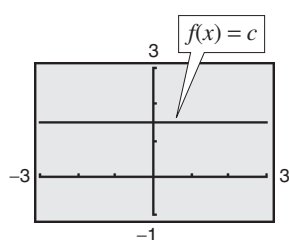


## 1.5 Shifting, Reflecting, and Stretching Graphs

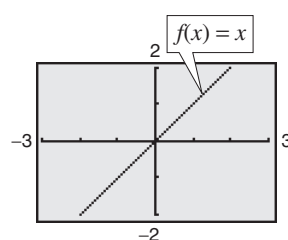
### Summary of Graphs of Parent Functions

One of the goals of this text is to enable you to build your intuition for the basic shapes of the graphs of different types of functions. For instance, from your study of lines in Section 1.2, you can determine the basic shape of the graph of the linear function  $f(x) = mx + b$ . Specifically, you know that the graph of this function is a line whose slope is  $m$  and whose  $y$ -intercept is  $(0, b)$ .

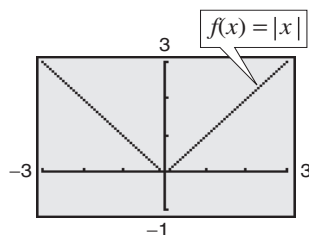
The six graphs shown in Figure 1.55 represent the most commonly used functions in algebra. Familiarity with the basic characteristics of these simple graphs will help you analyze the shapes of more complicated graphs.



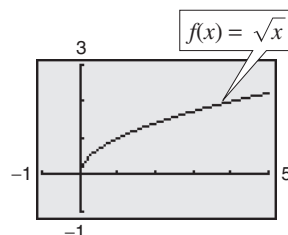
(a) Constant Function



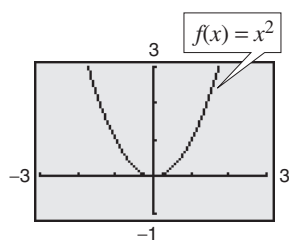
(b) Identity Function



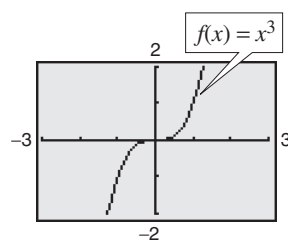
(c) Absolute Value Function



(d) Square Root Function



(e) Quadratic Function



(f) Cubic Function

Figure 1.55

Throughout this section, you will discover how many complicated graphs are derived by shifting, stretching, shrinking, or reflecting the parent graphs shown above. Shifts, stretches, shrinks, and reflections are called *transformations*. Many graphs of functions can be created from combinations of these transformations.

### What you should learn

- Recognize graphs of parent functions.
- Use vertical and horizontal shifts and reflections to graph functions.
- Use nonrigid transformations to graph functions.

### Why you should learn it

Recognizing the graphs of parent functions and knowing how to shift, reflect, and stretch graphs of functions can help you sketch a wide variety of simple functions by hand. This skill is useful in sketching graphs of functions that model real-life data. For example, in Exercise 57 on page 134, you are asked to sketch a function that models the amount of fuel used by vans, pickups, and sport utility vehicles from 1990 through 2003.



Tim Boyle/Getty Images

Emphasize that the graph of a function is related to a “family” of graphs, and if students learn these “families” of common graphs, graphing will be much easier. You can reinforce this concept with discovery methods such as graphing  $f(x) = x^2$ ,  $f(x) = x^2 + 2$ ,  $f(x) = (x - 1)^2$ , and  $f(x) = (x - 1)^2 + 2$  and noting similarities and differences.

## Vertical and Horizontal Shifts

Many functions have graphs that are simple transformations of the graphs of parent functions summarized in Figure 1.55. For example, you can obtain the graph of

$$h(x) = x^2 + 2$$

by shifting the graph of  $f(x) = x^2$  two units *upward*, as shown in Figure 1.56. In function notation,  $h$  and  $f$  are related as follows.

$$\begin{aligned} h(x) &= x^2 + 2 \\ &= f(x) + 2 \quad \text{Upward shift of two units} \end{aligned}$$

Similarly, you can obtain the graph of

$$g(x) = (x - 2)^2$$

by shifting the graph of  $f(x) = x^2$  two units to the *right*, as shown in Figure 1.57. In this case, the functions  $g$  and  $f$  have the following relationship.

$$\begin{aligned} g(x) &= (x - 2)^2 \\ &= f(x - 2) \quad \text{Right shift of two units} \end{aligned}$$

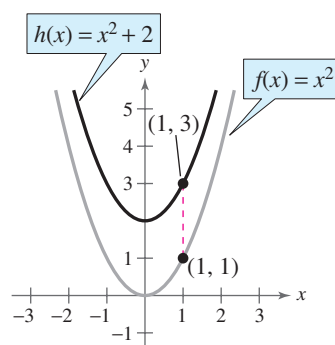


Figure 1.56 Vertical shift upward: two units

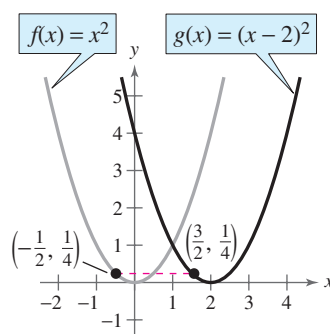


Figure 1.57 Horizontal shift to the right: two units

The following list summarizes vertical and horizontal shifts.

### Vertical and Horizontal Shifts

Let  $c$  be a positive real number. **Vertical and horizontal shifts** in the graph of  $y = f(x)$  are represented as follows.

- Vertical shift  $c$  units *upward*:  $h(x) = f(x) + c$
- Vertical shift  $c$  units *downward*:  $h(x) = f(x) - c$
- Horizontal shift  $c$  units to the *right*:  $h(x) = f(x - c)$
- Horizontal shift  $c$  units to the *left*:  $h(x) = f(x + c)$

In items 3 and 4, be sure you see that  $h(x) = f(x - c)$  corresponds to a *right* shift and  $h(x) = f(x + c)$  corresponds to a *left* shift for  $c > 0$ .

### Exploration

Use a graphing utility to display (in the same viewing window) the graphs of  $y = x^2 + c$ , where  $c = -2, 0, 2,$  and  $4$ . Use the results to describe the effect that  $c$  has on the graph.

Use a graphing utility to display (in the same viewing window) the graphs of  $y = (x + c)^2$ , where  $c = -2, 0, 2,$  and  $4$ . Use the results to describe the effect that  $c$  has on the graph.

You might also wish to illustrate simple transformations of functions numerically, using tables to emphasize what happens to individual ordered pairs. For instance, suppose you have

$$f(x) = x^2, \quad h(x) = x^2 + 2 = f(x) + 2$$

and

$$g(x) = (x - 2)^2 = f(x - 2).$$

You can illustrate these transformations with the following tables.

$x$	$f(x)$	$h(x) = f(x) + 2$
-2	4	$4 + 2 = 6$
-1	1	$1 + 2 = 3$
0	0	$0 + 2 = 2$
1	1	$1 + 2 = 3$
2	4	$4 + 2 = 6$

$x$	$x - 2$	$g(x) = f(x - 2)$
0	$0 - 2 = -2$	4
1	$1 - 2 = -1$	1
2	$2 - 2 = 0$	0
3	$3 - 2 = 1$	1
4	$4 - 2 = 2$	4

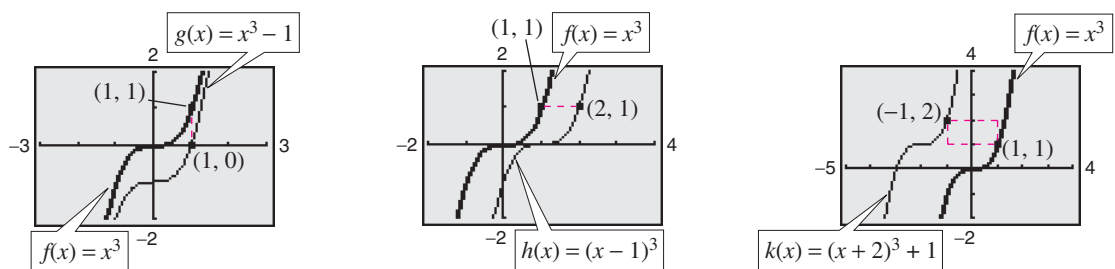
**Example 1** Shifts in the Graph of a Function

Compare the graph of each function with the graph of  $f(x) = x^3$ .

- a.  $g(x) = x^3 - 1$     b.  $h(x) = (x - 1)^3$     c.  $k(x) = (x + 2)^3 + 1$

**Solution**

- a. Graph  $f(x) = x^3$  and  $g(x) = x^3 - 1$  [see Figure 1.58(a)]. You can obtain the graph of  $g$  by shifting the graph of  $f$  one unit downward.
- b. Graph  $f(x) = x^3$  and  $h(x) = (x - 1)^3$  [see Figure 1.58(b)]. You can obtain the graph of  $h$  by shifting the graph of  $f$  one unit to the right.
- c. Graph  $f(x) = x^3$  and  $k(x) = (x + 2)^3 + 1$  [see Figure 1.58(c)]. You can obtain the graph of  $k$  by shifting the graph of  $f$  two units to the left and then one unit upward.



(a) Vertical shift: one unit downward

(b) Horizontal shift: one unit right

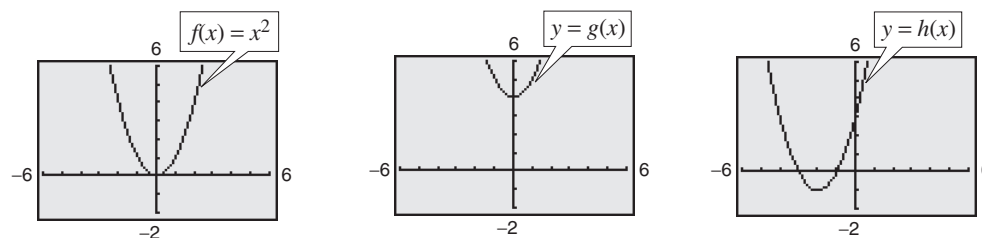
(c) Two units left and one unit upward

**Figure 1.58**

**CHECKPOINT** Now try Exercise 23.

**Example 2** Finding Equations from Graphs

The graph of  $f(x) = x^2$  is shown in Figure 1.59. Each of the graphs in Figure 1.60 is a transformation of the graph of  $f$ . Find an equation for each function.

**Figure 1.59****Figure 1.60****Solution**

- a. The graph of  $g$  is a vertical shift of four units upward of the graph of  $f(x) = x^2$ . So, the equation for  $g$  is  $g(x) = x^2 + 4$ .
- b. The graph of  $h$  is a horizontal shift of two units to the left, and a vertical shift of one unit downward, of the graph of  $f(x) = x^2$ . So, the equation for  $h$  is  $h(x) = (x + 2)^2 - 1$ .

**CHECKPOINT** Now try Exercise 17.

## Reflecting Graphs

Another common type of transformation is called a **reflection**. For instance, if you consider the  $x$ -axis to be a mirror, the graph of  $h(x) = -x^2$  is the mirror image (or reflection) of the graph of  $f(x) = x^2$  (see Figure 1.61).

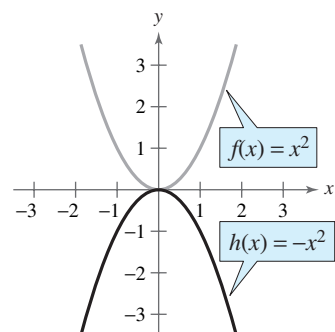


Figure 1.61

### Exploration

Compare the graph of each function with the graph of  $f(x) = x^2$  by using a graphing utility to graph the function and  $f$  in the same viewing window. Describe the transformation.

- $g(x) = -x^2$
- $h(x) = (-x)^2$

### Reflections in the Coordinate Axes

Reflections in the coordinate axes of the graph of  $y = f(x)$  are represented as follows.

- Reflection in the  $x$ -axis:  $h(x) = -f(x)$
- Reflection in the  $y$ -axis:  $h(x) = f(-x)$

### Example 3 Finding Equations from Graphs

The graph of  $f(x) = x^4$  is shown in Figure 1.62. Each of the graphs in Figure 1.63 is a transformation of the graph of  $f$ . Find an equation for each function.

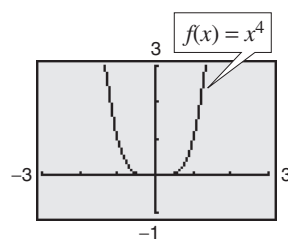
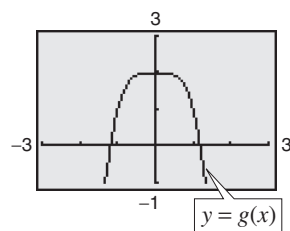
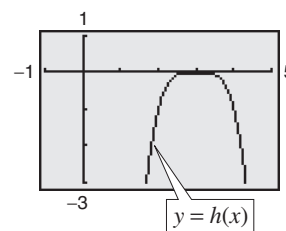


Figure 1.62

(a)  
Figure 1.63

(b)

### Solution

- The graph of  $g$  is a reflection in the  $x$ -axis followed by an upward shift of two units of the graph of  $f(x) = x^4$ . So, the equation for  $g$  is  $g(x) = -x^4 + 2$ .
- The graph of  $h$  is a horizontal shift of three units to the right followed by a reflection in the  $x$ -axis of the graph of  $f(x) = x^4$ . So, the equation for  $h$  is  $h(x) = -(x - 3)^4$ .

**CHECKPOINT** Now try Exercise 19.

**Example 4 Reflections and Shifts**

Compare the graph of each function with the graph of  $f(x) = \sqrt{x}$ .

- a.  $g(x) = -\sqrt{x}$     b.  $h(x) = \sqrt{-x}$     c.  $k(x) = -\sqrt{x+2}$

**Algebraic Solution**

- a. Relative to the graph of  $f(x) = \sqrt{x}$ , the graph of  $g$  is a reflection in the  $x$ -axis because

$$\begin{aligned} g(x) &= -\sqrt{x} \\ &= -f(x). \end{aligned}$$

- b. The graph of  $h$  is a reflection of the graph of  $f(x) = \sqrt{x}$  in the  $y$ -axis because

$$\begin{aligned} h(x) &= \sqrt{-x} \\ &= f(-x). \end{aligned}$$

- c. From the equation

$$\begin{aligned} k(x) &= -\sqrt{x+2} \\ &= -f(x+2) \end{aligned}$$

you can conclude that the graph of  $k$  is a left shift of two units, followed by a reflection in the  $x$ -axis, of the graph of  $f(x) = \sqrt{x}$ .

**Graphical Solution**

- a. Use a graphing utility to graph  $f$  and  $g$  in the same viewing window. From the graph in Figure 1.64, you can see that the graph of  $g$  is a reflection of the graph of  $f$  in the  $x$ -axis.
- b. Use a graphing utility to graph  $f$  and  $h$  in the same viewing window. From the graph in Figure 1.65, you can see that the graph of  $h$  is a reflection of the graph of  $f$  in the  $y$ -axis.
- c. Use a graphing utility to graph  $f$  and  $k$  in the same viewing window. From the graph in Figure 1.66, you can see that the graph of  $k$  is a left shift of two units of the graph of  $f$ , followed by a reflection in the  $x$ -axis.

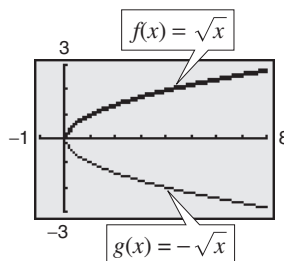


Figure 1.64

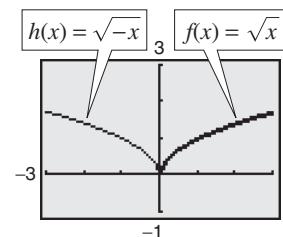


Figure 1.65

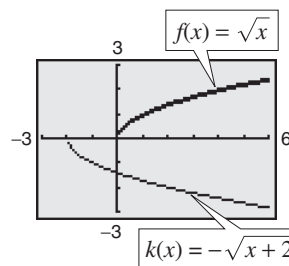


Figure 1.66

**CHECKPOINT** Now try Exercise 21.

When graphing functions involving square roots, remember that the domain must be restricted to exclude negative numbers inside the radical. For instance, here are the domains of the functions in Example 4.

$$\text{Domain of } g(x) = -\sqrt{x}: \quad x \geq 0$$

$$\text{Domain of } h(x) = \sqrt{-x}: \quad x \leq 0$$

$$\text{Domain of } k(x) = -\sqrt{x+2}: \quad x \geq -2$$

**Activity**

Does the graph of  $f(x) = -(x+1)^3 + 4$  represent a horizontal shift of one unit to the left, followed by a vertical shift of four units upward, followed by a reflection in the  $x$ -axis?

**Answer:** No, it represents a horizontal shift of one unit to the left, followed by a reflection in the  $x$ -axis, followed by a vertical shift of four units upward.

## Nonrigid Transformations

Horizontal shifts, vertical shifts, and reflections are called **rigid transformations** because the basic shape of the graph is unchanged. These transformations change only the *position* of the graph in the coordinate plane. **Nonrigid transformations** are those that cause a *distortion*—a change in the shape of the original graph. For instance, a nonrigid transformation of the graph of  $y = f(x)$  is represented by  $y = cf(x)$ , where the transformation is a **vertical stretch** if  $c > 1$  and a **vertical shrink** if  $0 < c < 1$ . Another nonrigid transformation of the graph of  $y = f(x)$  is represented by  $h(x) = f(cx)$ , where the transformation is a **horizontal shrink** if  $c > 1$  and a **horizontal stretch** if  $0 < c < 1$ .

### Example 5 Nonrigid Transformations

Compare the graph of each function with the graph of  $f(x) = |x|$ .

- $h(x) = 3|x|$
- $g(x) = \frac{1}{3}|x|$

#### Solution

- Relative to the graph of  $f(x) = |x|$ , the graph of

$$\begin{aligned} h(x) &= 3|x| \\ &= 3f(x) \end{aligned}$$

is a vertical stretch (each  $y$ -value is multiplied by 3) of the graph of  $f$ . (See Figure 1.67.)

- Similarly, the graph of

$$\begin{aligned} g(x) &= \frac{1}{3}|x| \\ &= \frac{1}{3}f(x) \end{aligned}$$

is a vertical shrink (each  $y$ -value is multiplied by  $\frac{1}{3}$ ) of the graph of  $f$ . (See Figure 1.68.)

**CHECKPOINT** Now try Exercise 31.

### Example 6 Nonrigid Transformations

Compare the graph of  $h(x) = f(\frac{1}{2}x)$  with the graph of  $f(x) = 2 - x^3$ .

#### Solution

Relative to the graph of  $f(x) = 2 - x^3$ , the graph of

$$h(x) = f\left(\frac{1}{2}x\right) = 2 - \left(\frac{1}{2}x\right)^3 = 2 - \frac{1}{8}x^3$$

is a horizontal stretch (each  $x$ -value is multiplied by 2) of the graph of  $f$ . (See Figure 1.69.)

**CHECKPOINT** Now try Exercise 39.

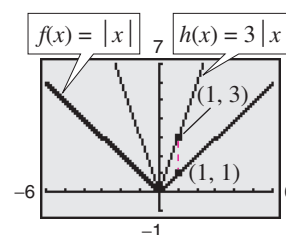


Figure 1.67

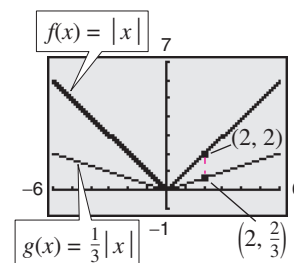


Figure 1.68

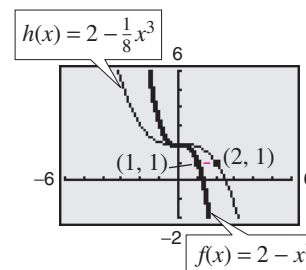


Figure 1.69

## 1.5 Exercises

See [www.CalcChat.com](http://www.CalcChat.com) for worked-out solutions to odd-numbered exercises.

## Vocabulary Check

In Exercises 1–5, fill in the blanks.

- The graph of a \_\_\_\_\_ is U-shaped.
- The graph of an \_\_\_\_\_ is V-shaped.
- Horizontal shifts, vertical shifts, and reflections are called \_\_\_\_\_.
- A reflection in the  $x$ -axis of  $y = f(x)$  is represented by  $h(x) = \underline{\hspace{2cm}}$ , while a reflection in the  $y$ -axis of  $y = f(x)$  is represented by  $h(x) = \underline{\hspace{2cm}}$ .
- A nonrigid transformation of  $y = f(x)$  represented by  $cf(x)$  is a vertical stretch if \_\_\_\_\_ and a vertical shrink if \_\_\_\_\_.
- Match the rigid transformation of  $y = f(x)$  with the correct representation, where  $c > 0$ .
 

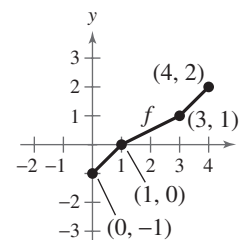
(a) $h(x) = f(x) + c$	(i) horizontal shift $c$ units to the left
(b) $h(x) = f(x) - c$	(ii) vertical shift $c$ units upward
(c) $h(x) = f(x - c)$	(iii) horizontal shift $c$ units to the right
(d) $h(x) = f(x + c)$	(iv) vertical shift $c$ units downward

In Exercises 1–12, sketch the graphs of the three functions by hand on the same rectangular coordinate system. Verify your result with a graphing utility.

- |   |  |
|---|--|
| <ol style="list-style-type: none"> <li><math>f(x) = x</math><br/><math>g(x) = x - 4</math><br/><math>h(x) = 3x</math></li> <li><math>f(x) = x^2</math><br/><math>g(x) = x^2 + 2</math><br/><math>h(x) = (x - 2)^2</math></li> <li><math>f(x) = -x^2</math><br/><math>g(x) = -x^2 + 1</math><br/><math>h(x) = -(x - 2)^2</math></li> <li><math>f(x) = x^2</math><br/><math>g(x) = \frac{1}{2}x^2</math><br/><math>h(x) = (2x)^2</math></li> <li><math>f(x) =  x </math><br/><math>g(x) =  x  - 1</math><br/><math>h(x) =  x - 3 </math></li> <li><math>f(x) = \sqrt{x}</math><br/><math>g(x) = \sqrt{x + 1}</math><br/><math>h(x) = \sqrt{x - 2} + 1</math></li> </ol> | <ol style="list-style-type: none"> <li><math>f(x) = \frac{1}{2}x</math><br/><math>g(x) = \frac{1}{2}x + 2</math><br/><math>h(x) = \frac{1}{2}(x - 2)</math></li> <li><math>f(x) = x^2</math><br/><math>g(x) = x^2 - 4</math><br/><math>h(x) = (x + 2)^2 + 1</math></li> <li><math>f(x) = (x - 2)^2</math><br/><math>g(x) = (x + 2)^2 + 2</math><br/><math>h(x) = -(x - 2)^2 - 1</math></li> <li><math>f(x) = x^2</math><br/><math>g(x) = \frac{1}{4}x^2 + 2</math><br/><math>h(x) = -\frac{1}{4}x^2</math></li> <li><math>f(x) =  x </math><br/><math>g(x) =  x + 3 </math><br/><math>h(x) = -2 x + 2  - 1</math></li> <li><math>f(x) = \sqrt{x}</math><br/><math>g(x) = \frac{1}{2}\sqrt{x}</math><br/><math>h(x) = -\sqrt{x + 4}</math></li> </ol> |
|---|--|

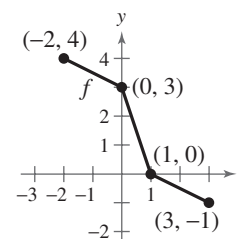
13. Use the graph of  $f$  to sketch each graph. To print an enlarged copy of the graph, go to the website [www.mathgraphs.com](http://www.mathgraphs.com).

- $y = f(x) + 2$
- $y = -f(x)$
- $y = f(x - 2)$
- $y = f(x + 3)$
- $y = 2f(x)$
- $y = f(-x)$
- $y = f(\frac{1}{2}x)$



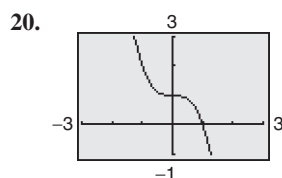
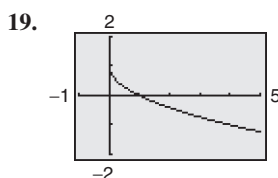
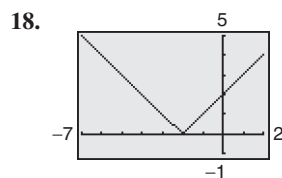
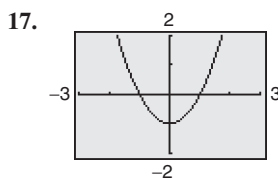
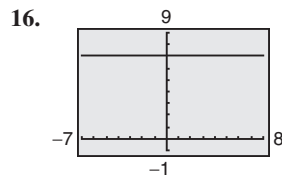
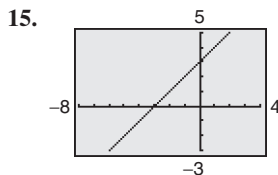
14. Use the graph of  $f$  to sketch each graph. To print an enlarged copy of the graph, go to the website [www.mathgraphs.com](http://www.mathgraphs.com).

- $y = f(x) - 1$
- $y = f(x + 1)$
- $y = f(x - 1)$
- $y = -f(x - 2)$
- $y = f(-x)$
- $y = \frac{1}{2}f(x)$
- $y = f(2x)$



## 134 Chapter 1 Functions and Their Graphs

In Exercises 15–20, identify the parent function and describe the transformation shown in the graph. Write an equation for the graphed function.



In Exercises 21–26, compare the graph of the function with the graph of  $f(x) = \sqrt{x}$ .

21.  $y = -\sqrt{x} - 1$

22.  $y = \sqrt{x} + 2$

23.  $y = \sqrt{x - 2}$

24.  $y = \sqrt{x + 4}$

25.  $y = 2\sqrt{x}$

26.  $y = \sqrt{-x + 3}$

In Exercises 27–32, compare the graph of the function with the graph of  $f(x) = |x|$ .

27.  $y = |x + 5|$

28.  $y = |x| - 3$

29.  $y = -|x|$

30.  $y = |-x|$

31.  $y = 4|x|$

32.  $y = \left|\frac{1}{2}x\right|$

In Exercises 33–38, compare the graph of the function with the graph of  $f(x) = x^3$ .

33.  $g(x) = 4 - x^3$

34.  $g(x) = -(x - 1)^3$

35.  $h(x) = \frac{1}{4}(x + 2)^3$

36.  $h(x) = -2(x - 1)^3 + 3$

37.  $p(x) = \left(\frac{1}{3}x\right)^3 + 2$

38.  $p(x) = [3(x - 2)]^3$

In Exercises 39–42, use a graphing utility to graph the three functions in the same viewing window. Describe the graphs of  $g$  and  $h$  relative to the graph of  $f$ .

39.  $f(x) = x^3 - 3x^2$

40.  $f(x) = x^3 - 3x^2 + 2$

$g(x) = f(x + 2)$

$g(x) = f(x - 1)$

$h(x) = \frac{1}{2}f(x)$

$h(x) = f(3x)$

41.  $f(x) = x^3 - 3x^2$

$g(x) = -\frac{1}{3}f(x)$

$h(x) = f(-x)$

42.  $f(x) = x^3 - 3x^2 + 2$

$g(x) = -f(x)$

$h(x) = f(2x)$

In Exercises 43–56,  $g$  is related to one of the six parent functions on page 127. (a) Identify the parent function  $f$ . (b) Describe the sequence of transformations from  $f$  to  $g$ . (c) Sketch the graph of  $g$  by hand. (d) Use function notation to write  $g$  in terms of the parent function  $f$ .

43.  $g(x) = 2 - (x + 5)^2$

44.  $g(x) = -(x + 10)^2 + 5$

45.  $g(x) = 3 + 2(x - 4)^2$

46.  $g(x) = -\frac{1}{4}(x + 2)^2 - 2$

47.  $g(x) = 3(x - 2)^3$

48.  $g(x) = -\frac{1}{2}(x + 1)^3$

49.  $g(x) = (x - 1)^3 + 2$

50.  $g(x) = -(x + 3)^3 - 10$

51.  $g(x) = |x + 4| + 8$

52.  $g(x) = |x + 3| + 9$

53.  $g(x) = -2|x - 1| - 4$

54.  $g(x) = \frac{1}{2}|x - 2| - 3$

55.  $g(x) = -\frac{1}{2}\sqrt{x + 3} - 1$

56.  $g(x) = -\sqrt{x + 1} - 6$

57. **Fuel Use** The amounts of fuel  $F$  (in billions of gallons) used by vans, pickups, and SUVs (sport utility vehicles) from 1990 through 2003 are shown in the table. A model for the data can be approximated by the function  $F(t) = 33.0 + 6.2\sqrt{t}$ , where  $t = 0$  represents 1990. (Source: U.S. Federal Highway Administration)



Year	Annual fuel use, $F$ (in billions of gallons)
1990	35.6
1991	38.2
1992	40.9
1993	42.9
1994	44.1
1995	45.6
1996	47.4
1997	49.4
1998	50.5
1999	52.8
2000	52.9
2001	53.5
2002	55.2
2003	56.3

- (a) Describe the transformation of the parent function  $f(t) = \sqrt{t}$ .
- (b) Use a graphing utility to graph the model and the data in the same viewing window.
- (c) Rewrite the function so that  $t = 0$  represents 2003. Explain how you got your answer.



## Section 1.5 Shifting, Reflecting, and Stretching Graphs 135

58. **Finance** The amounts  $M$  (in billions of dollars) of home mortgage debt outstanding in the United States from 1990 through 2004 can be approximated by the function

$$M(t) = 32.3t^2 + 3769$$

where  $t = 0$  represents 1990. (Source: Board of Governors of the Federal Reserve System)

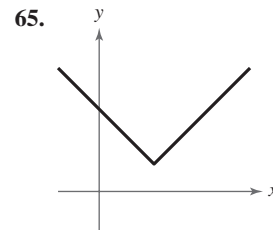
- Describe the transformation of the parent function  $f(t) = t^2$ .
- Use a graphing utility to graph the model over the interval  $0 \leq t \leq 14$ .
- According to the model, when will the amount of debt exceed 10 trillion dollars?
- Rewrite the function so that  $t = 0$  represents 2000. Explain how you got your answer.

### Synthesis

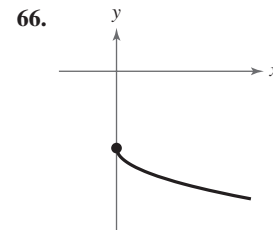
**True or False?** In Exercises 59 and 60, determine whether the statement is true or false. Justify your answer.

- The graph of  $y = f(-x)$  is a reflection of the graph of  $y = f(x)$  in the  $x$ -axis.
- The graph of  $y = -f(x)$  is a reflection of the graph of  $y = f(x)$  in the  $y$ -axis.
- Exploration** Use the fact that the graph of  $y = f(x)$  has  $x$ -intercepts at  $x = 2$  and  $x = -3$  to find the  $x$ -intercepts of the given graph. If not possible, state the reason.
  - $y = f(-x)$
  - $y = -f(x)$
  - $y = 2f(x)$
  - $y = f(x) + 2$
  - $y = f(x - 3)$
- Exploration** Use the fact that the graph of  $y = f(x)$  has  $x$ -intercepts at  $x = -1$  and  $x = 4$  to find the  $x$ -intercepts of the given graph. If not possible, state the reason.
  - $y = f(-x)$
  - $y = -f(x)$
  - $y = 2f(x)$
  - $y = f(x) - 1$
  - $y = f(x - 2)$
- Exploration** Use the fact that the graph of  $y = f(x)$  is increasing on the interval  $(-\infty, 2)$  and decreasing on the interval  $(2, \infty)$  to find the intervals on which the graph is increasing and decreasing. If not possible, state the reason.
  - $y = f(-x)$
  - $y = -f(x)$
  - $y = 2f(x)$
  - $y = f(x) - 3$
  - $y = f(x + 1)$
- Exploration** Use the fact that the graph of  $y = f(x)$  is increasing on the intervals  $(-\infty, -1)$  and  $(2, \infty)$  and decreasing on the interval  $(-1, 2)$  to find the intervals on which the graph is increasing and decreasing. If not possible, state the reason.
  - $y = f(-x)$
  - $y = -f(x)$
  - $y = \frac{1}{2}f(x)$
  - $y = -f(x - 1)$
  - $y = f(x - 2) + 1$

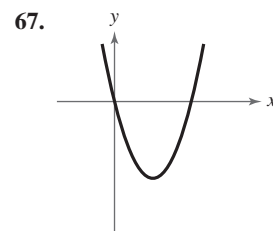
**Library of Parent Functions** In Exercises 65–68, determine which equation(s) may be represented by the graph shown. There may be more than one correct answer.



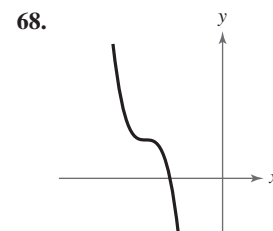
- $f(x) = |x + 2| + 1$
- $f(x) = |x - 1| + 2$
- $f(x) = |x - 2| + 1$
- $f(x) = 2 + |x - 2|$
- $f(x) = |(x - 2) + 1|$
- $f(x) = 1 - |x - 2|$



- $f(x) = -\sqrt{x} - 4$
- $f(x) = -4 - \sqrt{x}$
- $f(x) = -4 - \sqrt{-x}$
- $f(x) = \sqrt{-x} - 4$
- $f(x) = \sqrt{-x} + 4$
- $f(x) = \sqrt{x} - 4$



- $f(x) = (x - 2)^2 - 2$
- $f(x) = (x + 4)^2 - 4$
- $f(x) = (x - 2)^2 - 4$
- $f(x) = (x + 2)^2 - 4$
- $f(x) = 4 - (x - 2)^2$
- $f(x) = 4 - (x + 2)^2$



- $f(x) = -(x - 4)^3 + 2$
- $f(x) = -(x + 4)^3 + 2$
- $f(x) = -(x - 2)^3 + 4$
- $f(x) = (-x - 4)^3 + 2$
- $f(x) = (x + 4)^3 + 2$
- $f(x) = (-x + 4)^3 + 2$

### Skills Review

In Exercises 69 and 70, determine whether the lines  $L_1$  and  $L_2$  passing through the pairs of points are parallel, perpendicular, or neither.

- $L_1: (-2, -2), (2, 10)$   
 $L_2: (-1, 3), (3, 9)$
- $L_1: (-1, -7), (4, 3)$   
 $L_2: (1, 5), (-2, -7)$

In Exercises 71–74, find the domain of the function.

- $f(x) = \frac{4}{9 - x}$
- $f(x) = \frac{\sqrt{x - 5}}{x - 7}$
- $f(x) = \sqrt{100 - x^2}$
- $f(x) = \sqrt[3]{16 - x^2}$