What Did You Learn?

Key Terms

equation, *p. 77*
solution point, *p. 77*
intercepts, *p. 78*
slope, *p. 88*
point-slope form, *p. 90*
slope-intercept form, *p. 92*
parallel lines, *p. 94*
perpendicular lines, *p. 94*
function, *p. 101*
domain, *p. 101*
range, *p. 101*
independent variable, *p. 103*
dependent variable, *p. 103*
function notation, *p. 103*
graph of a function, *p. 115*
Vertical Line Test, *p. 116*
even function, *p. 121*
odd function, *p. 121*
rigid transformation, *p. 132*
inverse function, *p. 147*
one-to-one, *p. 151*
Horizontal Line Test, *p. 151*

Key Concepts

1.1 ■ Sketch graphs of equations

1. To sketch a graph by point plotting, rewrite the equation to isolate one of the variables on one side of the equation, make a table of values, plot these points on a rectangular coordinate system, and connect the points with a smooth curve or line.

2. To graph an equation using a graphing utility, rewrite the equation so that y is isolated on one side, enter the equation in the graphing utility, determine a viewing window that shows all important features, and graph the equation.

1.2 ■ Find and use the slopes of lines to write and graph linear equations

1. The slope m of the nonvertical line through (x_1, y_1) and (x_2, y_2), where $x_1 \neq x_2$, is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}.$$

2. The point-slope form of the equation of the line that passes through the point (x_1, y_1) and has a slope of m is $y - y_1 = m(x - x_1)$.

3. The graph of the equation $y = mx + b$ is a line whose slope is m and whose y-intercept is $(0, b)$.

1.3 ■ Evaluate functions and find their domains

1. To evaluate a function $f(x)$, replace the independent variable x with a value and simplify the expression.

2. The domain of a function is the set of all real numbers for which the function is defined.

1.4 ■ Analyze graphs of functions

1. The graph of a function may have intervals over which the graph increases, decreases, or is constant.

2. The points at which a function changes its increasing, decreasing, or constant behavior are the relative minimum and relative maximum values of the function.

3. An even function is symmetric with respect to the y-axis. An odd function is symmetric with respect to the origin.

1.5 ■ Identify and graph shifts, reflections, and nonrigid transformations of functions

1. Vertical and horizontal shifts of a graph are transformations in which the graph is shifted left, right, upward, or downward.

2. A reflection transformation is a mirror image of a graph in a line.

3. A nonrigid transformation distorts the graph by stretching or shrinking the graph horizontally or vertically.

1.6 ■ Find arithmetic combinations and compositions of functions

1. An arithmetic combination of functions is the sum, difference, product, or quotient of two functions. The domain of the arithmetic combination is the set of all real numbers that are common to the two functions.

2. The composition of the function f with the function g is $(f \circ g)(x) = f(g(x))$. The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f.

1.7 ■ Find inverse functions

1. If the point (a, b) lies on the graph of f, then the point (b, a) must lie on the graph of its inverse function f^{-1}, and vice versa. This means that the graph of f^{-1} is a reflection of the graph of f in the line $y = x$.

2. Use the Horizontal Line Test to decide if f has an inverse function. To find an inverse function algebraically, replace $f(x)$ by y, interchange the roles of x and y and solve for y, and replace y by $f^{-1}(x)$ in the new equation.

Review Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

1.1 In Exercises 1–4, complete the table. Use the resulting solution points to sketch the graph of the equation. Use a graphing utility to verify the graph.

1. $y = -\frac{1}{2}x + 2$

x	-2	0	2	3	4
y					
Solution point					

2. $y = x^2 - 3x$

x	-1	0	1	2	3
y					
Solution point					

3. $y = 4 - x^2$

x	-2	-1	0	1	2
y					
Solution point					

4. $y = \sqrt{x - 1}$

x	1	2	3	10	17
y					
Solution point					

In Exercises 5–12, use a graphing utility to graph the equation. Approximate any *x*- or *y*-intercepts.

5. $y = \frac{1}{4}(x + 1)^3$

6. $y = 4 - (x - 4)^2$

7. $y = \frac{1}{4}x^4 - 2x^2$

8. $y = \frac{1}{4}x^3 - 3x$

9. $y = x\sqrt{9 - x^2}$

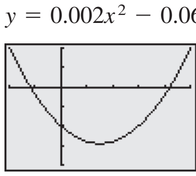
10. $y = x\sqrt{x + 3}$

11. $y = |x - 4| - 4$

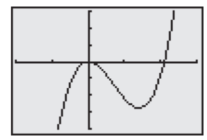
12. $y = |x + 2| + |3 - x|$

In Exercises 13 and 14, describe the viewing window of the graph shown.

13. $y = 0.002x^2 - 0.06x - 1$



14. $y = 10x^3 - 21x^2$



15. ***Consumerism*** You purchase a compact car for \$13,500. The depreciated value y after t years is

$y = 13{,}500 - 1100t,\ 0 \le t \le 6.$

(a) Use the constraints of the model to determine an appropriate viewing window.

(b) Use a graphing utility to graph the equation.

(c) Use the *zoom* and *trace* features of a graphing utility to determine the value of t when $y = \$9100$.

16. ***Data Analysis*** The table shows the sales for Best Buy from 1995 to 2004. (Source: Best Buy Company, Inc.)

Year	Sales, S (in billions of dollars)
1995	7.22
1996	7.77
1997	8.36
1998	10.08
1999	12.49
2000	15.33
2001	19.60
2002	20.95
2003	24.55
2004	27.43

A model for the data is $S = 0.1625t^2 - 0.702t + 6.04$, where S represents the sales (in billions of dollars) and t is the year, with $t = 5$ corresponding to 1995.

(a) Use the model and the *table* feature of a graphing utility to approximate the sales for Best Buy from 1995 to 2004.

(b) Use a graphing utility to graph the model and plot the data in the same viewing window. How well does the model fit the data?

(c) Use the model to predict the sales for the years 2008 and 2010. Do the values seem reasonable? Explain.

(d) Use the *zoom* and *trace* features to determine when sales exceeded 20 billion dollars. Confirm your result algebraically.

(e) According to the model, will sales ever reach 50 billion? If so, when?

1.2 In Exercises 17–22, plot the two points and find the slope of the line passing through the points.

17. $(-3, 2), (8, 2)$

18. $(7, -1), (7, 12)$

19. $\left(\frac{3}{2}, 1\right), \left(5, \frac{5}{2}\right)$

20. $\left(-\frac{3}{4}, \frac{5}{6}\right), \left(\frac{1}{2}, -\frac{5}{2}\right)$

21. $(-4.5, 6), (2.1, 3)$

22. $(-2.7, -6.3), (-1, -1.2)$

In Exercises 23–32, use the point on the line and the slope of the line to find the general form of the equation of the line, and find three additional points through which the line passes. (There are many correct answers.)

	Point	*Slope*
23.	$(2, -1)$	$m = \frac{1}{4}$
24.	$(-3, 5)$	$m = -\frac{3}{2}$
25.	$(0, -5)$	$m = \frac{3}{2}$
26.	$(3, 0)$	$m = -\frac{2}{3}$
27.	$\left(\frac{1}{5}, -5\right)$	$m = -1$
28.	$\left(0, \frac{7}{8}\right)$	$m = -\frac{4}{5}$
29.	$(-2, 6)$	$m = 0$
30.	$(-8, 8)$	$m = 0$
31.	$(10, -6)$	m is undefined.
32.	$(5, 4)$	m is undefined.

In Exercises 33–36, find the slope-intercept form of the equation of the line that passes through the points. Use a graphing utility to graph the line.

33. $(2, -1), (4, -1)$

34. $(0, 0), (0, 10)$

35. $(-1, 0), (6, 2)$

36. $(1, 6), (4, 2)$

Rate of Change **In Exercises 37 and 38, you are given the dollar value of a product in 2008 *and* the rate at which the value of the item is expected to change during the next 5 years. Use this information to write a linear equation that gives the dollar value V of the product in terms of the year t. (Let $t = 8$ represent 2008.)**

	2008 Value	*Rate*
37.	\$12,500	\$850 increase per year
38.	\$72.95	\$5.15 decrease per year

39. ***Sales*** During the second and third quarters of the year, an e-commerce business had sales of \$160,000 and \$185,000, respectively. The growth of sales follows a linear pattern. Estimate sales during the fourth quarter.

40. ***Depreciation*** The dollar value of a DVD player in 2006 is \$225. The product will decrease in value at an expected rate of \$12.75 per year.

(a) Write a linear equation that gives the dollar value V of the DVD player in terms of the year t. (Let $t = 6$ represent 2006.)

(b) Use a graphing utility to graph the equation found in part (a). Be sure to choose an appropriate viewing window. State the dimensions of your viewing window, and explain why you chose the values that you did.

(c) Use the *value* or *trace* feature of your graphing utility to estimate the dollar value of the DVD player in 2010. Confirm your answer algebraically.

(d) According to the model, when will the DVD player have no value?

In Exercises 41–44, write the slope-intercept forms of the equations of the lines through the given point (a) parallel to the given line and (b) perpendicular to the given line. Verify your result with a graphing utility (use a *square setting*).

	Point	*Line*
41.	$(3, -2)$	$5x - 4y = 8$
42.	$(-8, 3)$	$2x + 3y = 5$
43.	$(-6, 2)$	$x = 4$
44.	$(3, -4)$	$y = 2$

1.3 In Exercises 45 and 46, which sets of ordered pairs represent functions from A to B? Explain.

45. $A = \{10, 20, 30, 40\}$ and $B = \{0, 2, 4, 6\}$

(a) $\{(20, 4), (40, 0), (20, 6), (30, 2)\}$

(b) $\{(10, 4), (20, 4), (30, 4), (40, 4)\}$

(c) $\{(40, 0), (30, 2), (20, 4), (10, 6)\}$

(d) $\{(20, 2), (10, 0), (40, 4)\}$

46. $A = \{u, v, w\}$ and $B = \{-2, -1, 0, 1, 2\}$

(a) $\{(v, -1), (u, 2), (w, 0), (u, -2)\}$

(b) $\{(u, -2), (v, 2), (w, 1)\}$

(c) $\{(u, 2), (v, 2), (w, 1), (w, 1)\}$

(d) $\{(w, -2), (v, 0), (w, 2)\}$

In Exercises 47–50, determine whether the equation represents y as a function of x.

47. $16x^2 - y^2 = 0$

48. $2x - y - 3 = 0$

49. $y = \sqrt{1 - x}$

50. $|y| = x + 2$

In Exercises 51–54, evaluate the function at each specified value of the independent variable, and simplify.

51. $f(x) = x^2 + 1$
(a) $f(1)$ (b) $f(-3)$
(c) $f(b^3)$ (d) $f(x - 1)$

52. $g(x) = x^{4/3}$
(a) $g(8)$ (b) $g(t + 1)$
(c) $g(-27)$ (d) $g(-x)$

53. $h(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 + 2, & x > -1 \end{cases}$
(a) $h(-2)$ (b) $h(-1)$
(c) $h(0)$ (d) $h(2)$

54. $f(x) = \dfrac{3}{2x - 5}$
(a) $f(1)$ (b) $f(-2)$
(c) $f(t)$ (d) $f(10)$

In Exercises 55–60, find the domain of the function.

55. $f(x) = \dfrac{x - 1}{x + 2}$ **56.** $f(x) = \dfrac{x^2}{x^2 + 1}$

57. $f(x) = \sqrt{25 - x^2}$ **58.** $f(x) = \sqrt{x^2 - 16}$

59. $g(s) = \dfrac{5s + 5}{3s - 9}$ **60.** $f(x) = \dfrac{2x + 1}{3x + 4}$

61. ***Cost*** A hand tool manufacturer produces a product for which the variable cost is \$5.35 per unit and the fixed costs are \$16,000. The company sells the product for \$8.20 and can sell all that it produces.

(a) Write the total cost C as a function of x, the number of units produced.

(b) Write the profit P as a function of x.

62. ***Consumerism*** The retail sales R (in billions of dollars) of lawn care products and services in the United States from 1997 to 2004 can be approximated by the model

$$R(t) = \begin{cases} 0.126t^2 - 0.89t + 6.8, & 7 \le t < 11 \\ 0.1442t^3 - 5.611t^2 + 71.10t - 282.4, & 11 \le t \le 14 \end{cases}$$

where t represents the year, with $t = 7$ corresponding to 1997. Use the *table* feature of a graphing utility to approximate the retail sales of lawn care products and services for each year from 1997 to 2004. (Source: The National Gardening Association)

In Exercises 63 and 64, find the difference quotient and simplify your answer.

63. $f(x) = 2x^2 + 3x - 1,\quad \dfrac{f(x + h) - f(x)}{h},\quad h \ne 0$

64. $f(x) = x^3 - 5x^2 + x,\quad \dfrac{f(x + h) - f(x)}{h},\quad h \ne 0$

1.4 In Exercises 65–68, use a graphing utility to graph the function and estimate its domain and range. Then find the domain and range algebraically.

65. $f(x) = 3 - 2x^2$ **66.** $f(x) = \sqrt{2x^2 - 1}$

67. $h(x) = \sqrt{36 - x^2}$ **68.** $g(x) = |x + 5|$

In Exercises 69–72, (a) use a graphing utility to graph the equation and (b) use the Vertical Line Test to determine whether y is a function of x.

69. $y = \dfrac{x^2 + 3x}{6}$ **70.** $y = -\dfrac{2}{3}|x + 5|$

71. $3x + y^2 = 2$ **72.** $x^2 + y^2 = 49$

In Exercises 73–76, (a) use a graphing utility to graph the function and (b) determine the open intervals on which the function is increasing, decreasing, or constant.

73. $f(x) = x^3 - 3x$ **74.** $f(x) = \sqrt{x^2 - 9}$

75. $f(x) = x\sqrt{x - 6}$ **76.** $f(x) = \dfrac{|x + 8|}{2}$

In Exercises 77–80, use a graphing utility to approximate (to two decimal places) any relative minimum or relative maximum values of the function.

77. $f(x) = (x^2 - 4)^2$ **78.** $f(x) = x^2 - x - 1$

79. $h(x) = 4x^3 - x^4$ **80.** $f(x) = x^3 - 4x^2 - 1$

In Exercises 81–84, sketch the graph of the function by hand.

81. $f(x) = \begin{cases} 3x + 5, & x < 0 \\ x - 4, & x \ge 0 \end{cases}$

82. $f(x) = \begin{cases} x^2 + 7, & x < 1 \\ x^2 - 5x + 6, & x \ge 1 \end{cases}$

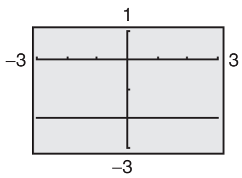
83. $f(x) = [\![x]\!] + 3$ **84.** $f(x) = [\![x + 2]\!]$

In Exercises 85 and 86, determine algebraically whether the function is even, odd, or neither. Verify your answer using a graphing utility.

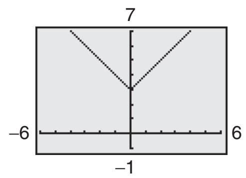
85. $f(x) = (x^2 - 8)^2$ **86.** $f(x) = 2x^3 - x^2$

1.5 In Exercises 87–90, identify the parent function and describe the transformation shown in the graph. Write an equation for the graphed function.

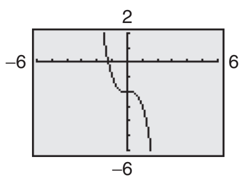
87.



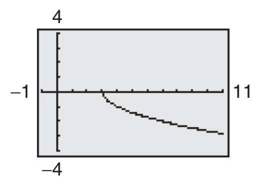
88.



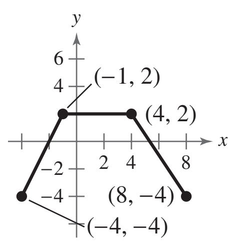
89.



90.



In Exercises 91–94, use the graph of $y = f(x)$ to graph the function.



91. $y = f(-x)$

92. $y = -f(x)$

93. $y = f(x) - 2$

94. $y = f(x - 1)$

In Exercises 95–100, h is related to one of the six parent functions on page 127. (a) Identify the parent function f. (b) Describe the sequence of transformations from f to h. (c) Sketch the graph of h by hand. (d) Use function notation to write h in terms of the parent function f.

95. $h(x) = |x| + 9$

96. $h(x) = (x - 2)^3 + 5$

97. $h(x) = -\sqrt{x} + 5$

98. $h(x) = -(x + 2)^2 - 8$

99. $h(x) = \frac{1}{2}(x - 3)^2 - 6$

100. $h(x) = 2\sqrt{x} + 5$

1.6 In Exercises 101–106, find two functions f and g such that $(f \circ g)(x) = h(x)$. (There are many correct answers.)

101. $h(x) = (x + 3)^2$

102. $h(x) = (1 - 2x)^3$

103. $h(x) = \sqrt{4x + 2}$

104. $h(x) = \sqrt[3]{(x + 2)^2}$

105. $h(x) = \dfrac{4}{x + 2}$

106. $h(x) = \dfrac{6}{(3x + 1)^3}$

Data Analysis **In Exercises 107 and 108, the numbers (in millions) of students taking the SAT (y_1) and ACT (y_2) for the years 1990 through 2004 can be modeled by $y_1 = 0.00204t^2 + 0.0015t + 1.021$ and $y_2 = 0.0274t + 0.785$, where t represents the year, with $t = 0$ corresponding to 1990.** (Source: College Entrance Examination Board and ACT, Inc.)

107. Use a graphing utility to graph y_1, y_2, and $y_1 + y_2$ in the same viewing window.

108. Use the model $y_1 + y_2$ to estimate the total number of students taking the SAT and ACT in 2008.

1.7 In Exercises 109 and 110, find the inverse function of f informally. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}f(x)) = x$.

109. $f(x) = \frac{1}{2}x + 3$

110. $f(x) = \dfrac{x - 4}{5}$

In Exercises 111 and 112, show that f and g are inverse functions (a) graphically and (b) numerically.

111. $f(x) = 3 - 4x, \quad g(x) = \dfrac{3 - x}{4}$

112. $f(x) = \sqrt{x + 1}, \quad g(x) = x^2 - 1, x \geq 0$

In Exercises 113–116, use a graphing utility to graph the function and use the Horizontal Line Test to determine whether the function is one-to-one and an inverse function exists.

113. $f(x) = \frac{1}{2}x - 3$

114. $f(x) = (x - 1)^2$

115. $h(t) = \dfrac{2}{t - 3}$

116. $g(x) = \sqrt{x + 6}$

In Exercises 117–122, find the inverse function of f algebraically.

117. $f(x) = \frac{1}{2}x - 5$

118. $f(x) = \dfrac{7x + 3}{8}$

119. $f(x) = 4x^3 - 3$

120. $f(x) = 5x^3 + 2$

121. $f(x) = \sqrt{x + 10}$

122. $f(x) = 4\sqrt{6 - x}$

Synthesis

True or False? **In Exercises 123–125, determine whether the statement is true or false. Justify your answer.**

123. If the graph of the common function $f(x) = x^2$ is moved six units to the right, moved three units upward, and reflected in the x-axis, then the point $(-1, 28)$ will lie on the graph of the transformation.

124. If $f(x) = x^n$ where n is odd, f^{-1} exists.

125. There exists no function f such that $f = f^{-1}$.

1 Chapter Test

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Take this test as you would take a test in class. After you are finished, check your work against the answers in the back of the book.

In Exercises 1–6, use the point-plotting method to graph the equation by hand and identify any *x*- and *y*-intercepts. Verify your results using a graphing utility.

1. $y = 2|x| - 1$

2. $y = 2x - \frac{8}{5}$

3. $y = 2x^2 - 4x$

4. $y = x^3 - x$

5. $y = -x^2 + 4$

6. $y = \sqrt{x - 2}$

7. Find equations of the lines that pass through the point $(0, 4)$ and are (a) parallel to and (b) perpendicular to the line $5x + 2y = 3$.

8. Find the slope-intercept form of the equation of the line that passes through the points $(2, -1)$ and $(-3, 4)$.

9. Does the graph at the right represent y as a function of x? Explain.

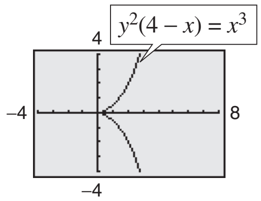


Figure for 9

10. Evaluate $f(x) = |x + 2| - 15$ at each value of the independent variable and simplify.

(a) $f(-8)$ (b) $f(14)$ (c) $f(t - 6)$

11. Find the domain of $f(x) = 10 - \sqrt{3 - x}$.

12. An electronics company produces a car stereo for which the variable cost is \$5.60 and the fixed costs are \$24,000. The product sells for \$99.50. Write the total cost C as a function of the number of units produced and sold, x. Write the profit P as a function of the number of units produced and sold, x.

In Exercises 13 and 14, determine algebraically whether the function is even, odd, or neither.

13. $f(x) = 2x^3 - 3x$

14. $f(x) = 3x^4 + 5x^2$

In Exercises 15 and 16, determine the open intervals on which the function is increasing, decreasing, or constant.

15. $h(x) = \frac{1}{4}x^4 - 2x^2$

16. $g(t) = |t + 2| - |t - 2|$

In Exercises 17 and 18, use a graphing utility to approximate (to two decimal places) any relative minimum or relative maximum values of the function.

17. $f(x) = -x^3 - 5x^2 + 12$

18. $f(x) = x^5 - x^3 + 2$

In Exercises 19–21, (a) identify the parent function *f*, (b) describe the sequence of transformations from *f* to *g*, and (c) sketch the graph of *g*.

19. $g(x) = -2(x - 5)^3 + 3$

20. $g(x) = \sqrt{-x - 7}$

21. $g(x) = 4|-x| - 7$

22. Use the functions $f(x) = x^2$ and $g(x) = \sqrt{2 - x}$ to find the specified function and its domain.

(a) $(f - g)(x)$ (b) $\left(\frac{f}{g}\right)(x)$ (c) $(f \circ g)(x)$ (d) $(g \circ f)(x)$

In Exercises 23–25, determine whether the function has an inverse function, and if so, find the inverse function.

23. $f(x) = x^3 + 8$

24. $f(x) = x^2 + 6$

25. $f(x) = \frac{3x\sqrt{x}}{8}$

Proofs in Mathematics

Conditional Statements

Many theorems are written in the **if-then form** "if p, then q," which is denoted by

$p \rightarrow q$ Conditional statement

where p is the **hypothesis** and q is the **conclusion.** Here are some other ways to express the conditional statement $p \rightarrow q$.

p implies q. p, only if q. p is sufficient for q.

Conditional statements can be either true or false. The conditional statement $p \rightarrow q$ is false only when p is true and q is false. To show that a conditional statement is true, you must prove that the conclusion follows for all cases that fulfill the hypothesis. To show that a conditional statement is false, you need only to describe a single **counterexample** that shows that the statement is not always true.

For instance, $x = -4$ is a counterexample that shows that the following statement is false.

If $x^2 = 16$, then $x = 4$.

The hypothesis "$x^2 = 16$" is true because $(-4)^2 = 16$. However, the conclusion "$x = 4$" is false. This implies that the given conditional statement is false.

For the conditional statement $p \rightarrow q$, there are three important associated conditional statements.

1. The **converse** of $p \rightarrow q$: $q \rightarrow p$
2. The **inverse** of $p \rightarrow q$: $\sim p \rightarrow \sim q$
3. The **contrapositive** of $p \rightarrow q$: $\sim q \rightarrow \sim p$

The symbol ~ means the **negation** of a statement. For instance, the negation of "The engine is running" is "The engine is not running."

Example Writing the Converse, Inverse, and Contrapositive

Write the converse, inverse, and contrapositive of the conditional statement "If I get a B on my test, then I will pass the course."

Solution

a. *Converse:* If I pass the course, then I got a B on my test.

b. *Inverse:* If I do not get a B on my test, then I will not pass the course.

c. *Contrapositive:* If I do not pass the course, then I did not get a B on my test.

In the example above, notice that neither the converse nor the inverse is logically equivalent to the original conditional statement. On the other hand, the contrapositive *is* logically equivalent to the original conditional statement.