

## What Did You Learn?

### Key Terms

extraneous solution, *p.* 167  
 mathematical modeling, *p.* 167  
 zero of a function, *p.* 177  
 point of intersection, *p.* 180  
 imaginary unit  $i$ , *p.* 187  
 complex number, *p.* 187

imaginary number, *p.* 187  
 complex conjugates, *p.* 190  
 quadratic equation, *p.* 195  
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 polynomial equation, *p.* 209  
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solution set of an inequality, *p.* 219  
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 critical numbers, *p.* 223  
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 positive correlation, *p.* 233  
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### Key Concepts

#### 2.1 ■ Solve and use linear equations

1. To solve an equation in  $x$  means to find all values of  $x$  for which the equation is true.
2. An equation that is true for every real number in the domain of the variable is called an identity.
3. An equation that is true for just some (or even none) of the real numbers in the domain of the variable is called a conditional equation.
4. To form a mathematical model, begin by using a verbal description of the problem to form a verbal model. Then, after assigning labels to the quantities in the verbal model, write the algebraic equation.

#### 2.2 ■ Find intercepts, zeros, and solutions of equations

1. The point  $(a, 0)$  is an  $x$ -intercept and the point  $(0, b)$  is a  $y$ -intercept of the graph of  $y = f(x)$ .
2. The number  $a$  is a zero of the function  $f$ .
3. The number  $a$  is a solution of the equation  $f(x) = 0$ .

#### 2.3 ■ Perform operations with complex numbers and plot complex numbers

1. If  $a$  and  $b$  are real numbers and  $i = \sqrt{-1}$ , the number  $a + bi$  is a complex number written in standard form.
2. Add:  $(a + bi) + (c + di) = (a + c) + (b + d)i$   
 Subtract:  $(a + bi) - (c + di) = (a - c) + (b - d)i$   
 Multiply:  $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$   
 Divide:  $\frac{a + bi}{c + di} \cdot \frac{c - di}{c - di} = \frac{ac + bd}{c^2 + d^2} + \left(\frac{bc - ad}{c^2 + d^2}\right)i$
3. The complex plane consists of a real (horizontal) axis and an imaginary (vertical) axis. The point that corresponds to the complex number  $a + bi$  is  $(a, b)$ .

#### 2.4 ■ Solve quadratic equations

1. Methods for solving quadratic equations include factoring, extracting square roots, completing the square, and using the Quadratic Formula.

2. Quadratic equations can have two real solutions, one repeated real solution, or two complex solutions.

#### 2.5 ■ Solve other types of equations

1. To solve a polynomial equation, factor if possible. Then use the methods used in solving linear and quadratic equations.
2. To solve an equation involving a radical, isolate the radical on one side of the equation, and raise each side to an appropriate power.
3. To solve an equation with a fraction, multiply each term by the LCD, then solve the resulting equation.
4. To solve an equation involving an absolute value, isolate the absolute value term on one side of the equation. Then set up two equations, one where the absolute value term is positive and one where the absolute value term is negative. Solve both equations.

#### 2.6 ■ Solve inequalities

1. To solve an inequality involving an absolute value, rewrite the inequality as a double inequality or as a compound inequality.
2. To solve a polynomial inequality, write the polynomial in general form, find all the real zeros (critical numbers) of the polynomial, and test the intervals bounded by the critical numbers to determine the intervals that are solutions to the polynomial inequality.
3. To solve a rational inequality, find the  $x$ -values for which the rational expression is 0 or undefined (critical numbers) and test the intervals bounded by the critical numbers to determine the intervals that are solutions to the rational inequality.

#### 2.7 ■ Use scatter plots and find linear models

1. A scatter plot is a graphical representation of data written as a set of ordered pairs.
2. The best-fitting linear model can be found using the *linear regression* feature of a graphing utility or a computer program.

## Review Exercises

See [www.CalcChat.com](http://www.CalcChat.com) for worked-out solutions to odd-numbered exercises.

**2.1** In Exercises 1 and 2, determine whether each value of  $x$  is a solution of the equation.

Equation	Values
1. $6 + \frac{3}{x-4} = 5$	(a) $x = 5$ (b) $x = 0$ (c) $x = -2$ (d) $x = 1$
2. $6 + \frac{2}{x+3} = \frac{6x+1}{3}$	(a) $x = -3$ (b) $x = 3$ (c) $x = 0$ (d) $x = -\frac{2}{3}$

In Exercises 3–12, solve the equation (if possible). Then use a graphing utility to verify your solution.

- |  |   |
|--|---|
| 3. $\frac{18}{x} = \frac{10}{x-4}$                     | 4. $\frac{2}{x} = \frac{5}{x-2}$        |
| 5. $\frac{5}{x-2} = \frac{13}{2x-3}$                   | 6. $\frac{10}{x+1} = \frac{12}{3x-2}$   |
| 7. $14 + \frac{2}{x-1} = 10$                           | 8. $10 + \frac{2}{x-1} = 4$             |
| 9. $6 - \frac{11}{x} = 3 + \frac{7}{x}$                | 10. $2 - \frac{1}{x} = 4 + \frac{3}{x}$ |
| 11. $\frac{9x}{3x-1} - \frac{4}{3x+1} = 3$             |   |
| 12. $\frac{5}{x-5} + \frac{1}{x+5} = \frac{2}{x^2-25}$ |   |

**13. Profit** In October, a greeting card company's total profit was 12% more than it was in September. The total profit for the two months was \$689,000. Find the profit for each month.

**14. Mixture Problem** A car radiator contains 10 liters of a 30% antifreeze solution. How many liters will have to be replaced with pure antifreeze if the resulting solution is to be 50% antifreeze?

**15. Height** To obtain the height of a tree, you measure the tree's shadow and find that it is 8 meters long. You also measure the shadow of a two-meter lamppost and find that it is 75 centimeters long.

- (a) Draw a diagram that illustrates the problem. Let  $h$  represent the height of the tree.  
(b) Find the height of the tree in meters.

**16. Investment** You invest \$12,000 in a fund paying  $2\frac{1}{2}\%$  simple interest and \$10,000 in a fund with a variable interest rate. At the end of the year, you were notified that the total interest for both funds was \$870. Find the equivalent simple interest rate on the variable-rate fund.

**17. Meteorology** The average daily temperature for the month of January in Juneau, Alaska is  $25.7^\circ\text{F}$ . What is Juneau's average daily temperature for the month of January in degrees Celsius? (Source: U.S. National Oceanic and Atmospheric Administration)

**18. Geometry** A basketball and a baseball have circumferences of 30 inches and  $9\frac{1}{4}$  inches, respectively. Find the volume of each.

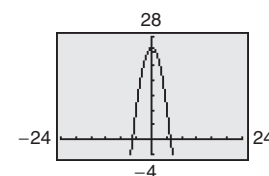
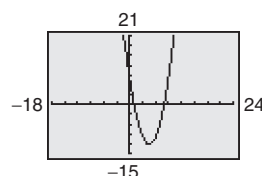
**2.2** In Exercises 19–22, find the  $x$ - and  $y$ -intercepts of the graph of the equation.

19.  $-x + y = 3$

20.  $x - 5y = 20$

21.  $y = x^2 - 9x + 8$

22.  $y = 25 - x^2$



In Exercises 23–28, use a graphing utility to approximate any solutions of the equation. [Remember to write the equation in the form  $f(x) = 0$ .]

23.  $5(x-2) - 1 = 0$

24.  $12 - 5(x-7) = 0$

25.  $3x^3 - 2x + 4 = 0$

26.  $\frac{1}{3}x^3 - x + 4 = 0$

27.  $x^4 - 3x + 1 = 0$

28.  $6 - \frac{1}{2}x^2 + \frac{5}{6}x^4 = 0$

In Exercises 29–32, use a graphing utility to approximate any points of intersection of the graphs of the equations. Check your results algebraically.

29.  $3x + 5y = -7$   
 $-x - 2y = 3$

30.  $x - y = 3$   
 $2x + y = 12$

31.  $x^2 + 2y = 14$   
 $3x + 4y = 1$

32.  $y = -x + 7$   
 $y = 2x^3 - x + 9$

**2.3** In Exercises 33–36, write the complex number in standard form.

33.  $6 + \sqrt{-25}$

34.  $-\sqrt{-12} + 3$

35.  $-2i^2 + 7i$

36.  $-i^2 - 4i$

In Exercises 37–48, perform the operations and write the result in standard form.

37.  $(7 + 5i) + (-4 + 2i)$

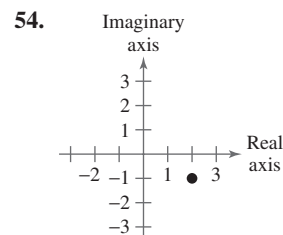
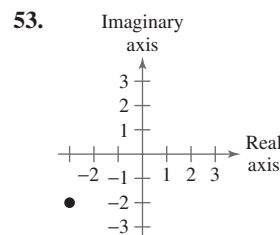
38.  $\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$

39.  $5i(13 - 8i)$  40.  $(1 + 6i)(5 - 2i)$   
 41.  $(\sqrt{-16} + 3)(\sqrt{-25} - 2)$   
 42.  $(5 - \sqrt{-4})(5 + \sqrt{-4})$   
 43.  $\sqrt{-9} + 3 + \sqrt{-36}$  44.  $7 - \sqrt{-81} + \sqrt{-49}$   
 45.  $(10 - 8i)(2 - 3i)$  46.  $i(6 + i)(3 - 2i)$   
 47.  $(3 + 7i)^2 + (3 - 7i)^2$  48.  $(4 - i)^2 - (4 + i)^2$

In Exercises 49–52, write the quotient in standard form.

49.  $\frac{6 + i}{i}$  50.  $\frac{4}{-3i}$   
 51.  $\frac{3 + 2i}{5 + i}$  52.  $\frac{1 - 7i}{2 + 3i}$

In Exercises 53 and 54, determine the complex number shown in the complex plane.



In Exercises 55–60, plot the complex number in the complex plane.

55.  $2 - 5i$  56.  $-1 + 4i$   
 57.  $-6i$  58.  $7i$   
 59.  $3$  60.  $-2$

**2.4** In Exercises 61–86, solve the equation using any convenient method. Use a graphing utility to verify your solution(s).

61.  $(2x - 1)(x + 3) = 0$  62.  $(2x - 5)(x + 2) = 0$   
 63.  $(3x - 2)(x - 5) = 0$  64.  $(3x - 1)(2x + 1) = 0$   
 65.  $6x = 3x^2$  66.  $16x^2 = 25$   
 67.  $x^2 - 4x = 5$  68.  $x^2 - 3x = 54$   
 69.  $x^2 - 3x = 4$  70.  $x^2 - 5x = 6$   
 71.  $2x^2 - x - 3 = 0$  72.  $2x^2 - x - 10 = 0$   
 73.  $15 + x - 2x^2 = 0$  74.  $1 + x - 2x^2 = 0$   
 75.  $(x + 4)^2 = 18$  76.  $(x + 1)^2 = 24$   
 77.  $x^2 - 12x + 30 = 0$  78.  $x^2 + 6x - 3 = 0$   
 79.  $2x^2 + 9x - 5 = 0$  80.  $4x^2 + x - 5 = 0$   
 81.  $-x^2 - x + 15 = 0$  82.  $2 - 3x - 2x^2 = 0$   
 83.  $x^2 + 4x + 10 = 0$  84.  $x^2 + 6x - 1 = 0$   
 85.  $2x^2 - 6x + 21 = 0$  86.  $2x^2 - 8x + 11 = 0$

**87. Medical Costs** The average costs per day  $C$  (in dollars) for hospital care from 1997 to 2003 in the U.S. can be approximated by the model  $C = 6.00t^2 - 62.9t + 1182$ ,  $7 \leq t \leq 13$ , where  $t$  is the year, with  $t = 7$  corresponding to 1997. (Source: Health Forum)

- (a) Use a graphing utility to graph the model in an appropriate viewing window.  
 (b) Use the *zoom* and *trace* features of a graphing utility to estimate when the cost per day reached \$1250.  
 (c) Algebraically find when the cost per day reached \$1250.  
 (d) According to the model, when will the cost per day reach \$1500 and \$2000?  
 (e) Do your answers seem reasonable? Explain.

**88. Auto Parts** The sales  $S$  (in millions of dollars) for Advanced Auto Parts from 2000 to 2006 can be approximated by the model  $S = -8.45t^2 + 439.0t + 2250$ ,  $0 \leq t \leq 6$ , where  $t$  is the year, with  $t = 0$  corresponding to 2000. (Source: Value Line)

- (a) Use a graphing utility to graph the model in an appropriate viewing window.  
 (b) Use the *zoom* and *trace* features of a graphing utility to estimate when the sales reached 3.5 billion dollars.  
 (c) Algebraically find when the sales reached 3.5 billion dollars.  
 (d) According to the model, when, if ever, will the sales reach 5.0 billion dollars? If sales will not reach that amount, explain why not.

**2.5** In Exercises 89–116, find all solutions of the equation algebraically. Use a graphing utility to verify the solutions graphically.

89.  $3x^3 - 26x^2 + 16x = 0$  90.  $36x^3 - x = 0$   
 91.  $5x^4 - 12x^3 = 0$  92.  $4x^3 - 6x^2 = 0$   
 93.  $x^4 - x^2 - 12 = 0$   
 94.  $x^4 - 4x^2 - 5 = 0$   
 95.  $2x^4 - 22x^2 = -56$   
 96.  $3x^4 + 18x^2 = -24$   
 97.  $\sqrt{x + 4} = 3$   
 98.  $\sqrt{x - 2} - 8 = 0$   
 99.  $2\sqrt{x} - 5 = 0$   
 100.  $\sqrt{3x - 2} = 4 - x$   
 101.  $\sqrt{2x + 3} + \sqrt{x - 2} = 2$   
 102.  $5\sqrt{x} - \sqrt{x - 1} = 6$   
 103.  $(x - 1)^{2/3} - 25 = 0$   
 104.  $(x + 2)^{3/4} = 27$   
 105.  $(x + 4)^{1/2} + 5x(x + 4)^{3/2} = 0$

## 244 Chapter 2 Solving Equations and Inequalities

106.  $8x^2(x^2 - 4)^{1/3} + (x^2 - 4)^{4/3} = 0$

107.  $\frac{x}{8} + \frac{3}{8} = \frac{1}{2x}$

108.  $\frac{3x}{2} = \frac{1}{x} - \frac{5}{2}$

109.  $3\left(1 - \frac{1}{5t}\right) = 0$

110.  $\frac{1}{x-2} = 3$

111.  $\frac{4}{(x-4)^2} = 1$

112.  $\frac{1}{(t+1)^2} = 1$

113.  $|x - 5| = 10$

114.  $|2x + 3| = 7$

115.  $|x^2 - 3| = 2x$

116.  $|x^2 - 6| = x$

117. **Cost Sharing** A group of farmers agree to share equally in the cost of a \$48,000 piece of machinery. If they can find two more farmers to join the group, each person's share of the cost will decrease by \$4000. How many farmers are presently in the group?

118. **Average Speed** You drove 56 miles one way on a service call. On the return trip, your average speed was 8 miles per hour greater and the trip took 10 fewer minutes. What was your average speed on the return trip?

119. **Mutual Funds** A deposit of \$1000 in a mutual fund reaches a balance of \$1196.95 after 6 years. What annual interest rate on a certificate of deposit compounded monthly would yield an equivalent return?

120. **Mutual Funds** A deposit of \$1500 in a mutual fund reaches a balance of \$2465.43 after 10 years. What annual interest rate on a certificate of deposit compounded quarterly would yield an equivalent return?

121. **City Population** The populations  $P$  (in millions) of New York City from 2000 to 2004 can be modeled by the equation

$$P = 18.310 + 0.1989\sqrt{t}, \quad 0 \leq t \leq 4$$

where  $t$  is the year, with  $t = 0$  corresponding to 2000.

(Source: U.S. Census Bureau)

(a) Use the *table* feature of a graphing utility to find the population of New York City for each year from 2000 to 2004.

(b) Use a graphing utility to graph the model in an appropriate viewing window.

(c) Use the *zoom* and *trace* features of a graphing utility to find when the population reached 18.5 million.

(d) Algebraically confirm your approximation in part (b).

(e) According to the model, when will the population reach 19 million? Does this answer seem reasonable?

(f) Do you believe the population will ever reach 20 million? Explain your reasoning.

122. **School Enrollment** The numbers of students  $N$  (in millions) enrolled in school at all levels in the United States from 1999 to 2003 can be modeled by the equation

$$N = \sqrt{23.649t^2 - 420.19t + 7090.1}, \quad 9 \leq t \leq 13$$

where  $t$  is the year, with  $t = 9$  corresponding to 1999.

(Source: U.S. Census Bureau)

(a) Use the *table* feature of a graphing utility to find the number of students enrolled for each year from 1999 to 2003.

(b) Use a graphing utility to graph the model in an appropriate viewing window.

(c) Use the *zoom* and *trace* features of a graphing utility to find when school enrollment reached 74 million.

(d) Algebraically confirm your approximation in part (c).

(e) According to the model, when will the enrollment reach 75 million? Does this answer seem reasonable?

(f) Do you believe the enrollment population will ever reach 100 million? Explain your reasoning.

**2.6 In Exercises 123–144, solve the inequality and sketch the solution on the real number line. Use a graphing utility to verify your solution graphically.**

123.  $8x - 3 < 6x + 15$

124.  $9x - 8 \leq 7x + 16$

125.  $\frac{1}{2}(3 - x) > \frac{1}{3}(2 - 3x)$

126.  $4(5 - 2x) \geq \frac{1}{2}(8 - x)$

127.  $-2 < -x + 7 \leq 10$

128.  $-6 \leq 3 - 2(x - 5) < 14$

129.  $|x - 2| < 1$

130.  $|x| \leq 4$

131.  $\left|x - \frac{3}{2}\right| \geq \frac{3}{2}$

132.  $|x - 3| > 4$

133.  $4|3 - 2x| \leq 16$

134.  $|x + 9| + 7 > 19$

135.  $x^2 - 2x \geq 3$

136.  $x^2 - 6x - 27 < 0$

137.  $4x^2 - 23x \leq 6$

138.  $6x^2 + 5x < 4$

139.  $x^3 - 16x \geq 0$

140.  $12x^3 - 20x^2 < 0$

141.  $\frac{x-5}{3-x} < 0$

142.  $\frac{2}{x+1} \leq \frac{3}{x-1}$

143.  $\frac{3x+8}{x-3} \leq 4$

144.  $\frac{x+8}{x+5} - 2 < 0$

**In Exercises 145–148, find the domain of  $x$  in the expression.**

145.  $\sqrt{x-4}$

146.  $\sqrt{x^2-25}$

147.  $\sqrt[3]{2-3x}$

148.  $\sqrt[3]{4x^2-1}$

**149. Accuracy of Measurement** You stop at a self-service gas station to buy 15 gallons of 87-octane gasoline at \$2.59 a gallon. The gas pump is accurate to within  $\frac{1}{10}$  of a gallon. How much might you be overcharged or undercharged?

**150. Meteorology** An electronic device is to be operated in an environment with relative humidity  $h$  in the interval defined by  $|h - 50| \leq 30$ . What are the minimum and maximum relative humidities for the operation of this device?

### 2.7

**151. Education** The following ordered pairs give the entrance exam scores  $x$  and the grade-point averages  $y$  after 1 year of college for 10 students.

(75, 2.3), (82, 3.0), (90, 3.6), (65, 2.0), (70, 2.1), (88, 3.5), (93, 3.9), (69, 2.0), (80, 2.8), (85, 3.3)

(a) Create a scatter plot of the data.

(b) Does the relationship between  $x$  and  $y$  appear to be approximately linear? Explain.


**152. Stress Test** A machine part was tested by bending it  $x$  centimeters 10 times per minute until it failed ( $y$  equals the time to failure in hours). The results are given as the following ordered pairs.

(3, 61), (6, 56), (9, 53), (12, 55), (15, 48), (18, 35), (21, 36), (24, 33), (27, 44), (30, 23)

(a) Create a scatter plot of the data.

(b) Does the relationship between  $x$  and  $y$  appear to be approximately linear? If not, give some possible explanations.

**153. Falling Object** In an experiment, students measured the speed  $s$  (in meters per second) of a ball  $t$  seconds after it was released. The results are shown in the table.



Time, $t$	Speed, $s$
0	0
1	11.0
2	19.4
3	29.2
4	39.4

(a) Sketch a scatter plot of the data.

(b) Find the equation of the line that seems to best fit the data.

(c) Use the *regression* feature of a graphing utility to find a linear model for the data. Compare this model with the model from part (b).

(d) Use the model from part (c) to estimate the speed of the ball after 2.5 seconds.

**154. Sports** The following ordered pairs  $(x, y)$  represent the Olympic year  $x$  and the winning time  $y$  (in minutes) in the men's 400-meter freestyle swimming event. (Source: *The World Almanac 2005*)

(1964, 4.203)	(1980, 3.855)	(1996, 3.800)
(1968, 4.150)	(1984, 3.854)	(2000, 3.677)
(1972, 4.005)	(1988, 3.783)	(2004, 3.718)
(1976, 3.866)	(1992, 3.750)	

(a) Use the *regression* feature of a graphing utility to find a linear model for the data and to identify the correlation coefficient. Let  $x$  represent the year, with  $x = 4$  corresponding to 1964.

(b) Use a graphing utility to create a scatter plot of the data.

(c) Graph the model with the data in the same viewing window.

(d) Is the model a good fit for the data? Explain.

(e) Is the model appropriate for predicting the winning times in future Olympics? Explain.

### Synthesis

**True or False?** In Exercises 155–157, determine whether the statement is true or false. Justify your answer.

**155.** The graph of a function may have two distinct  $y$ -intercepts.

**156.** The sum of two complex numbers cannot be a real number.

**157.** The sign of the slope of a regression line is always positive.

**158. Writing** In your own words, explain the difference between an identity and a conditional equation.

**159. Writing** Describe the relationship among the  $x$ -intercepts of a graph, the zeros of a function, and the solutions of an equation.

**160.** Consider the linear equation  $ax + b = 0$ .

(a) What is the sign of the solution if  $ab > 0$ ?

(b) What is the sign of the solution if  $ab < 0$ ?

**161. Error Analysis** Describe the error.

$$\sqrt{-6}\sqrt{-6} = \sqrt{(-6)(-6)} = \sqrt{36} = 6$$

**162. Error Analysis** Describe the error.

$$\begin{aligned} -i(\sqrt{-4} - 1) &= -i(4i - 1) \\ &= -4i^2 - i \\ &= 4 - i \end{aligned}$$

**163.** Write each of the powers of  $i$  as  $i$ ,  $-i$ ,  $1$ , or  $-1$ .

(a)  $i^{40}$  (b)  $i^{25}$  (c)  $i^{50}$  (d)  $i^{67}$

## 2 Chapter Test

See [www.CalcChat.com](http://www.CalcChat.com) for worked-out solutions to odd-numbered exercises.

Take this test as you would take a test in class. After you are finished, check your work against the answers given in the back of the book.

In Exercises 1 and 2, solve the equation (if possible). Then use a graphing utility to verify your solution.

1.  $\frac{12}{x} - 7 = -\frac{27}{x} + 6$

2.  $\frac{4}{3x-2} - \frac{9x}{3x+2} = -3$

In Exercises 3–6, perform the operations and write the result in standard form.

3.  $(-8 - 3i) + (-1 - 15i)$

4.  $(10 + \sqrt{-20}) - (4 - \sqrt{-14})$

5.  $(2 + i)(6 - i)$

6.  $(4 + 3i)^2 - (5 + i)^2$

In Exercises 7–9, write the quotient in standard form.

7.  $\frac{8 + 5i}{6 - i}$

8.  $\frac{5i}{2 + i}$

9.  $(2i - 1) \div (3i + 2)$

10. Plot the complex number  $3 - 2i$  in the complex plane.

In Exercises 11–14, use a graphing utility to approximate any solutions of the equation. [Remember to write the equation in the form  $f(x) = 0$ .]

11.  $3x^2 - 6 = 0$

12.  $8x^2 - 2 = 0$

13.  $x^3 + 5x = 4x^2$

14.  $x = x^3$

In Exercises 15–18, solve the equation using any convenient method. Use a graphing utility to verify the solutions graphically.

15.  $x^2 - 10x + 9 = 0$

16.  $x^2 + 12x - 2 = 0$

17.  $4x^2 - 81 = 0$

18.  $5x^2 + 14x - 3 = 0$

In Exercises 19–22, find all solutions of the equation algebraically. Use a graphing utility to verify the solutions graphically.

19.  $3x^3 - 4x^2 - 12x + 16 = 0$

20.  $x + \sqrt{22 - 3x} = 6$

21.  $(x^2 + 6)^{2/3} = 16$

22.  $|8x - 1| = 21$

In Exercises 23–26, solve the inequality and sketch the solution on the real number line. Use a graphing utility to verify your solution graphically.


23.  $8x - 1 > 3x - 10$

24.  $2|x - 8| < 10$

25.  $6x^2 + 5x + 1 \geq 0$

26.  $\frac{3 - 5x}{2 + 3x} < -2$

27. The table shows the numbers of cellular phone subscribers  $S$  (in millions) in the United States from 1999 through 2004, where  $t$  represents the year, with  $t = 9$  corresponding to 1999. Use the *regression* feature of a graphing utility to find a linear model for the data and to identify the correlation coefficient. Use the model to find the year in which the number of subscribers exceeded 200 million. (Source: Cellular Telecommunications & Internet Association)



Year, $t$	Subscribers, $S$
9	86.0
10	109.5
11	128.4
12	140.8
13	158.7
14	182.1

Table for 27

## P–2 Cumulative Test

See [www.CalcChat.com](http://www.CalcChat.com) for worked-out solutions to odd-numbered exercises.

Take this test to review the material in Chapters P–2. After you are finished, check your work against the answers in the back of the book.

In Exercises 1–3, simplify the expression.

1.  $\frac{14x^2y^{-3}}{32x^{-1}y^2}$

2.  $8\sqrt{60} - 2\sqrt{135} - \sqrt{15}$

3.  $\sqrt{28x^4y^3}$

In Exercises 4–6, perform the operation and simplify the result.

4.  $4x - [2x + 5(2 - x)]$

5.  $(x - 2)(x^2 + x - 3)$

6.  $\frac{2}{x+3} - \frac{1}{x+1}$

In Exercises 7–9, factor the expression completely.

7.  $25 - (x - 2)^2$

8.  $x - 5x^2 - 6x^3$

9.  $54 - 16x^3$

10. Find the midpoint of the line segment connecting the points  $(-\frac{7}{2}, 4)$  and  $(\frac{5}{2}, -8)$ . Then find the distance between the points.

11. Write the standard form of the equation of a circle with center  $(-\frac{1}{2}, -8)$  and a radius of 4.

In Exercises 12–14, use point plotting to sketch the graph of the equation.

12.  $x - 3y + 12 = 0$

13.  $y = x^2 - 9$

14.  $y = \sqrt{4 - x}$

In Exercises 15–17, (a) write the general form of the equation of the line that satisfies the given conditions and (b) find three additional points through which the line passes.

15. The line contains the points  $(-5, 8)$  and  $(-1, 4)$ .

16. The line contains the point  $(-\frac{1}{2}, 1)$  and has a slope of  $-2$ .

17. The line has an undefined slope and contains the point  $(-\frac{3}{7}, \frac{1}{8})$ .

18. Find the equation of the line that passes through the point  $(2, 3)$  and is (a) parallel to and (b) perpendicular to the line  $6x - y = 4$ .

In Exercises 19 and 20, evaluate the function at each value of the independent variable and simplify.

19.  $f(x) = \frac{x}{x-2}$

20.  $f(x) = \begin{cases} 3x - 8, & x < 0 \\ x^2 + 4, & x \geq 0 \end{cases}$

(a)  $f(5)$  (b)  $f(2)$  (c)  $f(5 + 4s)$  (a)  $f(-8)$  (b)  $f(0)$  (c)  $f(4)$

In Exercises 21–24, find the domain of the function.

21.  $f(x) = (x + 2)(3x - 4)$

22.  $f(t) = \sqrt{5 + 7t}$

23.  $g(s) = \sqrt{9 - s^2}$

24.  $h(x) = \frac{4}{5x + 2}$

25. Determine if the function given by  $g(x) = 3x - x^3$  is even, odd, or neither.







## Proofs in Mathematics

### Biconditional Statements

Recall from the Proofs in Mathematics in Chapter 1 that a conditional statement is a statement of the form “if  $p$ , then  $q$ .” A statement of the form “ $p$  if and only if  $q$ ” is called a **biconditional statement**. A biconditional statement, denoted by

$$p \leftrightarrow q \quad \text{Biconditional statement}$$

is the conjunction of the conditional statement  $p \rightarrow q$  and its converse  $q \rightarrow p$ .

A biconditional statement can be either true or false. To be true, *both* the conditional statement and its converse must be true.

#### Example 1 Analyzing a Biconditional Statement

Consider the statement  $x = 3$  if and only if  $x^2 = 9$ .

- a.** Is the statement a biconditional statement?    **b.** Is the statement true?

#### Solution

- a.** The statement is a biconditional statement because it is of the form “ $p$  if and only if  $q$ .”  
**b.** The statement can be rewritten as the following conditional statement and its converse.

*Conditional statement:* If  $x = 3$ , then  $x^2 = 9$ .

*Converse:* If  $x^2 = 9$ , then  $x = 3$ .

The first of these statements is true, but the second is false because  $x$  could also equal  $-3$ . So, the biconditional statement is false.

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Knowing how to use biconditional statements is an important tool for reasoning in mathematics.

#### Example 2 Analyzing a Biconditional Statement

Determine whether the biconditional statement is true or false. If it is false, provide a counterexample.

A number is divisible by 5 if and only if it ends in 0.

#### Solution

The biconditional statement can be rewritten as the following conditional statement and its converse.

*Conditional statement:* If a number is divisible by 5, then it ends in 0.

*Converse:* If a number ends in 0, then it is divisible by 5.

The conditional statement is false. A counterexample is the number 15, which is divisible by 5 but does not end in 0. So, the biconditional statement is false.

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## Progressive Summary (Chapters P–2)

This chart outlines the topics that have been covered so far in this text. Progressive Summary charts appear after Chapters 2, 4, and 7. In each progressive summary, new topics encountered for the first time appear in red.

Algebraic Functions	Transcendental Functions	Other Topics
Polynomial, Rational, Radical		
<div><div>■ Rewriting</div><div>Polynomial form ↔ Factored form Operations with polynomials Rationalize denominators Simplify rational expressions Exponent form ↔ Radical form Operations with complex numbers</div></div>	<div><div>■ Rewriting</div></div>	<div><div>■ Rewriting</div></div>
<div><div>■ Solving</div><div><div>Equation</div><div>Strategy</div><div>Linear . . . . . Isolate variable</div><div>Quadratic . . . . . Factor, set to zero</div><div>Extract square roots</div><div>Complete the square</div><div>Quadratic Formula</div><div>Polynomial . . . . . Factor, set to zero</div><div>Rational Zero Test</div><div>Rational . . . . . Multiply by LCD</div><div>Radical . . . . . Isolate, raise to power</div><div>Absolute Value . . . . Isolate, form two equations</div></div></div>	<div><div>■ Solving</div></div>	<div><div>■ Solving</div></div>
<div><div>■ Analyzing</div><div><div>Graphically</div><div>Algebraically</div><div>Intercepts</div><div>Domain, Range</div><div>Symmetry</div><div>Transformations</div><div>Slope</div><div>Composition</div><div>Asymptotes</div></div><div><div>Numerically</div><div>Table of values</div></div></div>	<div><div>■ Analyzing</div></div>	<div><div>■ Analyzing</div></div>