x in terms of the rent p.

(b) Use a graphing utility to graph the demand equation and use the *trace* feature to estimate the number of units occupied when the rent is \$655. Verify your answer algebraically.

(c) Use the demand equation to predict the number of units occupied when the rent is lowered to \$595. Verify your answer graphically.

87. ***Education*** In 1991, Penn State University had an enrollment of 75,349 students. By 2005, the enrollment had increased to 80,124. (Source: Penn State Fact Book)

(a) What was the average annual change in enrollment from 1991 to 2005?

(b) Use the average annual change in enrollment to estimate the enrollments in 1984, 1997, and 2000.

(c) Write the equation of a line that represents the given data. What is its slope? Interpret the slope in the context of the problem.

88. ***Writing*** Using the results of Exercise 87, write a short paragraph discussing the concepts of *slope* and *average rate of change*.

Synthesis

True or False? **In Exercises 89 and 90, determine whether the statement is true or false. Justify your answer.**

89. The line through $(-8, 2)$ and $(-1, 4)$ and the line through $(0, -4)$ and $(-7, 7)$ are parallel.

90. If the points $(10, -3)$ and $(2, -9)$ lie on the same line, then the point $\left(-12, -\frac{37}{2}\right)$ also lies on that line.

Exploration **In Exercises 91–94, use a graphing utility to graph the equation of the line in the form**

$$\frac{x}{a} + \frac{y}{b} = 1, \qquad a \neq 0, b \neq 0.$$

Use the graphs to make a conjecture about what *a* and *b* represent. Verify your conjecture.

91. $\frac{x}{5} + \frac{y}{-3} = 1$ **92.** $\frac{x}{-6} + \frac{y}{2} = 1$

93. $\frac{x}{4} + \frac{y}{-\frac{2}{3}} = 1$ **94.** $\frac{x}{\frac{1}{2}} + \frac{y}{5} = 1$

In Exercises 95–98, use the results of Exercises 91–94 to write an equation of the line that passes through the points.

95. x-intercept: $(2, 0)$
y-intercept: $(0, 3)$

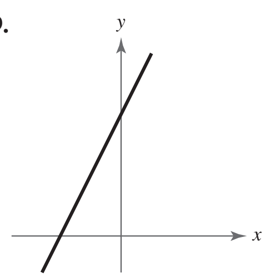
96. x-intercept: $(-5, 0)$
y-intercept: $(0, -4)$

97. x-intercept: $\left(-\frac{1}{6}, 0\right)$
y-intercept: $\left(0, -\frac{2}{3}\right)$

98. x-intercept: $\left(\frac{3}{4}, 0\right)$
y-intercept: $\left(0, \frac{4}{5}\right)$

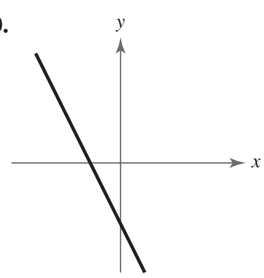
Library of Parent Functions **In Exercises 99 and 100, determine which equation(s) may be represented by the graph shown. (There may be more than one correct answer.)**

99.



(a) $2x - y = -10$
(b) $2x + y = 10$
(c) $x - 2y = 10$
(d) $x + 2y = 10$

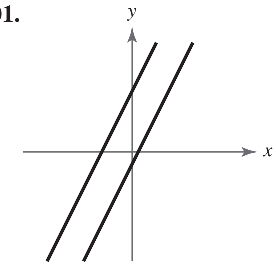
100.



(a) $2x + y = 5$
(b) $2x + y = -5$
(c) $x - 2y = 5$
(d) $x - 2y = -5$

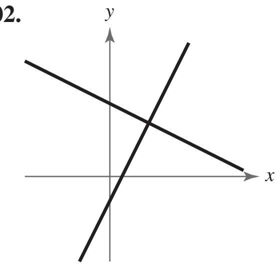
Library of Parent Functions **In Exercises 101 and 102, determine which pair of equations may be represented by the graphs shown.**

101.



(a) $2x - y = 5$
$2x - y = 1$
(b) $2x + y = -5$
$2x + y = 1$
(c) $2x - y = -5$
$2x - y = 1$
(d) $x - 2y = -5$
$x - 2y = -1$

102.



(a) $2x - y = 2$
$x + 2y = 12$
(b) $x - y = 1$
$x + y = 6$
(c) $2x + y = 2$
$x - 2y = 12$
(d) $x - 2y = 2$
$x + 2y = 12$

103. ***Think About It*** Does every line have both an x-intercept and a y-intercept? Explain.

104. ***Think About It*** Can every line be written in slope-intercept form? Explain.

105. ***Think About It*** Does every line have an infinite number of lines that are parallel to the given line? Explain.

106. ***Think About It*** Does every line have an infinite number of lines that are perpendicular to the given line? Explain.

Skills Review

In Exercises 107–112, determine whether the expression is a polynomial. If it is, write the polynomial in standard form.

107. $x + 20$ **108.** $3x - 10x^2 + 1$

109. $4x^2 + x^{-1} - 3$ **110.** $2x^2 - 2x^4 - x^3 + 2$

111. $\frac{x^2 + 3x + 4}{x^2 - 9}$ **112.** $\sqrt{x^2 + 7x + 6}$

In Exercises 113–116, factor the trinomial.

113. $x^2 - 6x - 27$ **114.** $x^2 - 11x + 28$

115. $2x^2 + 11x - 40$ **116.** $3x^2 - 16x + 5$

117. ***Make a Decision*** To work an extended application analyzing the numbers of bachelor's degrees earned by women in the United States from 1985 to 2005, visit this textbook's *Online Study Center*. (Data Source: U.S. Census Bureau)

The *Make a Decision* exercise indicates a multipart exercise using large data sets. Go to this textbook's *Online Study Center* to view these exercises.