2.2 Solving Equations Graphically

Intercepts, Zeros, and Solutions

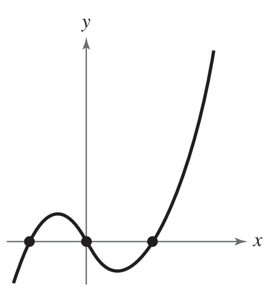
In Section 1.1, you learned that the intercepts of a graph are the points at which the graph intersects the x- or y-axis.

Definitions of Intercepts

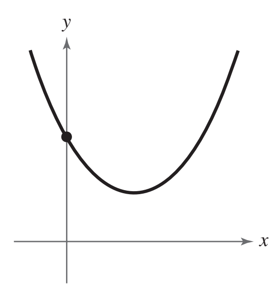
1. The point $(a, 0)$ is called an ***x*-intercept** of the graph of an equation if it is a solution point of the equation. To find the x-intercept(s), set y equal to 0 and solve the equation for x.
2. The point $(0, b)$ is called a ***y*-intercept** of the graph of an equation if it is a solution point of the equation. To find the y-intercept(s), set x equal to 0 and solve the equation for y.

Sometimes it is convenient to denote the x-intercept as simply the x-coordinate of the point $(a, 0)$ rather than the point itself. Unless it is necessary to make a distinction, "intercept" will be used to mean either the point or the coordinate.

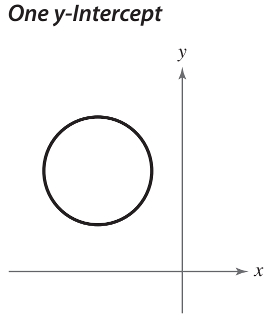
It is possible for a graph to have no intercepts, one intercept, or several intercepts. For instance, consider the four graphs shown in Figure 2.6.



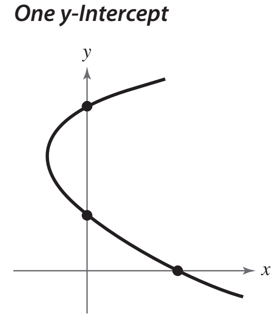
Three x-Intercepts
One y-Intercept



No x-Intercepts
One y-Intercept



No Intercepts



One x-Intercept
Two y-Intercepts

Figure 2.6

As you study this section, you will see the connection among intercepts, zeros, and solutions of functions.

What you should learn

- Find x- and y-intercepts of graphs of equations.
- Find solutions of equations graphically.
- Find the points of intersection of two graphs.

Why you should learn it

Because some real-life problems involve equations that are difficult to solve algebraically, it is helpful to use a graphing utility to approximate the solutions of such equations. For instance, you can use a graphing utility to find the intersection point of the equations in Example 7 on page 182 to determine the year during which the number of morning newspapers in the United States exceeded the number of evening newspapers.

Simon Bottomley/Getty Images

Example 1 Finding x- and y-Intercepts

Find the x- and y-intercepts of the graph of $2x + 3y = 5$.

Solution

To find the x-intercept, let $y = 0$ and solve for x. This produces

$$2x = 5 \quad \Rightarrow \quad x = \frac{5}{2}$$

which implies that the graph has one x-intercept: $\left(\frac{5}{2}, 0\right)$. To find the y-intercept, let $x = 0$ and solve for y. This produces

$$3y = 5 \quad \Rightarrow \quad y = \frac{5}{3}$$

which implies that the graph has one y-intercept: $\left(0, \frac{5}{3}\right)$. See Figure 2.7.

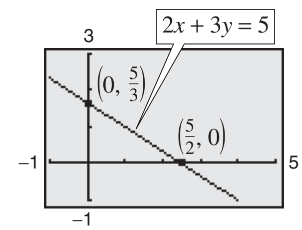


Figure 2.7

CHECKPOINT Now try Exercise 1.

A **zero** of a function $y = f(x)$ is a number a such that $f(a) = 0$. So, to find the zeros of a function, you must solve the equation $f(x) = 0$.

The concepts of x-intercepts, zeros of functions, and solutions of equations are closely related. In fact, the following statements are equivalent.

1. The point $(a, 0)$ is an *x-intercept* of the graph of $y = f(x)$.
2. The number a is a *zero* of the function f.
3. The number a is a *solution* of the equation $f(x) = 0$.

TECHNOLOGY SUPPORT

For instructions on how to use the *table* feature, see Appendix A. For specific keystrokes, go to this textbook's *Online Study Center.*

Example 2 Verifying Zeros of Functions

Verify that the real numbers -2 and 3 are zeros of the function $f(x) = x^2 - x - 6$.

Algebraic Solution

To verify that -2 is a zero of f, check that $f(-2) = 0$.

$$f(x) = x^2 - x - 6 \qquad \text{Write original function.}$$

$$f(-2) = (-2)^2 - (-2) - 6 \qquad \text{Substitute } -2 \text{ for } x.$$

$$= 4 + 2 - 6 \qquad \text{Simplify.}$$

$$= 0 \qquad -2 \text{ is a zero. } \checkmark$$

To verify that 3 is a zero of f, check that $f(3) = 0$.

$$f(x) = x^2 - x - 6 \qquad \text{Write original function.}$$

$$f(3) = (3)^2 - 3 - 6 \qquad \text{Substitute 3 for } x.$$

$$= 9 - 3 - 6 \qquad \text{Simplify.}$$

$$= 0 \qquad 3 \text{ is a zero. } \checkmark$$

CHECKPOINT Now try Exercise 17.

Numerical Solution

You can use the *table* feature of a graphing utility to verify that the real numbers -2 and 3 are zeros of the function $f(x) = x^2 - x - 6$. Enter the equation $y_1 = x^2 - x - 6$ into the graphing utility. Then use the *table* feature to create a table, as shown in Figure 2.8. Note that when $x = -2$ and $x = 3$, the entries for y_1 are 0. You can now conclude that -2 and 3 are zeros of the function $f(x) = x^2 - x - 6$.

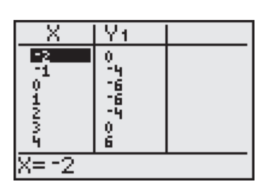


Figure 2.8

The close connection among x-intercepts, zeros, and solutions is crucial to your study of algebra. You can take advantage of this connection in two ways. Use your algebraic "equation-solving skills" to find the x-intercepts of a graph and your "graphing skills" to approximate the solutions of an equation.

Finding Solutions Graphically

Polynomial equations of degree 1 or 2 can be solved in relatively straightforward ways. Solving polynomial equations of higher degrees can, however, be quite difficult, especially if you rely only on algebraic techniques. For such equations, a graphing utility can be very helpful.

Graphical Approximations of Solutions of an Equation

1. Write the equation in *general form*, $f(x) = 0$, with the nonzero terms on one side of the equation and zero on the other side.
2. Use a graphing utility to graph the function $y = f(x)$. Be sure the viewing window shows all the relevant features of the graph.
3. Use the *zero* or *root* feature or the *zoom* and *trace* features of the graphing utility to approximate the x-intercepts of the graph of f.

In Chapter 3 you will learn techniques for determining the number of solutions of a polynomial equation. For now, you should know that a polynomial equation of degree n cannot have more than n different solutions.

Example 3 Finding Solutions of an Equation Graphically

Use a graphing utility to approximate the solutions of $2x^3 - 3x + 2 = 0$.

Solution

Graph the function $y = 2x^3 - 3x + 2$. You can see from the graph that there is one x-intercept. It lies between -2 and -1 and is approximately -1.5. By using the *zero* or *root* feature of a graphing utility, you can improve the approximation. Choose a left bound of $x = -2$ (see Figure 2.9) and a right bound of $x = -1$ (see Figure 2.10). To two-decimal-place accuracy, the solution is $x \approx -1.48$, as shown in Figure 2.11. Check this approximation on your calculator. You will find that the value of y is $y = 2(-1.48)^3 - 3(-1.48) + 2 \approx -0.04$.

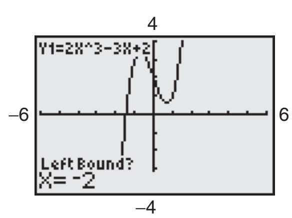


Figure 2.9

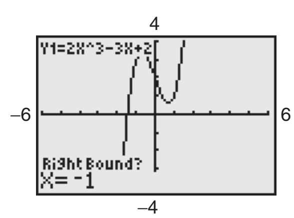


Figure 2.10

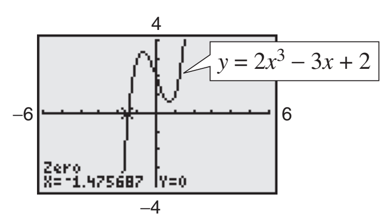


Figure 2.11

CHECKPOINT Now try Exercise 37.

Exploration

In Chapter 3 you will learn that a cubic equation such as

$$24x^3 - 36x + 17 = 0$$

can have up to three real solutions. Use a graphing utility to graph

$$y = 24x^3 - 36x + 17.$$

Describe a viewing window that enables you to determine the number of real solutions of the equation

$$24x^3 - 36x + 17 = 0.$$

Use the same technique to determine the number of real solutions of

$$97x^3 - 102x^2 - 200x - 63 = 0.$$

TECHNOLOGY TIP You can also use a graphing utility's *zoom* and *trace* features to approximate the solution of an equation. Here are some suggestions for using the *zoom-in* feature of a graphing utility.

1. With each successive zoom-in, adjust the x-scale (if necessary) so that the resulting viewing window shows at least the two scale marks between which the solution lies.
2. The accuracy of the approximation will always be such that the error is less than the distance between two scale marks.
3. If you have a *trace* feature on your graphing utility, you can generally add one more decimal place of accuracy without changing the viewing window.

Unless stated otherwise, this book will approximate all real solutions with an error of *at most* 0.01.

TECHNOLOGY SUPPORT

For instructions on how to use the *zoom* and *trace* features and the *zero* or *root* feature, see Appendix A; for specific keystrokes, go to this textbook's *Online Study Center.*

Example 4 Approximating Solutions of an Equation Graphically

Use a graphing utility to approximate the solutions of

$$x^2 + 3 = 5x.$$

Solution

In general form, this equation is

$$x^2 - 5x + 3 = 0. \qquad \text{Equation in general form}$$

So, you can begin by graphing

$$y = x^2 - 5x + 3 \qquad \text{Function to be graphed}$$

as shown in Figure 2.12. This graph has two x-intercepts, and by using the *zoom* and *trace* features you can approximate the corresponding solutions to be $x \approx 0.70$ and $x \approx 4.30$, as shown in Figures 2.13 and 2.14.

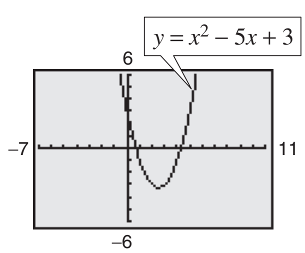


Figure 2.12

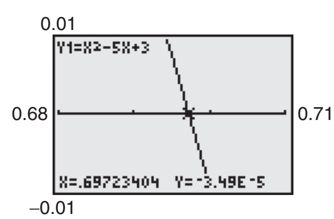


Figure 2.13

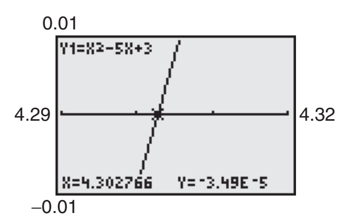


Figure 2.14

✓CHECKPOINT Now try Exercise 41.

TECHNOLOGY TIP Remember from Example 3 that the built-in *zero* and *root* features of a graphing utility will approximate solutions of equations or x-intercepts of graphs. If your graphing utility has such features, try using them to approximate the solutions in Example 4.

TECHNOLOGY TIP

Remember that the more decimal places in the solution, the more accurate the solution is. You can reach the desired accuracy when zooming in as follows.

- To approximate the zero to the nearest hundredth, set the x-scale to 0.01.
- To approximate the zero to the nearest thousandth, set the x-scale to 0.001.

Points of Intersection of Two Graphs

An ordered pair that is a solution of two different equations is called a **point of intersection** of the graphs of the two equations. For instance, in Figure 2.15 you can see that the graphs of the following equations have two points of intersection.

$y = x + 2$ Equation 1

$y = x^2 - 2x - 2$ Equation 2

The point $(-1, 1)$ is a solution of both equations, and the point $(4, 6)$ is a solution of both equations. To check this algebraically, substitute $x = -1$ and $x = 4$ into each equation.

Check that $(-1, 1)$ *is a solution.*

Equation 1: $y = -1 + 2 = 1$ Solution checks. ✓

Equation 2: $y = (-1)^2 - 2(-1) - 2 = 1$ Solution checks. ✓

Check that $(4, 6)$ *is a solution.*

Equation 1: $y = 4 + 2 = 6$ Solution checks. ✓

Equation 2: $y = (4)^2 - 2(4) - 2 = 6$ Solution checks. ✓

To find the points of intersection of the graphs of two equations, solve each equation for y (or x) and set the two results equal to each other. The resulting equation will be an equation in one variable, which can be solved using standard procedures, as shown in Example 5.

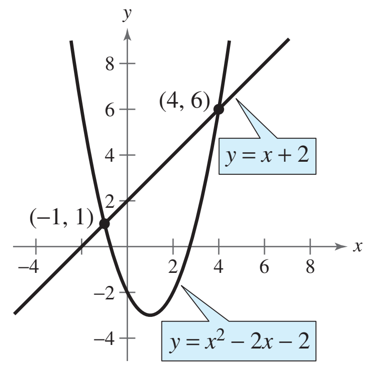


Figure 2.15

TECHNOLOGY SUPPORT

For instructions on how to use the *intersect* feature, see Appendix A; for specific keystrokes, go to this textbook's *Online Study Center.*

Example 5 Finding Points of Intersection

Find the points of intersection of the graphs of $2x - 3y = -2$ and $4x - y = 6$.

Algebraic Solution

To begin, solve each equation for y to obtain

$$y = \frac{2}{3}x + \frac{2}{3} \quad \text{and} \quad y = 4x - 6.$$

Next, set the two expressions for y equal to each other and solve the resulting equation for x, as follows.

$\frac{2}{3}x + \frac{2}{3} = 4x - 6$ Equate expressions for y.

$2x + 2 = 12x - 18$ Multiply each side by 3.

$-10x = -20$ Subtract $12x$ and 2 from each side.

$x = 2$ Divide each side by -10.

When $x = 2$, the y-value of each of the original equations is 2. So, the point of intersection is $(2, 2)$.

CHECKPOINT Now try Exercise 65.

Graphical Solution

To begin, solve each equation for y to obtain $y_1 = \frac{2}{3}x + \frac{2}{3}$ and $y_2 = 4x - 6$. Then use a graphing utility to graph both equations in the same viewing window. In Figure 2.16, the graphs appear to have one point of intersection. Use the *intersect* feature of the graphing utility to approximate the point of intersection to be $(2, 2)$.

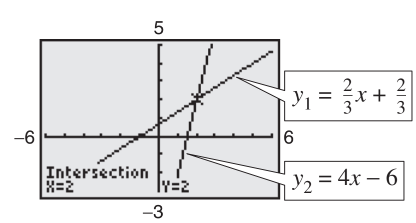


Figure 2.16

TECHNOLOGY TIP Another way to approximate the points of intersection of two graphs is to graph both equations with a graphing utility and use the *zoom* and *trace* features to find the point or points at which the two graphs intersect.

Example 6 Approximating Points of Intersection Graphically

Approximate the point(s) of intersection of the graphs of the following equations.

$y = x^2 - 3x - 4$ Equation 1 (quadratic function)

$y = x^3 + 3x^2 - 2x - 1$ Equation 2 (cubic function)

Solution

Begin by using a graphing utility to graph both functions, as shown in Figure 2.17. From this display, you can see that the two graphs have only one point of intersection. Then, using the *zoom* and *trace* features, approximate the point of intersection to be $(-2.17, 7.25)$, as shown in Figure 2.18.

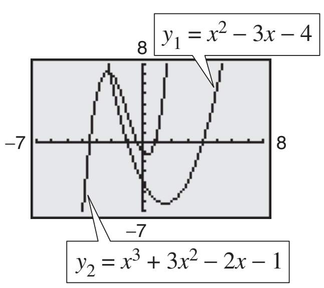


Figure 2.17

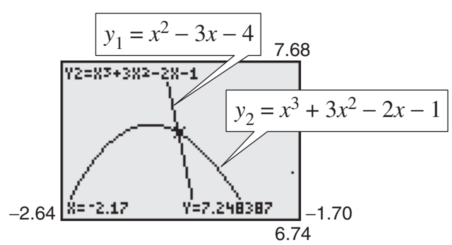


Figure 2.18

To test the reasonableness of this approximation, you can evaluate both functions at $x = -2.17$.

Quadratic Function:

$$y = (-2.17)^2 - 3(-2.17) - 4$$
$$\approx 7.22$$

Cubic Function:

$$y = (-2.17)^3 + 3(-2.17)^2 - 2(-2.17) - 1$$
$$\approx 7.25$$

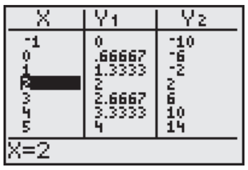
Because both functions yield approximately the same y-value, you can conclude that the approximate coordinates of the point of intersection are $x \approx -2.17$ and $y \approx 7.25$.

CHECKPOINT Now try Exercise 69.

TECHNOLOGY TIP If you choose to use the *intersect* feature of your graphing utility to find the point of intersection of the graphs in Example 6, you will see that it yields the same result.

TECHNOLOGY TIP

The table shows some points on the graphs of the equations in Example 5. Find the points of intersection of the graphs by finding the value(s) of x for which y_1 and y_2 are equal.



Additional Example

Approximate the point(s) of intersection of the graphs of $y = x^3 - 6x^2 + 6x + 3$ and $y = -x^2 + 7x - 2$.

Solution

Use a graphing utility to graph both functions in the same viewing window. Use the *zoom* and *trace* features or the *intersect* feature to approximate the points of intersection to be $(1, 4)$, $(5, 8)$, and $(-1, -10)$.

Prerequisite Skills

Review the techniques for evaluating functions in Section 1.3, if you have difficulty with this example.

The method shown in Example 6 gives a nice graphical picture of the points of intersection of two graphs. However, for actual approximation purposes, it is better to use the algebraic procedure described in Example 5. That is, the point of intersection of $y = x^2 - 3x - 4$ and $y = x^3 + 3x^2 - 2x - 1$ coincides with the solution of the equation

$$x^3 + 3x^2 - 2x - 1 = x^2 - 3x - 4 \qquad \text{Equate } y\text{-values.}$$

$$x^3 + 2x^2 + x + 3 = 0. \qquad \text{Write in general form.}$$

By graphing $y = x^3 + 2x^2 + x + 3$ on a graphing utility and using the *zoom* and *trace* features (or the *zero* or *root* feature), you can approximate the solution of this equation to be $x \approx -2.17$.

Example 7 A Historical Look at Newspapers

Between 1990 and 2004, the number of morning newspapers M in the United States was *increasing* and the number of evening newspapers E was *decreasing*. Two models that approximate the numbers of newspapers are

$$M = 18.5t + 564, \quad 0 \le t \le 14 \qquad \text{Morning newspapers}$$

$$E = -30.9t + 1054, \quad 0 \le t \le 14 \qquad \text{Evening newspapers}$$

where t represents the year, with $t = 0$ corresponding to 1990. According to these two models, when would you expect the number of morning newspapers to have exceeded the number of evening newspapers? (Source: Editor & Publisher Co.)

Algebraic Solution

Set the two expressions equal to each other and solve the resulting equation for t, as follows.

$$18.5t + 564 = -30.9t + 1054 \qquad \text{Equate expressions.}$$

$$49.4t + 564 = 1054 \qquad \text{Add } 30.9t \text{ to each side.}$$

$$49.4t = 490 \qquad \text{Subtract 564 from each side.}$$

$$t = \frac{490}{49.4} \qquad \text{Divide each side by 49.4.}$$

$$t \approx 9.92 \qquad \text{Use a calculator.}$$

So, from the given models, you would expect that the number of morning newspapers exceeded the number of evening newspapers sometime during 1999.

CHECKPOINT Now try Exercise 81.

Graphical Solution

Use a graphing utility to graph both equations in the same viewing window. From Figure 2.19, the graphs appear to have one point of intersection. Use the *intersect* feature of the graphing utility to approximate the point of intersection to be (9.92, 747.50). So, you would expect that the number of morning newspapers exceeded the number of evening newspapers sometime during 1999.

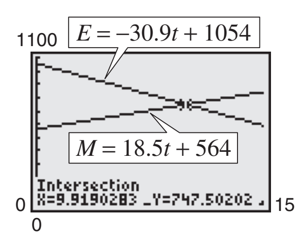


Figure 2.19

TECHNOLOGY TIP If you choose to use the *zoom* and *trace* features of your graphing utility to find the point of intersection of the graphs in Example 7, you will see that these features yield the same result.

2.2 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

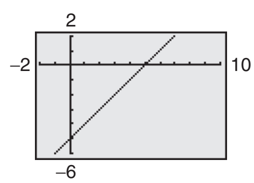
Vocabulary Check

Fill in the blanks.

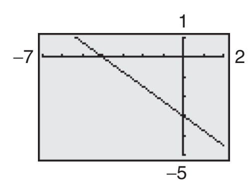
1. The points $(a, 0)$ and $(0, b)$ are called the _______ and _______ , respectively, of the graph of an equation.
2. A _______ of a function is a number a such that $f(a) = 0$.
3. An ordered pair that is a solution of two different equations is called a _______ of the graphs of the two equations.

In Exercises 1–10, find the x- and y-intercepts of the graph of the equation.

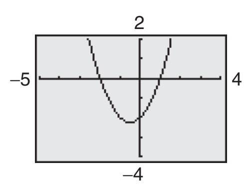
1. $y = x - 5$



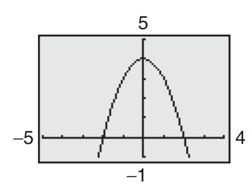
2. $y = -\frac{3}{4}x - 3$



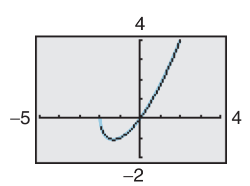
3. $y = x^2 + x - 2$



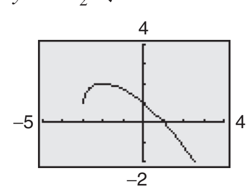
4. $y = 4 - x^2$



5. $y = x\sqrt{x + 2}$



6. $y = -\frac{1}{2}x\sqrt{x + 3} + 1$



7. $y = \dfrac{4}{x}$

8. $y = \dfrac{3x - 1}{4x}$

9. $xy - 2y - x + 1 = 0$

10. $xy - x + 4y = 0$

Graphical Analysis **In Exercises 11–14, use a graphing utility to graph the equation and approximate any x- and y-intercepts. Verify your results algebraically.**

11. $y = 2(x - 1) - 4$

12. $y = 4(x + 3) - 2$

13. $y = 20 - (3x - 10)$

14. $y = 10 + 2(x - 2)$

In Exercises 15–20, the zero(s) of the function are given. Verify the zero(s) both algebraically and graphically.

	Function	*Zero(s)*
15.	$f(x) = 5(4 - x)$	$x = 4$
16.	$f(x) = 3(x - 5) + 9$	$x = 2$
17.	$f(x) = x^3 - 6x^2 + 5x$	$x = 0, 5, 1$
18.	$f(x) = x^3 - 9x^2 + 18x$	$x = 0, 3, 6$
19.	$f(x) = \dfrac{x + 2}{3} - \dfrac{x - 1}{5} - 1$	$x = 1$
20.	$f(x) = x - 3 - \dfrac{10}{x}$	$x = -2, 5$

In Exercises 21–34, solve the equation algebraically. Then write the equation in the form $f(x) = 0$ and use a graphing utility to verify the algebraic solution.

21. $2.7x - 0.4x = 1.2$

22. $3.5x - 8 = 0.5x$

23. $25(x - 3) = 12(x + 2) - 10$

24. $1200 = 300 + 2(x - 500)$

25. $\dfrac{3x}{2} + \dfrac{1}{4}(x - 2) = 10$

26. $\dfrac{2x}{3} + \dfrac{1}{2}(x - 5) = 6$

27. $0.60x + 0.40(100 - x) = 1.2$

28. $0.75x + 0.2(80 - x) = 20$

29. $\dfrac{x - 3}{3} = \dfrac{3x - 5}{2}$

30. $\dfrac{x - 3}{25} = \dfrac{x - 5}{12}$

31. $\dfrac{x - 5}{4} + \dfrac{x}{2} = 10$

32. $\dfrac{x - 5}{10} - \dfrac{x - 3}{5} = 1$

33. $(x + 2)^2 = x^2 - 6x + 1$

34. $(x + 1)^2 + 2(x - 2) = (x + 1)(x - 2)$

In Exercises 35–54, use a graphing utility to approximate any solutions of the equation. [Remember to write the equation in the form $f(x) = 0$.]

35. $\frac{1}{4}(x^2 - 10x + 17) = 0$

36. $-\frac{1}{2}(x^2 - 6x + 6) = 0$

37. $x^3 + x + 4 = 0$

38. $\frac{1}{9}x^3 + x + 4 = 0$

39. $2x^3 - x^2 - 18x + 9 = 0$

40. $4x^3 + 12x^2 - 26x - 24 = 0$

41. $x^4 = 2x^3 + 1$

42. $x^5 = 3 + 2x^3$

43. $\dfrac{2}{x+2} = 3$

44. $\dfrac{1}{x-3} = 2$

45. $\dfrac{5}{x} = 1 + \dfrac{3}{x+2}$

46. $\dfrac{3}{x} + 1 = \dfrac{3}{x-1}$

47. $|x - 3| = 4$

48. $|x + 1| = 6$

49. $|3x - 2| - 1 = 4$

50. $|4x + 1| + 2 = 8$

51. $\sqrt{x-2} = 3$

52. $\sqrt{x-4} = 8$

53. $2 + \sqrt{x-5} = 6$

54. $1 + \sqrt{x+3} = 6$

55. ***Exploration***

(a) Use a graphing utility to complete the table. Determine the interval in which the solution to the equation $3.2x - 5.8 = 0$ is located. Explain your reasoning.

x	-1	0	1	2	3	4
$3.2x - 5.8$						

(b) Use a graphing utility to complete the table. Determine the interval in which the solution to the equation $3.2x - 5.8 = 0$ is located. Explain how this process can be used to approximate the solution to any desired degree of accuracy.

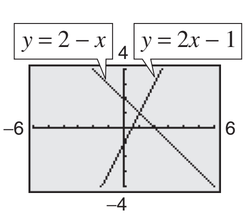
x	1.5	1.6	1.7	1.8	1.9	2
$3.2x - 5.8$						

(c) Use a graphing utility to verify graphically the solution to $3.2x - 5.8 = 0$ found in part (b).

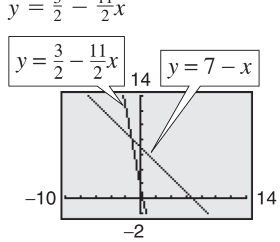
56. ***Exploration*** Use the procedure in Exercise 55 to approximate the solution of the equation $0.3(x - 1.5) - 2 = 0$ accurate to two decimal places.

In Exercises 57–64, determine any point(s) of intersection algebraically. Then verify your result numerically by creating a table of values for each function.

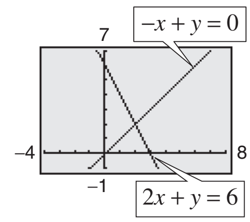
57. $y = 2 - x$
$y = 2x - 1$



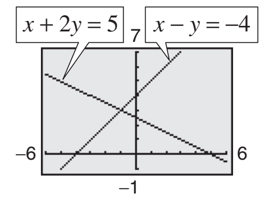
58. $y = 7 - x$
$y = \frac{3}{2} - \frac{11}{2}x$



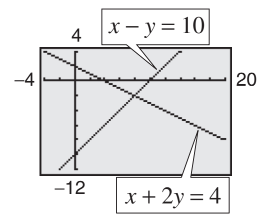
59. $2x + y = 6$
$-x + y = 0$



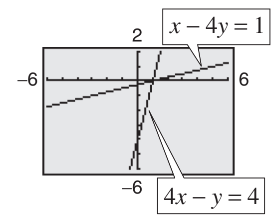
60. $x - y = -4$
$x + 2y = 5$



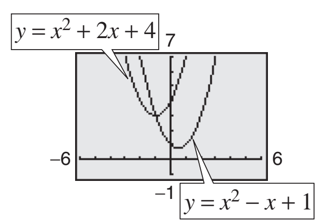
61. $x - y = 10$
$x + 2y = 4$



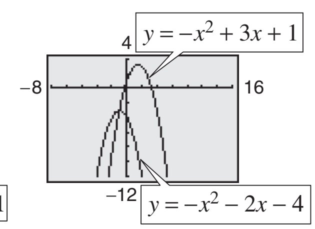
62. $4x - y = 4$
$x - 4y = 1$



63. $y = x^2 - x + 1$
$y = x^2 + 2x + 4$



64. $y = -x^2 + 3x + 1$
$y = -x^2 - 2x - 4$



In Exercises 65–70, use a graphing utility to approximate any points of intersection of the graphs of the equations. Check your results algebraically.

65. $y = 9 - 2x$
$y = x - 3$

66. $x - 3y = -2$
$5x - 2y = 11$

67. $y = 4 - x^2$
$y = 2x - 1$

68. $x^3 - y = 3$
$2x + y = 5$

69. $y = 2x^2$
$y = x^4 - 2x^2$

70. $y = -x$
$y = 2x - x^2$

In Exercises 71 and 72, evaluate the expression in two ways. (a) Calculate entirely on your calculator by storing intermediate results and then rounding the final answer to two decimal places. (b) Round both the numerator and denominator to two decimal places before dividing, and then round the final answer to two decimal places. Does the method in part (b) decrease the accuracy? Explain.

71. $\dfrac{1 + 0.73205}{1 - 0.73205}$

72. $\dfrac{1 + 0.86603}{1 - 0.86603}$

73. ***Travel Time*** On the first part of a 280-mile trip, a salesperson averaged 63 miles per hour. The salesperson averaged only 54 miles per hour on the last part of the trip because of an increased volume of traffic.

(a) Write the total time t for the trip as a function of the distance x traveled at an average speed of 63 miles per hour.

(b) Use a graphing utility to graph the time function. What is the domain of the function?

(c) Approximate the number of miles traveled at 63 miles per hour when the total time is 4 hours and 45 minutes.

74. ***Production*** An electronics company has fixed costs of \$25,000 per month and a variable cost of \$18.65 per 13-inch TV/VCR combination manufactured. (*Fixed costs* are those that occur regardless of the level of production.)

(a) Write the total monthly costs C as a function of the number of units x produced.

(b) Use a graphing utility to graph the cost function.

(c) Use the graph from part (b) to approximate the number of units that can be produced per month if total costs cannot exceed \$200,000. Verify algebraically. Is this problem better solved algebraically or graphically? Explain.

75. ***Mixture Problem*** A 55-gallon barrel contains a mixture with a concentration of 33% sodium chloride. You remove x gallons of this mixture and replace it with 100% sodium chloride.

(a) Write the amount A of sodium chloride in the final mixture as a function of x.

(b) Use a graphing utility to graph the concentration function. What is the domain of the function?

(c) Approximate (accurate to one decimal place) the value of x when the final mixture is 60% sodium chloride.

76. ***Geometry*** A rectangular horse corral with a perimeter of 230 meters has a length of x.

(a) Draw a diagram that gives a visual representation of the problem.

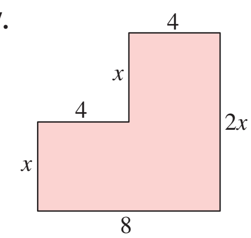
(b) Write the corral's area A as a function of x.

(c) Use a graphing utility to graph the area function. What is the domain of the function?

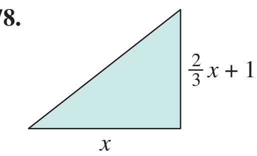
(d) Approximate (accurate to one decimal place) the dimensions of the corral when its area is 2000 square meters.

Geometry **In Exercises 77 and 78, (a) write a function for the area of the region, (b) use a graphing utility to graph the function, and (c) approximate the value of x when the area of the region is 200 square units.**

77.



78.



79. ***Income Tax*** The following information describes a possible negative income tax for a family consisting of two adults and two children. The plan would guarantee the poor a minimum income while encouraging a family to increase its private income ($0 \le x \le 20{,}000$). (A *subsidy* is a grant of money.)

Family's earned income: $I = x$

Subsidy: $S = 10{,}000 - \frac{1}{2}x$

Total income: $T = I + S$

(a) Write the total income T in terms of x.

(b) Use a graphing utility to find the earned income x when the subsidy is \$6600. Verify your answer algebraically.

(c) Use a graphing utility to find the earned income x when the total income is \$13,800. Verify your answer algebraically.

(d) Find the subsidy S graphically when the total income is \$12,500.

80. ***Hospitals*** The numbers y of hospitals in the United States from 1990 to 2003 can be modeled by the linear model $y = -77.6t + 6671$, $0 \le t \le 13$, where t is the year, with $t = 0$ corresponding to 1990. (Source: Health Forum)

(a) According to the model, when did the number of hospitals drop to 6000?

(b) What is the slope of the model and what does it tell you about the number of hospitals in the United States?

(c) Do you think the model can be used to predict the numbers of hospitals in the Unites States for years beyond 2003? If so, for what time period? Explain.

(d) Explain, both algebraically and graphically, how you could find when the number of hospitals drops to 5000 according to the model.

81. ***State Populations*** The populations (in thousands) of South Carolina S and Arizona A from 1980 to 2004 can be modeled by

$$S = 45.2t + 3087, \quad 0 \le t \le 24$$
$$A = 128.2t + 2533, \quad 0 \le t \le 24$$

where t represents the year, with $t = 0$ corresponding to 1980. (Source: U.S. Census Bureau)

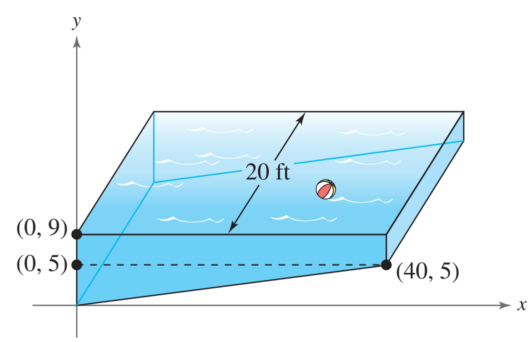
(a) Use a graphing utility to graph each model in the same viewing window over the appropriate domain. Approximate the point of intersection. Round your result to one decimal place. Explain the meaning of the coordinates of the point.

(b) Find the point of intersection algebraically. Round your result to one decimal place. What does the point of intersection represent?

(c) Explain the meaning of the slopes of both models and what it tells you about the population growth rates.

(d) Use the models to estimate the population of each state in 2010. Do the values seem reasonable? Explain.

82. ***Geometry*** Consider the swimming pool in the figure. (When finding its volume, use the fact that the volume is the area of the region on the vertical sidewall times the width of the pool.)



(a) Find the volume of the pool.

(b) Find an equation of the line representing the base of the pool.

(c) The depth of the water at the deep end of the pool is d feet. Show that the volume of water is

$$V(d) = \begin{cases} 80d^2, & 0 \le d \le 5 \\ 800d - 2000, & 5 < d \le 9 \end{cases}.$$

(d) Graph the volume function.

(e) Use a graphing utility to complete the table.

d	3	5	7	8
V				

(f) Approximate the depth of the water at the deep end when the volume is 4800 cubic feet.

(g) How many gallons of water are in the pool? (There are 7.48 gallons of water in 1 cubic foot.)

Synthesis

True or False? **In Exercises 83–85, determine whether the statement is true or false. Justify your answer.**

83. To find the y-intercept of a graph, let $x = 0$ and solve the equation for y.

84. Every linear equation has at least one y-intercept or x-intercept.

85. Two linear equations can have either one point of intersection or no points of intersection.

86. ***Writing*** You are solving the equation

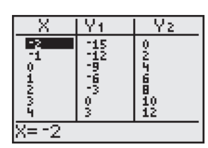
$$\frac{x}{x-1} - \frac{99}{100} = 0$$

for x, and you obtain $x = -99.1$ as your solution. Substituting this value back into the equation produces

$$\frac{-99.1}{-99.1 - 1} - \frac{99}{100} = 0.00000999 = 9.99 \times 10^{-6} \approx 0.$$

Is -99.1 a good approximation of the solution? Write a short paragraph explaining why or why not.

Exploration **In Exercises 87–90, use the table to solve each linear equation where $y_1 = f(x)$ and $y_2 = g(x)$.**



X	Y1	Y2
-2	-15	0
-1	-12	2
0	-9	4
1	-6	6
2	-3	8
3	0	10
4	3	12

X=-2

87. $f(x) = 0$

88. $g(x) = 0$

89. $g(x) = -f(x)$

90. $g(x) = 4f(x)$

Skills Review

In Exercises 91–94, rationalize the denominator.

91. $\dfrac{12}{5\sqrt{3}}$

92. $\dfrac{4}{\sqrt{10} - 2}$

93. $\dfrac{3}{8 + \sqrt{11}}$

94. $\dfrac{14}{3\sqrt{10} - 1}$

In Exercises 95–98, find the product.

95. $(x + 6)(3x - 5)$

96. $(3x + 13)(4x - 7)$

97. $(2x - 9)(2x + 9)$

98. $(4x + 1)^2$