2.1 Linear Equations and Problem Solving

Equations and Solutions of Equations

An **equation** in x is a statement that two algebraic expressions are equal. For example, $3x - 5 = 7$, $x^2 - x - 6 = 0$, and $\sqrt{2x} = 4$ are equations. To **solve** an equation in x means to find all values of x for which the equation is true. Such values are **solutions.** For instance, $x = 4$ is a solution of the equation $3x - 5 = 7$, because $3(4) - 5 = 7$ is a true statement.

The solutions of an equation depend on the kinds of numbers being considered. For instance, in the set of rational numbers, $x^2 = 10$ has no solution because there is no rational number whose square is 10. However, in the set of real numbers the equation has the two solutions $x = \sqrt{10}$ and $x = -\sqrt{10}$.

An equation that is true for *every* real number in the domain of the variable is called an **identity.** For example, $x^2 - 9 = (x + 3)(x - 3)$ is an identity because it is a true statement for any real value of x, and $x/(3x^2) = 1/(3x)$, where $x \neq 0$, is an identity because it is true for any nonzero real value of x.

An equation that is true for just *some* (or even none) of the real numbers in the domain of the variable is called a **conditional equation.** For example, the equation $x^2 - 9 = 0$ is conditional because $x = 3$ and $x = -3$ are the only values in the domain that satisfy the equation. The equation $2x + 1 = 2x - 3$ is also conditional because there are no real values of x for which the equation is true. Learning to solve conditional equations is the primary focus of this chapter.

A **linear equation in one variable x** is an equation that can be written in the standard form $ax + b = 0$, where a and b are real numbers, with $a \neq 0$. For a review of solving one- and two-step linear equations, see Appendix D.

To solve an equation involving fractional expressions, find the least common denominator (LCD) of all terms in the equation and multiply every term by this LCD. This procedure clears the equation of fractions, as demonstrated in Example 1.

What you should learn

- Solve equations involving fractional expressions.
- Write and use mathematical models to solve real-life problems.
- Use common formulas to solve real-life problems.

Why you should learn it

Linear equations are useful for modeling situations in which you need to find missing information. For instance, Exercise 68 on page 174 shows how to use a linear equation to determine the height of a flagpole by measuring its shadow.

Sergio Piumatti

Example 1 Solving an Equation Involving Fractions

Solve $\dfrac{x}{3} + \dfrac{3x}{4} = 2$.

Solution

$$\frac{x}{3} + \frac{3x}{4} = 2 \qquad \text{Write original equation.}$$

$$(12)\frac{x}{3} + (12)\frac{3x}{4} = (12)2 \qquad \text{Multiply each term by the LCD of 12.}$$

$$4x + 9x = 24 \qquad \text{Divide out and multiply.}$$

$$13x = 24 \qquad \text{Combine like terms.}$$

$$x = \frac{24}{13} \qquad \text{Divide each side by 13.}$$

CHECKPOINT Now try Exercise 23.

STUDY TIP

After solving an equation, you should check each solution in the original equation. For instance, you can check the solution to Example 1 as follows.

$$\frac{x}{3} + \frac{3x}{4} = 2$$

$$\frac{\frac{24}{13}}{3} + \frac{3\left(\frac{24}{13}\right)}{4} \stackrel{?}{=} 2$$

$$\frac{8}{13} + \frac{18}{13} \stackrel{?}{=} 2$$

$$2 = 2 \checkmark$$

When multiplying or dividing an equation by a *variable* expression, it is possible to introduce an **extraneous solution**—one that does not satisfy the original equation. The next example demonstrates the importance of checking your solution when you have multiplied or divided by a variable expression.

As you cover this chapter, you should point out to your students that some equations are best solved algebraically, whereas others are best solved with a graphing utility.

Example 2 An Equation with an Extraneous Solution

Solve $\dfrac{1}{x-2} = \dfrac{3}{x+2} - \dfrac{6x}{x^2-4}$.

Algebraic Solution

The LCD is

$$x^2 - 4 = (x+2)(x-2).$$

Multiplying each term by the LCD and simplifying produces the following.

$$\frac{1}{x-2}(x+2)(x-2)$$

$$= \frac{3}{x+2}(x+2)(x-2) - \frac{6x}{x^2-4}(x+2)(x-2)$$

$$x + 2 = 3(x-2) - 6x, \quad x \neq \pm 2$$

$$x + 2 = 3x - 6 - 6x$$

$$4x = -8$$

$$x = -2 \qquad \text{Extraneous solution}$$

A check of $x = -2$ in the original equation shows that it yields a denominator of zero. So, $x = -2$ is an extraneous solution, and the original equation has *no solution*.

CHECKPOINT Now try Exercise 39.

Graphical Solution

Use a graphing utility (in *dot* mode) to graph the left and right sides of the equation

$$y_1 = \frac{1}{x-2} \quad \text{and} \quad y_2 = \frac{3}{x+2} - \frac{6x}{x^2-4}$$

in the same viewing window, as shown in Figure 2.1. The graphs of the equations do not appear to intersect. This means that there is no point for which the left side of the equation y_1 is equal to the right side of the equation y_2. So, the equation appears to have *no solution*.

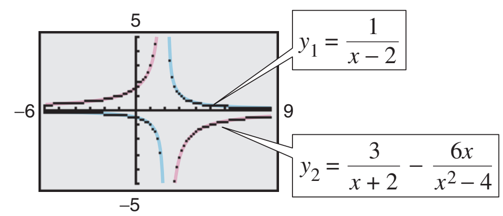


Figure 2.1

Using Mathematical Models to Solve Problems

One of the primary goals of this text is to learn how algebra can be used to solve problems that occur in real-life situations. This procedure, introduced in Chapter 1, is called **mathematical modeling.**

A good approach to mathematical modeling is to use two stages. Begin by using the verbal description of the problem to form a *verbal model*. Then, after assigning labels to the quantities in the verbal model, form a *mathematical model* or an *algebraic equation*.

Verbal Description → Verbal Model → Algebraic Equation

When you are trying to construct a verbal model, it is helpful to look for a *hidden equality*—a statement that two algebraic expressions are equal. These two expressions might be explicitly stated as being equal, or they might be known to be equal (based on prior knowledge or experience).

TECHNOLOGY TIP

Notice in Figure 2.1 that the equations were graphed using the *dot* mode of a graphing utility. In this text, a blue or light red curve is placed behind the graphing utility's display to indicate where the graph should appear. You will learn more about how graphing utilities graph these types of equations in Section 3.6.

Example 3 Finding the Dimensions of a Room

A rectangular family room is twice as long as it is wide, and its perimeter is 84 feet. Find the dimensions of the family room.

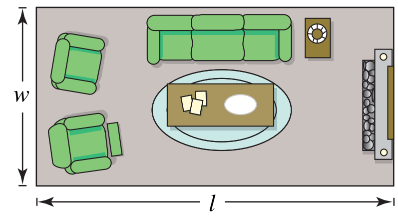


Figure 2.2

The figure provided with Example 3 is not really essential to the solution. Nevertheless, strongly encourage your students to develop the habit of including sketches with their solutions even if they are not required.

Solution

For this problem, it helps to draw a diagram, as shown in Figure 2.2.

Verbal Model: $2 \cdot$ Length $+ 2 \cdot$ Width $=$ Perimeter

Labels: Perimeter $= 84$ (feet)
Width $= w$ (feet)
Length $= l = 2w$ (feet)

Equation:

$$2(2w) + 2w = 84 \quad \text{Original equation}$$

$$6w = 84 \quad \text{Combine like terms.}$$

$$w = 14 \quad \text{Divide each side by 6.}$$

Because the length is twice the width, you have

$$l = 2w \quad \text{Length is twice width.}$$

$$= 2(14) \quad \text{Substitute 14 for } w.$$

$$= 28. \quad \text{Simplify.}$$

So, the dimensions of the room are 14 feet by 28 feet.

CHECKPOINT Now try Exercise 59.

Example 4 A Distance Problem

A plane is flying nonstop from New York to San Francisco, a distance of about 2600 miles, as shown in Figure 2.3. After $1\frac{1}{2}$ hours in the air, the plane flies over Chicago (a distance of about 800 miles from New York). Estimate the time it will take the plane to fly from New York to San Francisco.

Solution

Verbal Model: Distance $=$ Rate $\cdot$ Time

Labels: Distance $= 2600$ (miles)

Rate $= \dfrac{\text{Distance to Chicago}}{\text{Time to Chicago}} = \dfrac{800}{1.5}$ (miles per hour)

Time $= t$ (hours)

Equation:

$$2600 = \frac{800}{1.5}t$$

$$4.875 = t$$

The trip will take about 4.875 hours, or about 4 hours and 53 minutes.

CHECKPOINT Now try Exercise 63.

STUDY TIP

Students sometimes say that although a solution looks easy when it is worked out in class, they don't see where to begin when solving a problem alone. Keep in mind that no one—not even great mathematicians—can expect to look at every mathematical problem and know immediately where to begin. Many problems involve some trial and error before a solution is found. To make algebra work for you, put in a lot of time, expect to try solution methods that end up not working, and learn from both your successes and your failures.

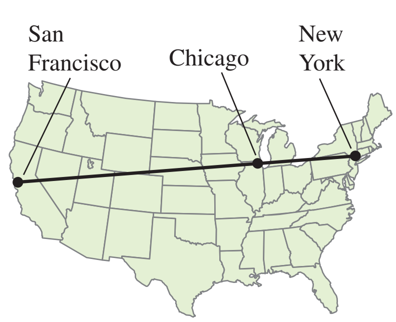


Figure 2.3

Example 5 Height of a Building

To determine the height of the Aon Center Building (in Chicago), you measure the shadow cast by the building and find it to be 142 feet long, as shown in Figure 2.4. Then you measure the shadow cast by a 48-inch post and find it to be 6 inches long. Estimate the building's height.

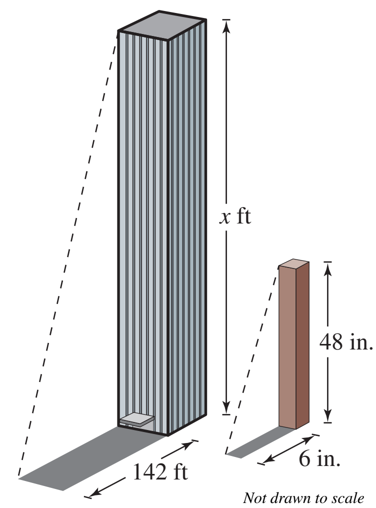


Figure 2.4

Solution

To solve this problem, you use a result from geometry that states that the ratios of corresponding sides of similar triangles are equal.

Verbal Model: $\dfrac{\text{Height of building}}{\text{Length of building's shadow}} = \dfrac{\text{Height of post}}{\text{Length of post's shadow}}$

Labels: Height of building $= x$ (feet)
Length of building's shadow $= 142$ (feet)
Height of post $= 48$ (inches)
Length of post's shadow $= 6$ (inches)

Equation: $\dfrac{x}{142} = \dfrac{48}{6} \quad \Rightarrow \quad x = 1136$

So, the Aon Center Building is about 1136 feet high.

CHECKPOINT Now try Exercise 67.

Example 6 An Inventory Problem

A store has \$30,000 of inventory in 13-inch and 19-inch color televisions. The profit on a 13-inch set is 22% and the profit on a 19-inch set is 40%. The profit for the entire stock is 35%. How much was invested in each type of television?

Solution

Verbal Model: Profit from 13-inch sets + Profit from 19-inch sets = Total profit

Labels: Inventory of 13-inch sets $= x$ (dollars)
Inventory of 19-inch sets $= 30{,}000 - x$ (dollars)
Profit from 13-inch sets $= 0.22x$ (dollars)
Profit from 19-inch sets $= 0.40(30{,}000 - x)$ (dollars)
Total profit $= 0.35(30{,}000) = 10{,}500$ (dollars)

Equation:
$$0.22x + 0.40(30{,}000 - x) = 10{,}500$$
$$-0.18x = -1500$$
$$x \approx 8333.33$$

So, \$8333.33 was invested in 13-inch sets and $30{,}000 - x$, or \$21,666.67, was invested in 19-inch sets.

CHECKPOINT Now try Exercise 73.

STUDY TIP

Notice in the solution of Example 6 that percents are expressed as decimals. For instance, 22% is written as 0.22.

You might want to remind your students that words and phrases such as *is*, *are*, *will be*, and *represents* indicate equality; *sum*, *plus*, *greater than*, *increased by*, *more than*, *exceeds*, and *total of* indicate addition; *difference*, *minus*, *less than*, *decreased by*, *subtracted from*, *reduced by*, and *the remainder* indicate subtraction; *product*, *multiplied by*, *twice*, *times*, and *percent of* indicate multiplication; and *quotient*, *divided by*, *ratio*, and *per* indicate division.

Common Formulas

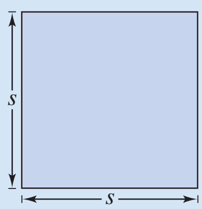
Many common types of geometric, scientific, and investment problems use ready-made equations called **formulas.** Knowing these formulas will help you translate and solve a wide variety of real-life applications.

Common Formulas for Area A, Perimeter P, Circumference C, and Volume V

Square

$A = s^2$

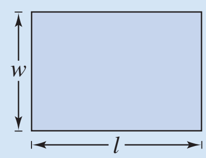
$P = 4s$



Rectangle

$A = lw$

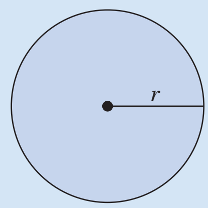
$P = 2l + 2w$



Circle

$A = \pi r^2$

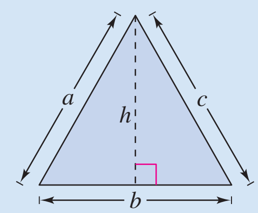
$C = 2\pi r$



Triangle

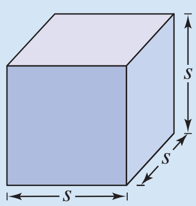
$A = \frac{1}{2}bh$

$P = a + b + c$



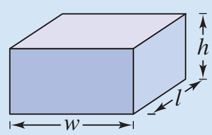
Cube

$V = s^3$



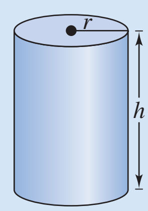
Rectangular Solid

$V = lwh$



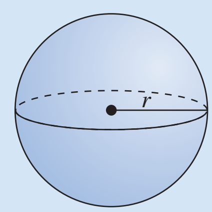
Circular Cylinder

$V = \pi r^2 h$



Sphere

$V = \frac{4}{3}\pi r^3$



Miscellaneous Common Formulas

Temperature:	$F = \frac{9}{5}C + 32$	F = degrees Fahrenheit, C = degrees Celsius
Simple Interest:	$I = Prt$	I = interest, P = principal (original deposit), r = annual interest rate, t = time in years
Compound Interest:	$A = P\left(1 + \frac{r}{n}\right)^{nt}$	A = balance, P = principal (original deposit), r = annual interest rate, n = compoundings (number of times interest is calculated) per year, t = time in years
Distance:	$d = rt$	d = distance traveled, r = rate, t = time

When working with applied problems, you may find it helpful to rewrite a common formula. For instance, the formula for the perimeter of a rectangle, $P = 2l + 2w$, can be solved for w as $w = \frac{1}{2}(P - 2l)$.

Example 7 Using a Formula

A cylindrical can has a volume of 600 cubic centimeters and a radius of 4 centimeters, as shown in Figure 2.5. Find the height of the can.

Figure 2.5

Solution

The formula for the volume of a cylinder is $V = \pi r^2 h$. To find the height of the can, solve for h.

$$h = \frac{V}{\pi r^2}$$

Then, using $V = 600$ and $r = 4$, find the height.

$$h = \frac{600}{\pi(4)^2} = \frac{600}{16\pi} \approx 11.94$$

You can use unit analysis to check that your answer is reasonable.

$$\frac{600 \text{ cm}^{\cancel{3}}}{16\pi \cancel{\text{cm}^2}} \approx 11.94 \text{ cm}$$

CHECKPOINT Now try Exercise 77.

Example 8 Using a Formula

The average daily temperature in San Diego, California is 64.4°F. What is San Diego's average daily temperature in degrees Celsius? (Source: U.S. National Oceanic and Atmospheric Administration)

Solution

First solve for C in the formula for temperature. Then use $F = 64.4$ to find the temperature in degrees Celsius.

$$\begin{aligned} F &= \frac{9}{5}C + 32 && \text{Formula for temperature} \\ F - 32 &= \frac{9}{5}C && \text{Subtract 32 from each side.} \\ \frac{5}{9}(F - 32) &= C && \text{Multiply each side by } \tfrac{5}{9}. \\ \frac{5}{9}(64.4 - 32) &= C && \text{Substitute 64.4 for } F. \\ 18 &= C && \text{Simplify.} \end{aligned}$$

The average daily temperature in San Diego is 18°C.

CHECKPOINT Now try Exercise 81.

STUDY TIP

Once you have rewritten the formula for temperature, you can easily find other Celsius values. Simply substitute other Fahrenheit values and evaluate.

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2.1 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blanks.

1. A(n) _______ is a statement that equates two algebraic expressions.
2. To find all values that satisfy an equation is to _______ the equation.
3. There are two types of equations, _______ and _______ .
4. A linear equation in one variable is an equation that can be written in the standard form _______ .
5. When solving an equation, it is possible to introduce an _______ solution, which is a value that does not satisfy the original equation.
6. _______ is a procedure used in algebra to solve problems that occur in real-life situations.
7. Many real-life problems can be solved using ready-made equations called _______ .

In Exercises 1–6, determine whether each value of x is a solution of the equation.

Equation	*Values*	
1. $\frac{5}{2x} - \frac{4}{x} = 3$	(a) $x = -\frac{1}{2}$	(b) $x = 4$
	(c) $x = 0$	(d) $x = \frac{1}{4}$
2. $\frac{x}{2} + \frac{6x}{7} = \frac{19}{14}$	(a) $x = -2$	(b) $x = 1$
	(c) $x = \frac{1}{2}$	(d) $x = 7$
3. $3 + \frac{1}{x+2} = 4$	(a) $x = -1$	(b) $x = -2$
	(c) $x = 0$	(d) $x = 5$
4. $\frac{(x+5)(x-3)}{2} = 24$	(a) $x = -3$	(b) $x = -2$
	(c) $x = 7$	(d) $x = 9$
5. $\frac{\sqrt{x+4}}{6} + 3 = 4$	(a) $x = -3$	(b) $x = 0$
	(c) $x = 21$	(d) $x = 32$
6. $\frac{\sqrt[3]{x-8}}{3} = -\frac{2}{3}$	(a) $x = -16$	(b) $x = 0$
	(c) $x = 9$	(d) $x = 16$

In Exercises 7–12, determine whether the equation is an identity or a conditional equation.

7. $2(x - 1) = 2x - 2$

8. $-7(x - 3) + 4x = 3(7 - x)$

9. $x^2 - 8x + 5 = (x - 4)^2 - 11$

10. $x^2 + 2(3x - 2) = x^2 + 6x - 4$

11. $3 + \frac{1}{x+1} = \frac{4x}{x+1}$

12. $\frac{5}{x} + \frac{3}{x} = 24$

In Exercises 13–16, solve the equation using two methods. Then explain which method is easier.

13. $\frac{3x}{8} - \frac{4x}{3} = 4$

14. $\frac{3z}{8} - \frac{z}{10} = 6$

15. $\frac{2x}{5} + 5x = \frac{4}{3}$

16. $\frac{4y}{3} - 2y = \frac{16}{5}$

In Exercises 17–40, solve the equation (if possible).

17. $3x - 5 = 2x + 7$

18. $5x + 3 = 6 - 2x$

19. $4y + 2 - 5y = 7 - 6y$

20. $5y + 1 = 8y - 5 + 6y$

21. $3(y - 5) = 3 + 5y$

22. $5(z - 4) + 4z = 5 - 6z$

23. $\frac{x}{5} - \frac{x}{2} = 3$

24. $\frac{5x}{4} + \frac{1}{2} = x - \frac{1}{2}$

25. $\frac{3}{2}(z + 5) - \frac{1}{4}(z + 24) = 0$

26. $\frac{3x}{2} + \frac{1}{4}(x - 2) = 10$

27. $\frac{2(z-4)}{5} + 5 = 10z$

28. $\frac{5}{3} + 2(y + 1) = \frac{10}{3}$

29. $\frac{100 - 4u}{3} = \frac{5u + 6}{4} + 6$

30. $\frac{17 + y}{y} + \frac{32 + y}{y} = 100$

31. $\frac{5x - 4}{5x + 4} = \frac{2}{3}$

32. $\frac{10x + 3}{5x + 6} = \frac{1}{2}$

33. $\frac{1}{x-3} + \frac{1}{x+3} = \frac{10}{x^2 - 9}$

34. $\frac{1}{x-2} + \frac{3}{x+3} = \frac{4}{x^2 + x - 6}$

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35. $\dfrac{7}{2x+1} - \dfrac{8x}{2x-1} = -4$

36. $\dfrac{x}{x+4} + \dfrac{4}{x+4} + 2 = 0$

37. $\dfrac{1}{x} + \dfrac{2}{x-5} = 0$

38. $3 = 2 + \dfrac{2}{z+2}$

39. $\dfrac{3}{x^2-3x} + \dfrac{4}{x} = \dfrac{1}{x-3}$

40. $\dfrac{6}{x} - \dfrac{2}{x+3} = \dfrac{3(x+5)}{x(x+3)}$

In Exercises 41–56, solve for the indicated variable.

41. ***Area of a Triangle***

Solve for h: $A = \frac{1}{2}bh$

42. ***Area of a Trapezoid***

Solve for b: $A = \frac{1}{2}(a+b)h$

43. ***Investment at Compound Interest***

Solve for P: $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$

44. ***Investment at Simple Interest***

Solve for r: $A = P + Prt$

45. ***Geometric Progression***

Solve for r: $S = \dfrac{rL - a}{r - 1}$

46. ***Arithmetic Progression***

Solve for n: $L = a + (n-1)d$

47. ***Volume of an Oblate Spheroid***

Solve for b: $V = \dfrac{4}{3}\pi a^2 b$

48. ***Volume of a Spherical Segment***

Solve for r: $V = \dfrac{1}{3}\pi h^2(3r - h)$

49. ***Perimeter of a Rectangle***

Solve for w: $P = 2l + 2w$

50. ***Sum of a Convergent Geometric Series***

Solve for r: $S = \dfrac{a}{1 - r}$

51. ***Volume of a Right Circular Cylinder***

Solve for h: $V = \pi r^2 h$

52. ***Volume of a Right Circular Cone***

Solve for h: $V = \frac{1}{3}\pi r^2 h$

53. ***Lateral Surface Area of a Right Circular Cylinder***

Solve for r: $S = 2\pi rh$

54. ***Velocity of a Free-Falling Object***

Solve for t: $v = -gt + v_0$

55. ***Ideal Gas Law***

Solve for R: $PV = nRT$

56. ***Resistors in Parallel***

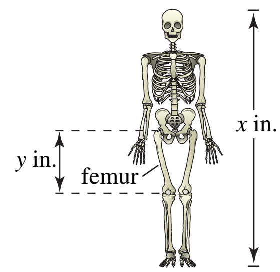
Solve for R_1: $\dfrac{1}{R_T} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

Anthropology **In Exercises 57 and 58, use the following information. The relationship between the length of an adult's femur (thigh bone) and the height of the adult can be approximated by the linear equations**

$y = 0.432x - 10.44$ **Female**

$y = 0.449x - 12.15$ **Male**

where y is the length of the femur in inches and x is the height of the adult in inches (see figure).



57. An anthropologist discovers a femur belonging to an adult human female. The bone is 16 inches long. Estimate the height of the female.

58. From the foot bones of an adult human male, an anthropologist estimates that the person's height was 69 inches. A few feet away from the site where the foot bones were discovered, the anthropologist discovers a male adult femur that is 19 inches long. Is it likely that both the foot bones and the thigh bone came from the same person?

59. ***Geometry*** A room is 1.5 times as long as it is wide, and its perimeter is 25 meters.

(a) Draw a diagram that gives a visual representation of the problem. Identify the length as l and the width as w.

(b) Write l in terms of w and write an equation for the perimeter in terms of w.

(c) Find the dimensions of the room.

60. ***Geometry*** A picture frame has a total perimeter of 3 meters. The height of the frame is $\frac{2}{3}$ times its width.

(a) Draw a diagram that gives a visual representation of the problem. Identify the width as w and the height as h.

(b) Write h in terms of w and write an equation for the perimeter in terms of w.

(c) Find the dimensions of the picture frame.

61. ***Course Grade*** To get an A in a course, you must have an average of at least 90 on four tests of 100 points each. The scores on your first three tests were 87, 92, and 84.

(a) Write a verbal model for the test average for the course.

(b) What must you score on the fourth test to get an A for the course?

62. ***Course Grade*** You are taking a course that has four tests. The first three tests are 100 points each and the fourth test is 200 points. To get an A in the course, you must have an average of at least 90% on the four tests. Your scores on the first three tests were 87, 92, and 84. What must you score on the fourth test to get an A for the course?

63. ***Travel Time*** You are driving on a Canadian freeway to a town that is 300 kilometers from your home. After 30 minutes you pass a freeway exit that you know is 50 kilometers from your home. Assuming that you continue at the same constant speed, how long will it take for the entire trip?

64. ***Travel Time*** On the first part of a 317-mile trip, a salesperson averaged 58 miles per hour. The salesperson averaged only 52 miles per hour on the last part of the trip because of an increased volume of traffic. The total time of the trip was 5 hours and 45 minutes. Find the amount of time at each of the two speeds.

65. ***Average Speed*** A truck driver traveled at an average speed of 55 miles per hour on a 200-mile trip to pick up a load of freight. On the return trip (with the truck fully loaded), the average speed was 40 miles per hour. Find the average speed for the round trip.

66. ***Wind Speed*** An executive flew in the corporate jet to a meeting in a city 1500 kilometers away. After traveling the same amount of time on the return flight, the pilot mentioned that they still had 300 kilometers to go. The air speed of the plane was 600 kilometers per hour. How fast was the wind blowing? (Assume that the wind direction was parallel to the flight path and constant all day.)

67. ***Height*** To obtain the height of a barn silo, you measure the silo's shadow and find that it is 80 feet long. You also measure the shadow of a four-foot stake and find that it is $3\frac{1}{2}$ feet long.

(a) Draw a diagram that illustrates the problem. Let h represent the height of the silo.

(b) Find the height of the silo.

68. ***Height*** A person who is 6 feet tall walks away from a flagpole toward the tip of the shadow of the flagpole. When the person is 30 feet from the flagpole, the tips of the person's shadow and the shadow cast by the flagpole coincide at a point 5 feet in front of the person.

(a) Draw a diagram that illustrates the problem. Let h represent the height of the flagpole.

(b) Find the height of the flagpole.

69. ***Simple Interest*** Find the interest on a \$5000 bond that pays an annual percentage rate of $6\frac{1}{2}\%$ for 6 years.

70. ***Simple Interest*** A certificate of deposit with an initial deposit of \$8000 accumulates \$400 interest in 2 years. Find the annual interest rate.

71. ***Investment*** You plan to invest \$12,000 in two funds paying $4\frac{1}{2}\%$ and 5% simple interest. (There is more risk in the 5% fund.) Your goal is to obtain a total annual interest income of \$560 from the investments. What is the smallest amount you can invest in the 5% fund in order to meet your objective?

72. ***Investment*** You plan to invest \$25,000 in two funds paying 3% and $4\frac{1}{2}\%$ simple interest. (There is more risk in the $4\frac{1}{2}\%$ fund.) Your goal is to obtain a total annual interest income of \$1000 from the investments. What is the smallest amount you can invest in the $4\frac{1}{2}\%$ fund in order to meet your objective?

73. ***Inventory*** A store has \$50,000 of inventory in DVD players and VCRs. The profit on a DVD player is 30% and the profit on a VCR is 25%. The profit on the entire stock is 29%. How much is invested in DVD players and how much is invested in VCRs?

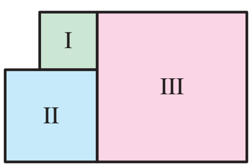
74. ***Inventory*** A store has \$4500 of inventory in 8×10 picture frames and 5×7 picture frames. The profit on an 8×10 frame is 25% and the profit on a 5×7 frame is 22%. The profit on the entire stock is 24%. How much is invested in the 8×10 picture frames and how much is invested in the 5×7 picture frames?

75. ***Mixture Problem*** A grocer mixes peanuts that cost \$2.49 per pound and walnuts that cost \$3.89 per pound to make 100 pounds of a mixture that costs \$3.19 per pound. How much of each kind of nut is put into the mixture?

76. ***Mixture Problem*** A forester mixes gasoline and oil to make 2 gallons of mixture for his two-cycle chainsaw engine. This mixture is 32 parts gasoline and 1 part oil. How much gasoline must be added to bring the mixture to 40 parts gasoline and 1 part oil?

77. ***Height*** A triangular sail has an area of 182.25 square feet. The sail has a base of 13.5 feet. Find the height of the sail.

78. ***Area*** The figure shows three squares. The perimeter of square I is 20 inches and the perimeter of square II is 32 inches. Find the area of square III.



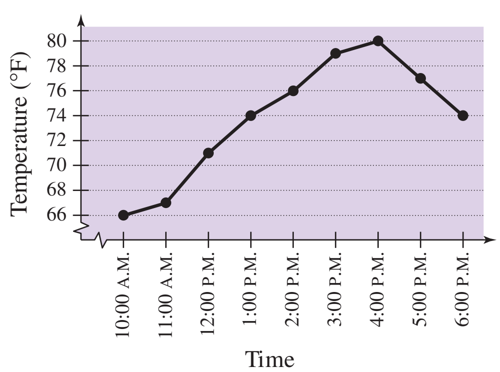
79. ***Geometry*** The volume of a rectangular package is 2304 cubic inches. The length of the package is 3 times its width, and the height is $1\frac{1}{2}$ times its width.

(a) Draw a diagram that illustrates the problem. Label the height, width, and length accordingly.

(b) Find the dimensions of the package.

80. ***Geometry*** The volume of a globe is about 47,712.94 cubic centimeters. Use a graphing utility to find the radius of the globe. Round your result to two decimal places.

81. ***Meteorology*** The line graph shows the temperatures (in degrees Fahrenheit) on a summer day in Buffalo, New York from 10:00 A.M. to 6:00 P.M. Create a new line graph showing the temperatures throughout the day in degrees Celsius.

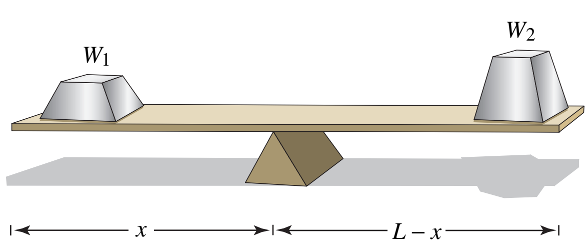


82. ***Meteorology*** The average daily temperature in San Francisco, California is 57.3°F. What is San Francisco's average daily temperature in degrees Celsius? (U.S. National Oceanic and Atmospheric Administration)

Statics **In Exercises 83 and 84, you have a uniform beam of length L with a fulcrum x feet from one end (see figure). Objects with weights W_1 and W_2 are placed at opposite ends of the beam. The beam will balance when**

$$W_1x = W_2(L - x).$$

Find x such that the beam will balance.



83. Two children weighing 50 pounds and 75 pounds are going to play on a seesaw that is 10 feet long.

84. A person weighing 200 pounds is attempting to move a 550-pound rock with a bar that is 5 feet long.

Synthesis

True or False? **In Exercises 85 and 86, determine whether the statement is true or false. Justify your answer.**

85. The equation

$$x(3 - x) = 10$$

is a linear equation.

86. The volume of a cube with a side length of 9.5 inches is greater than the volume of a sphere with a radius of 5.9 inches.

In Exercises 87 and 88, write a linear equation that has the given solution. (There are many correct answers.)

87. $x = -3$

88. $x = \frac{1}{4}$

89. ***Think About It*** What is meant by *equivalent equations*? Give an example of two equivalent equations.

90. ***Writing*** In your own words, describe how to clear an equation of fractions.

91. ***Think About It*** Find c such that $x = 3$ is a solution to the linear equation $2x - 5c = 10 + 3c - 3x$.

92. ***Think About It*** Find c such that $x = 2$ is a solution to the linear equation $5x + 2c = 12 + 4x - 2c$.

Skills Review

In Exercises 93–98, sketch the graph of the equation by hand. Verify using a graphing utility.

93. $y = \frac{5}{8}x - 2$

94. $y = \dfrac{3x - 5}{2} + 2$

95. $y = (x - 3)^2 + 7$

96. $y = \frac{1}{3}x^2 - 4$

97. $y = -\frac{1}{2}|x + 4| - 1$

98. $y = |x - 2| + 10$

In Exercises 99–104, evaluate the combination of functions for $f(x) = -x^2 + 4$ and $g(x) = 6x - 5$.

99. $(f + g)(-3)$

100. $(g - f)(-1)$

101. $(fg)(8)$

102. $\left(\dfrac{f}{g}\right)\left(\dfrac{1}{2}\right)$

103. $(f \circ g)(4)$

104. $(g \circ f)(2)$