2.5 Solving Other Types of Equations Algebraically

Polynomial Equations

In this section, the techniques for solving equations are extended to nonlinear and nonquadratic equations. At this point in the text, you have only four basic methods for solving nonlinear equations—*factoring*, *extracting square roots*, *completing the square*, and the *Quadratic Formula*. So the main goal of this section is to learn to *rewrite* nonlinear equations in a form to which you can apply one of these methods.

Example 1 shows how to use factoring to solve a **polynomial equation,** which is an equation that can be written in the general form

$$a_nx^n + a_{n-1}x^{n-1} + \cdot\cdot\cdot + a_2x^2 + a_1x + a_0 = 0.$$

Example 1 Solving a Polynomial Equation by Factoring

Solve $3x^4 = 48x^2$.

Solution

First write the polynomial equation in general form with zero on one side, factor the other side, and then set each factor equal to zero and solve.

$3x^4 = 48x^2$		Write original equation.
$3x^4 - 48x^2 = 0$		Write in general form.
$3x^2(x^2 - 16) = 0$		Factor out common factor.
$3x^2(x + 4)(x - 4) = 0$		Factor completely.
$3x^2 = 0$	$\Rightarrow$ $x = 0$	Set 1st factor equal to 0.
$x + 4 = 0$	$\Rightarrow$ $x = -4$	Set 2nd factor equal to 0.
$x - 4 = 0$	$\Rightarrow$ $x = 4$	Set 3rd factor equal to 0.

You can check these solutions by substituting in the original equation, as follows.

Check

$3(0)^4 \stackrel{?}{=} 48(0)^2$	Substitute 0 for x.
$0 = 0$	0 checks. ✓
$3(-4)^4 \stackrel{?}{=} 48(-4)^2$	Substitute -4 for x.
$768 = 768$	-4 checks. ✓
$3(4)^4 \stackrel{?}{=} 48(4)^2$	Substitute 4 for x.
$768 = 768$	4 checks. ✓

So, you can conclude that the solutions are $x = 0$, $x = -4$, and $x = 4$.

✓CHECKPOINT Now try Exercise 1.

What you should learn

- Solve polynomial equations of degree three or greater.
- Solve equations involving radicals.
- Solve equations involving fractions or absolute values.
- Use polynomial equations and equations involving radicals to model and to solve real-life problems.

Why you should learn it

Polynomial equations, radical equations, and absolute value equations can be used to model and solve real-life problems. For instance, in Exercise 79 on page 217, a radical equation can be used to model the total monthly cost of airplane flights between Chicago and Denver.

© Ralf-Finn Hestoft/Corbis

STUDY TIP

A common mistake that is made in solving an equation such as that in Example 1 is to divide each side of the equation by the variable factor x^2. This loses the solution $x = 0$. When solving an equation, always write the equation in general form, then factor the equation and set each factor equal to zero. Do not divide each side of an equation by a variable factor in an attempt to simplify the equation.

Example 2 Solving a Polynomial Equation by Factoring

Solve $2x^3 - 6x^2 + 6x - 18 = 0$.

Solution

This equation has a common factor of 2. You can simplify the equation by first dividing each side of the equation by 2.

$$2x^3 - 6x^2 + 6x - 18 = 0 \qquad \text{Write original equation.}$$
$$x^3 - 3x^2 + 3x - 9 = 0 \qquad \text{Divide each side by 2.}$$
$$x^2(x - 3) + 3(x - 3) = 0 \qquad \text{Group terms.}$$
$$(x - 3)(x^2 + 3) = 0 \qquad \text{Factor by grouping.}$$
$$x - 3 = 0 \Rightarrow x = 3 \qquad \text{Set 1st factor equal to 0.}$$
$$x^2 + 3 = 0 \Rightarrow x = \pm\sqrt{3}i \qquad \text{Set 2nd factor equal to 0.}$$

The equation has three solutions: $x = 3$, $x = \sqrt{3}i$, and $x = -\sqrt{3}i$. Check these solutions in the original equation. Figure 2.38 verifies the solutions graphically.

CHECKPOINT Now try Exercise 5.

Prerequisite Skills

Review the techniques for factoring polynomial expressions in Section P.3, if you have difficulty with this example.

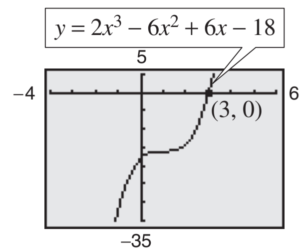

Figure 2.38

Occasionally, mathematical models involve equations that are of **quadratic type.** In general, an equation is of quadratic type if it can be written in the form

$$au^2 + bu + c = 0$$

where $a \neq 0$ and u is an algebraic expression.

Example 3 Solving an Equation of Quadratic Type

Solve $x^4 - 3x^2 + 2 = 0$.

Solution

This equation is of quadratic type with $u = x^2$. To solve this equation, you can use the Quadratic Formula.

$$x^4 - 3x^2 + 2 = 0 \qquad \text{Write original equation.}$$
$$(x^2)^2 - 3(x^2) + 2 = 0 \qquad \text{Write in quadratic form.}$$
$$x^2 = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(2)}}{2(1)} \qquad \text{Quadratic Formula}$$
$$x^2 = \frac{3 \pm 1}{2} \qquad \text{Simplify.}$$
$$x^2 = 2 \Rightarrow x = \pm\sqrt{2} \qquad \text{Solutions}$$
$$x^2 = 1 \Rightarrow x = \pm 1 \qquad \text{Solutions}$$

The equation has four solutions: $x = -1$, $x = 1$, $x = \sqrt{2}$, and $x = -\sqrt{2}$. Check these solutions in the original equation. Figure 2.39 verifies the solutions graphically.

CHECKPOINT Now try Exercise 7.

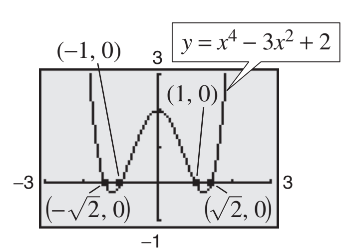

Figure 2.39

Equations Involving Radicals

An equation involving a radical expression can often be cleared of radicals by raising each side of the equation to an appropriate power. When using this procedure, remember to check for extraneous solutions.

To show why the radical should be isolated, have students square each side of the equation before isolating the radical. Compare problem-solving strategies to help convince students of the need to isolate the radical.

Example 4 Solving an Equation Involving a Radical

Solve $\sqrt{2x+7} - x = 2$.

Algebraic Solution

$\sqrt{2x+7} - x = 2$	Write original equation.
$\sqrt{2x+7} = x + 2$	Isolate radical.
$2x + 7 = x^2 + 4x + 4$	Square each side.
$x^2 + 2x - 3 = 0$	Write in general form.
$(x+3)(x-1) = 0$	Factor.
$x + 3 = 0 \Rightarrow x = -3$	Set 1st factor equal to 0.
$x - 1 = 0 \Rightarrow x = 1$	Set 2nd factor equal to 0.

By substituting into the original equation, you can determine that $x = -3$ is extraneous, whereas $x = 1$ is valid. So, the equation has only one real solution: $x = 1$.

CHECKPOINT Now try Exercise 29.

Graphical Solution

First rewrite the equation as $\sqrt{2x+7} - x - 2 = 0$. Then use a graphing utility to graph $y = \sqrt{2x+7} - x - 2$, as shown in Figure 2.40. Notice that the domain is $x \geq -\frac{7}{2}$ because the expression under the radical cannot be negative. There appears to be one solution near $x = 1$. Use the *zoom* and *trace* features, as shown in Figure 2.41, to approximate the only solution to be $x = 1$.

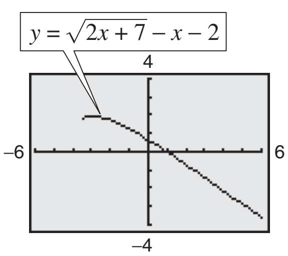

Figure 2.40

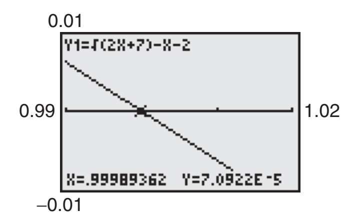

Figure 2.41

Example 5 Solving an Equation Involving Two Radicals

$\sqrt{2x+6} - \sqrt{x+4} = 1$	Original equation
$\sqrt{2x+6} = 1 + \sqrt{x+4}$	Isolate $\sqrt{2x+6}$.
$2x + 6 = 1 + 2\sqrt{x+4} + (x+4)$	Square each side.
$x + 1 = 2\sqrt{x+4}$	Isolate $2\sqrt{x+4}$.
$x^2 + 2x + 1 = 4(x+4)$	Square each side.
$x^2 - 2x - 15 = 0$	Write in general form.
$(x-5)(x+3) = 0$	Factor.
$x - 5 = 0 \Rightarrow x = 5$	Set 1st factor equal to 0.
$x + 3 = 0 \Rightarrow x = -3$	Set 2nd factor equal to 0.

By substituting into the original equation, you can determine that $x = -3$ is extraneous, whereas $x = 5$ is valid. Figure 2.42 verifies that $x = 5$ is the only solution.

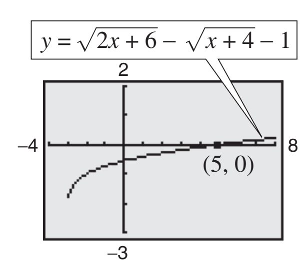

Figure 2.42

CHECKPOINT Now try Exercise 38.

Example 6 Solving an Equation with Rational Exponents

Solve $(x + 1)^{2/3} = 4$.

Algebraic Solution

$$(x + 1)^{2/3} = 4 \qquad \text{Write original equation.}$$

$$\sqrt[3]{(x + 1)^2} = 4 \qquad \text{Rewrite in radical form.}$$

$$(x + 1)^2 = 64 \qquad \text{Cube each side.}$$

$$x + 1 = \pm 8 \qquad \text{Take square root of each side.}$$

$$x = 7,\ x = -9 \qquad \text{Subtract 1 from each side.}$$

Substitute $x = 7$ and $x = -9$ into the original equation to determine that both are valid solutions.

Graphical Solution

Use a graphing utility to graph $y_1 = \sqrt[3]{(x + 1)^2}$ and $y_2 = 4$ in the same viewing window. Use the *intersect* feature of the graphing utility to approximate the solutions to be $x = -9$ and $x = 7$, as shown in Figure 2.43.

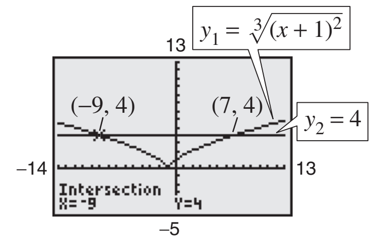

Figure 2.43

CHECKPOINT Now try Exercise 41.

Equations Involving Fractions or Absolute Values

As demonstrated in Section 2.1, you can solve an equation involving fractions algebraically by multiplying each side of the equation by the least common denominator of all terms in the equation to clear the equation of fractions.

Example 7 Solving an Equation Involving Fractions

Solve $\dfrac{2}{x} = \dfrac{3}{x - 2} - 1$.

Solution

For this equation, the least common denominator of the three terms is $x(x - 2)$, so you can begin by multiplying each term of the equation by this expression.

$$\frac{2}{x} = \frac{3}{x - 2} - 1 \qquad \text{Write original equation.}$$

$$x(x - 2)\frac{2}{x} = x(x - 2)\frac{3}{x - 2} - x(x - 2)(1) \qquad \text{Multiply each term by the LCD.}$$

$$2(x - 2) = 3x - x(x - 2), \quad x \neq 0, 2 \qquad \text{Simplify.}$$

$$x^2 - 3x - 4 = 0 \qquad \text{Write in general form.}$$

$$(x - 4)(x + 1) = 0 \qquad \text{Factor.}$$

$$x - 4 = 0 \quad \Rightarrow \quad x = 4 \qquad \text{Set 1st factor equal to 0.}$$

$$x + 1 = 0 \quad \Rightarrow \quad x = -1 \qquad \text{Set 2nd factor equal to 0.}$$

The equation has two solutions: $x = 4$ and $x = -1$. Check these solutions in the original equation. Use a graphing utility to verify these solutions graphically.

CHECKPOINT Now try Exercise 53.

Exploration

Using *dot* mode, graph the equations

$$y_1 = \frac{2}{x}$$

and

$$y_2 = \frac{3}{x - 2} - 1$$

in the same viewing window. How many times do the graphs of the equations intersect? What does this tell you about the solution to Example 7?

TECHNOLOGY TIP

Graphs of functions involving variable denominators can be tricky because of the way graphing utilities skip over points at which the denominator is zero. You will study graphs of such functions in Sections 3.5 and 3.6.

Example 8 Solving an Equation Involving an Absolute Value

Solve $|x^2 - 3x| = -4x + 6$.

Solution

Begin by writing the equation as $|x^2 - 3x| + 4x - 6 = 0$. From the graph of $y = |x^2 - 3x| + 4x - 6$ in Figure 2.44, you can estimate the solutions to be $x = -3$ and $x = 1$. These can be verified by substitution into the equation. To solve an equation involving an absolute value *algebraically*, you must consider the fact that the expression inside the absolute value symbols can be positive or negative. This results in *two* separate equations, each of which must be solved.

First Equation:

$x^2 - 3x = -4x + 6$	Use positive expression.
$x^2 + x - 6 = 0$	Write in general form.
$(x + 3)(x - 2) = 0$	Factor.
$x + 3 = 0 \Rightarrow x = -3$	Set 1st factor equal to 0.
$x - 2 = 0 \Rightarrow x = 2$	Set 2nd factor equal to 0.

Second Equation:

$-(x^2 - 3x) = -4x + 6$	Use negative expression.
$x^2 - 7x + 6 = 0$	Write in general form.
$(x - 1)(x - 6) = 0$	Factor.
$x - 1 = 0 \Rightarrow x = 1$	Set 1st factor equal to 0.
$x - 6 = 0 \Rightarrow x = 6$	Set 2nd factor equal to 0.

Check

$\lvert(-3)^2 - 3(-3)\rvert \stackrel{?}{=} -4(-3) + 6$	Substitute -3 for x.
$18 = 18$	-3 checks. ✓
$\lvert 2^2 - 3(2)\rvert \stackrel{?}{=} -4(2) + 6$	Substitute 2 for x.
$2 \neq -2$	2 does not check.
$\lvert 1^2 - 3(1)\rvert \stackrel{?}{=} -4(1) + 6$	Substitute 1 for x.
$2 = 2$	1 checks. ✓
$\lvert 6^2 - 3(6)\rvert \stackrel{?}{=} -4(6) + 6$	Substitute 6 for x.
$18 \neq -18$	6 does not check.

The equation has only two solutions: $x = -3$ and $x = 1$, just as you obtained by graphing.

CHECKPOINT Now try Exercise 66.

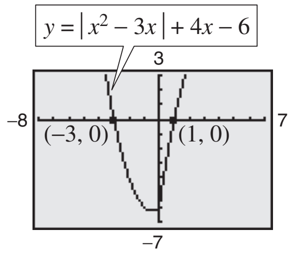

Figure 2.44

Prerequisite Skills

If you have difficulty with this example, review the discussion of absolute value in Section P.1.

Additional Example

$$\frac{3x}{x+1} = \frac{12}{x^2-1} + 2$$

$$(x^2-1)\frac{3x}{x+1} = (x^2-1)\frac{12}{x^2-1} + (x^2-1)2$$

$$3x(x-1) = 12 + (x^2-1)2$$

$$3x^2 - 3x = 12 + 2x^2 - 2$$

$$x^2 - 3x - 10 = 0$$

$$(x+2)(x-5) = 0$$

$$x + 2 = 0 \rightarrow x = -2$$

$$x - 5 = 0 \rightarrow x = 5$$

Exploration

In Figure 2.44, the graph of $y = |x^2 - 3x| + 4x - 6$ appears to be a straight line to the right of the y-axis. Is it? Explain your reasoning.

Applications

It would be impossible to categorize the many different types of applications that involve nonlinear and nonquadratic models. However, from the few examples and exercises that are given, you will gain some appreciation for the variety of applications that can occur.

Example 9 Reduced Rates

A ski club chartered a bus for a ski trip at a cost of \$480. In an attempt to lower the bus fare per skier, the club invited nonmembers to go along. After five nonmembers joined the trip, the fare per skier decreased by \$4.80. How many club members are going on the trip?

Solution

Begin the solution by creating a verbal model and assigning labels.

Verbal Model: $\boxed{\text{Cost per skier}} \cdot \boxed{\text{Number of skiers}} = \boxed{\text{Cost of trip}}$

Labels:

Cost of trip $= 480$	(dollars)
Number of ski club members $= x$	(people)
Number of skiers $= x + 5$	(people)
Original cost per member $= \dfrac{480}{x}$	(dollars per person)
Cost per skier $= \dfrac{480}{x} - 4.80$	(dollars per person)

Equation:

$$\left(\frac{480}{x} - 4.80\right)(x + 5) = 480$$

$$\left(\frac{480 - 4.8x}{x}\right)(x + 5) = 480 \qquad \text{Write } \left(\frac{480}{x} - 4.80\right) \text{ as a fraction.}$$

$$(480 - 4.8x)(x + 5) = 480x \qquad \text{Multiply each side by } x.$$

$$480x + 2400 - 4.8x^2 - 24x = 480x \qquad \text{Multiply.}$$

$$-4.8x^2 - 24x + 2400 = 0 \qquad \text{Subtract } 480x \text{ from each side.}$$

$$x^2 + 5x - 500 = 0 \qquad \text{Divide each side by } -4.8.$$

$$(x + 25)(x - 20) = 0 \qquad \text{Factor.}$$

$$x + 25 = 0 \Rightarrow x = -25$$

$$x - 20 = 0 \Rightarrow x = 20$$

Choosing the positive value of x, you can conclude that 20 ski club members are going on the trip. Check this in the original statement of the problem, as follows.

$$\left(\frac{480}{20} - 4.80\right)(20 + 5) \stackrel{?}{=} 480 \qquad \text{Substitute 20 for } x.$$

$$(24 - 4.80)25 \stackrel{?}{=} 480 \qquad \text{Simplify.}$$

$$480 = 480 \qquad \text{20 checks. } \checkmark$$

CHECKPOINT Now try Exercise 71.

Interest in a savings account is calculated by one of three basic methods: simple interest, interest compounded n times per year, and interest compounded continuously. The next example uses the formula for interest that is compounded n times per year.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

In this formula, A is the balance in the account, P is the principal (or original deposit), r is the annual interest rate (in decimal form), n is the number of compoundings per year, and t is the time in years. In Chapter 4, you will study a derivation of the formula above for interest compounded continuously.

Example 10 Compound Interest

When you were born, your grandparents deposited \$5000 in a long-term investment in which the interest was compounded quarterly. Today, on your 25th birthday, the value of the investment is \$25,062.59. What is the annual interest rate for this investment?

Solution

Formula: $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$

Labels:

Balance $= A = 25{,}062.59$	(dollars)
Principal $= P = 5000$	(dollars)
Time $= t = 25$	(years)
Compoundings per year $= n = 4$	(compoundings per year)
Annual interest rate $= r$	(percent in decimal form)

Equation: $25{,}062.59 = 5000\left(1 + \dfrac{r}{4}\right)^{4(25)}$

$$\frac{25{,}062.59}{5000} = \left(1 + \frac{r}{4}\right)^{100}$$ Divide each side by 5000.

$$5.0125 \approx \left(1 + \frac{r}{4}\right)^{100}$$ Use a calculator.

$$(5.0125)^{1/100} = 1 + \frac{r}{4}$$ Raise each side to reciprocal power.

$$1.01625 \approx 1 + \frac{r}{4}$$ Use a calculator.

$$0.01625 = \frac{r}{4}$$ Subtract 1 from each side.

$$0.065 = r$$ Multiply each side by 4.

The annual interest rate is about 0.065, or 6.5%. Check this in the original statement of the problem.

CHECKPOINT Now try Exercise 75.

Activities

1. Solve $9x^4 - 9x^2 + 2 = 0$.
   Answer: $x = \pm\frac{\sqrt{3}}{3}, x = \pm\frac{\sqrt{6}}{3}$
2. Solve $\sqrt{3x + 10} - x = 4$.
   Answer: $x = -2, x = -3$
3. Solve $|5x + 2| = 22$.
   Answer: $x = -\frac{24}{5}, x = 4$

2.5 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blanks.

1. The equation $a_nx^n + a_{n-1}x^{n-1} + \cdot \cdot \cdot + a_2x^2 + a_1x + a_0 = 0$ is a polynomial equation of degree _______ written in standard form.
2. Squaring each side of an equation, multiplying each side of an equation by a variable quantity, and raising both sides of an equation to a rational power are all operations that can introduce _______ solutions to a given equation.
3. The equation $x^4 - 5x^2 + 4 = 0$ is an equation that is said to be of _______ type.

In Exercises 1–14, find all solutions of the equation algebraically. Use a graphing utility to verify the solutions graphically.

1. $4x^4 - 16x^2 = 0$
2. $8x^4 - 18x^2 = 0$
3. $5x^3 + 30x^2 + 45x = 0$
4. $9x^4 - 24x^3 + 16x^2 = 0$
5. $x^3 + 3 = 3x^2 + x$
6. $x^4 - 2x^3 = 16 + 8x - 4x^3$
7. $x^4 - 4x^2 + 3 = 0$
8. $x^4 + 5x^2 - 36 = 0$
9. $4x^4 - 65x^2 + 16 = 0$
10. $36t^4 + 29t^2 - 7 = 0$
11. $\dfrac{1}{t^2} + \dfrac{8}{t} + 15 = 0$
12. $6 - \dfrac{1}{x} - \dfrac{1}{x^2} = 0$
13. $6\left(\dfrac{s}{s+1}\right)^2 + 5\left(\dfrac{s}{s+1}\right) - 6 = 0$
14. $8\left(\dfrac{t}{t-1}\right)^2 - 2\left(\dfrac{t}{t-1}\right) - 3 = 0$

Graphical Analysis **In Exercises 15–18, (a) use a graphing utility to graph the equation, (b) use the graph to approximate any x-intercepts of the graph, (c) set $y = 0$ and solve the resulting equation, and (d) compare the result of part (c) with the x-intercepts of the graph.**

15. $y = x^3 - 2x^2 - 3x$
16. $y = 2x^4 - 15x^3 + 18x^2$
17. $y = x^4 - 10x^2 + 9$
18. $y = x^4 - 29x^2 + 100$

In Exercises 19–48, find all solutions of the equation algebraically. Use a graphing utility to verify the solutions graphically.

19. $3\sqrt{x} - 10 = 0$
20. $4\sqrt{x} - 3 = 0$
21. $\sqrt{x - 10} - 4 = 0$
22. $\sqrt{2x + 5} + 3 = 0$
23. $\sqrt[3]{2x + 5} + 3 = 0$
24. $\sqrt[3]{3x + 1} - 5 = 0$
25. $\sqrt[3]{2x + 1} + 8 = 0$
26. $\sqrt[3]{4x - 3} + 2 = 0$
27. $\sqrt{5x - 26} + 4 = x$
28. $x - \sqrt{8x - 31} = 5$
29. $\sqrt{x + 1} - 3x = 1$
30. $\sqrt{x + 5} - 2x = 3$
31. $\sqrt{x + 1} = \sqrt{3x + 1}$
32. $\sqrt{x + 5} = \sqrt{2x - 5}$
33. $2x + 9\sqrt{x} - 5 = 0$
34. $6x - 7\sqrt{x} - 3 = 0$
35. $\sqrt{x} - \sqrt{x - 5} = 1$
36. $\sqrt{x} + \sqrt{x - 20} = 10$
37. $3\sqrt{x - 5} - \sqrt{x - 1} = 0$
38. $4\sqrt{x - 3} - \sqrt{6x - 17} = 3$
39. $3x^{1/3} + 2x^{2/3} = 5$
40. $9t^{2/3} + 24t^{1/3} + 16 = 0$
41. $(x - 5)^{2/3} = 16$
42. $(x - 1)^{3/2} = 8$
43. $(x - 8)^{2/3} = 25$
44. $(x - 2)^{2/3} = 9$
45. $(x^2 - 5x - 2)^{1/3} = -2$
46. $(x^2 - x - 22)^{4/3} = 16$
47. $3x(x - 1)^{1/2} + 2(x - 1)^{3/2} = 0$
48. $4x^2(x - 1)^{1/3} + 6x(x - 1)^{4/3} = 0$

Graphical Analysis **In Exercises 49–52, (a) use a graphing utility to graph the equation, (b) use the graph to approximate any x-intercepts of the graph, (c) set $y = 0$ and solve the resulting equation, and (d) compare the result of part (c) with the x-intercepts of the graph.**

49. $y = \sqrt{11x - 30} - x$
50. $y = 2x - \sqrt{15 - 4x}$
51. $y = \sqrt{7x + 36} - \sqrt{5x + 16} - 2$
52. $y = 3\sqrt{x} - \dfrac{4}{\sqrt{x}} - 4$

In Exercises 53–66, find all solutions of the equation. Use a graphing utility to verify your solutions graphically.

53. $x = \dfrac{3}{x} + \dfrac{1}{2}$
54. $\dfrac{4}{x} - \dfrac{5}{3} = \dfrac{x}{6}$
55. $\dfrac{1}{x} - \dfrac{1}{x + 1} = 3$
56. $\dfrac{4}{x + 1} - \dfrac{3}{x + 2} = 1$

57. $\dfrac{20 - x}{x} = x$

58. $4x + 1 = \dfrac{3}{x}$

59. $\dfrac{x}{x^2 - 4} + \dfrac{1}{x + 2} = 3$

60. $\dfrac{x - 2}{x} - \dfrac{1}{x + 2} = 0$

61. $|2x - 1| = 5$

62. $|3x + 2| = 7$

63. $|x| = x^2 + x - 3$

64. $|x - 10| = x^2 - 10x$

65. $|x + 1| = x^2 - 5$

66. $x - 10 = |x^2 - 10x|$

Graphical Analysis **In Exercises 67–70, (a) use a graphing utility to graph the equation, (b) use the graph to approximate any *x*-intercepts of the graph, (c) set $y = 0$ and solve the resulting equation, and (d) compare the result of part (c) with the *x*-intercepts of the graph.**

67. $y = \dfrac{1}{x} - \dfrac{4}{x - 1} - 1$

68. $y = x + \dfrac{9}{x + 1} - 5$

69. $y = |x + 1| - 2$

70. $y = |x - 2| - 3$

71. ***Chartering a Bus*** A college charters a bus for \$1700 to take a group to a museum. When six more students join the trip, the cost per student drops by \$7.50. How many students were in the original group?

72. ***Renting an Apartment*** Three students are planning to rent an apartment for a year and share equally in the cost. By adding a fourth person, each person could save \$75 a month. How much is the monthly rent?

73. ***Airspeed*** An airline runs a commuter flight between Portland, Oregon and Seattle, Washington, which are 145 miles apart. If the average speed of the plane could be increased by 40 miles per hour, the travel time would be decreased by 12 minutes. What airspeed is required to obtain this decrease in travel time? Round your result to one decimal place.

74. ***Average Speed*** A family drove 1080 miles to their vacation lodge. Because of increased traffic density, their average speed on the return trip was decreased by 6 miles per hour and the trip took $2\frac{1}{2}$ hours longer. Determine their average speed on the way to the lodge.

75. ***Mutual Funds*** A deposit of \$2500 in a mutual fund reaches a balance of \$3052.49 after 5 years. What annual interest rate on a certificate of deposit compounded monthly would yield an equivalent return?

76. ***Mutual Funds*** A sales representative for a mutual funds company describes a "guaranteed investment fund" that the company is offering to new investors. You are told that if you deposit \$10,000 in the fund you will be guaranteed a return of at least \$25,000 after 20 years. (Assume the interest is compounded quarterly.)

(a) What is the annual interest rate if the investment only meets the minimum guaranteed amount?

(b) After 20 years, you receive \$32,000. What is the annual interest rate?

77. ***Transplants*** The numbers N of lung transplants in the United States from 1990 to 2004 can be approximated by the model, $N = 206 + 248.7\sqrt{t}$, $0 \le t \le 14$, where t is the year, with $t = 0$ corresponding to 1990. (Source: U.S. Department of Health and Human Services)

(a) Use the *table* feature of a graphing utility to find the number of transplants each year from 1990 to 2004.

(b) According to the table, when did the number of transplants each year reach 500 and 1000?

(c) Verify your answers in part (b) algebraically.

(d) Use the *zoom* and *trace* features of a graphing utility to verify your answers in parts (b) and (c).

(e) According to the model, when will the number of transplants reach 1500 and 2000 per year? Do your answers seem reasonable? Explain.

78. ***Juvenile Crime*** The table shows the numbers (in thousands) of violent crimes committed by juveniles from 1995 to 2003. (Source: U.S. Federal Bureau of Investigation)

Year	Number of crimes (in millions)
1995	123.131
1996	104.455
1997	100.273
1998	90.201
1999	81.715
2000	78.450
2001	78.443
2002	71.059
2003	69.060

(a) Use a graphing utility to create a scatter plot of the data, with $t = 5$ corresponding to 1995.

(b) A model that approximates the data is

$$N = \frac{1000}{0.8255t + 4.240}, \quad 5 \le t \le 13$$

where N is the number of crimes (in millions) and t is the year, with $t = 5$ corresponding to 1995. Graph the model and the data in the same window. Is the model a good fit? Explain.

(c) According to the model, when, if ever, will the number of violent crimes committed by juveniles decrease to 50 million? 25 million?

79. ***Airline Passengers*** An airline offers daily flights between Chicago and Denver. The total monthly cost C (in millions of dollars) of these flights is modeled by $C = \sqrt{0.2x + 1}$, where x is the number of passengers (in thousands). The total cost of the flights for June is 2.5 million dollars. How many passengers flew in June?

80. ***Demand*** The demand equation for a video game is modeled by

$$p = 40 - \sqrt{0.01x + 1}$$

where x is the number of units demanded per day and p is the price per unit. Approximate the demand when the price is \$37.55.

81. ***Demand*** The demand equation for a high-definition television set is modeled by

$$p = 800 - \sqrt{0.01x + 1}$$

where x is the number of units demanded per month and p is the price per unit. Approximate the demand when the price is \$750.

82. ***Baseball*** A baseball diamond has the shape of a square in which the distance from home plate to second base is approximately $127\frac{1}{2}$ feet. Approximate the distance between the bases.

83. ***Saturated Steam*** The temperature T (in degrees Fahrenheit) of saturated steam increases as pressure increases. This relationship is approximated by the model

$$T = 75.82 - 2.11x + 43.51\sqrt{x}, \quad 5 \le x \le 40$$

where x is the absolute pressure (in pounds per square inch).

(a) Use a graphing utility to graph the function over the specified domain.

(b) The temperature of steam at sea level ($x = 14.696$) is 212°F. Evaluate the model at this pressure and verify the result graphically.

(c) Use the model to approximate the pressure for a steam temperature of 240°F.

84. ***Meteorology*** A meteorologist is positioned 100 feet from the point at which a weather balloon is launched. When the balloon is at height h, the distance d (in feet) between the meteorologist and the balloon is $d = \sqrt{100^2 + h^2}$.

(a) Use a graphing utility to graph the equation. Use the *trace* feature to approximate the value of h when $d = 200$.

(b) Complete the table. Use the table to approximate the value of h when $d = 200$.

h	160	165	170	175	180	185
d						

(c) Find h algebraically when $d = 200$.

(d) Compare the results of each method. In each case, what information did you gain that wasn't revealed by another solution method?

Synthesis

True or False? **In Exercises 85 and 86, determine whether the statement is true or false. Justify your answer.**

85. An equation can never have more than one extraneous solution.

86. When solving an absolute value equation, you will always have to check more than one solution.

Think About It **In Exercises 87–92, find an equation having the given solutions. (There are many correct answers.)**

87. $\pm\sqrt{2}, 4$

88. $2, \pm\sqrt{5}$

89. $2, -1, \frac{1}{2}, -3$

90. $-1, \frac{3}{2}, \pm 2$

91. $\pm 2, \pm i$

92. $\pm 4i, \pm 6$

Think About It **In Exercises 93 and 94, find x such that the distance between the points is 13.**

93. $(1, 2), (x, -10)$

94. $(-8, 0), (x, 5)$

In Exercises 95 and 96, consider an equation of the form $x + |x - a| = b$, where a and b are constants.

95. Find a and b when the solution of the equation is $x = 9$. (There are many correct answers.)

96. ***Writing*** Write a short paragraph listing the steps required to solve this equation involving absolute values and explain why it is important to check your solutions.

In Exercises 97 and 98, consider an equation of the form $x + \sqrt{x - a} = b$, where a and b are constants.

97. Find a and b when the solution of the equation is $x = 20$. (There are many correct answers.)

98. ***Writing*** Write a short paragraph listing the steps required to solve this equation involving radicals and explain why it is important to check your solutions.

Skills Review

In Exercises 99–102, simplify the expression.

99. $\frac{8}{3x} + \frac{3}{2x}$

100. $\frac{2}{x^2 - 4} - \frac{1}{x^2 - 3x + 2}$

101. $\frac{2}{z + 2} - \left(3 - \frac{2}{z}\right)$

102. $25y^2 \div \frac{xy}{5}$

In Exercises 103 and 104, find all real solutions of the equation.

103. $x^2 - 22x + 121 = 0$

104. $x(x - 20) + 3(x - 20) = 0$