3.6 Graphs of Rational Functions

The Graph of a Rational Function

To sketch the graph of a rational function, use the following guidelines.

Guidelines for Graphing Rational Functions

Let $f(x) = N(x)/D(x)$, where $N(x)$ and $D(x)$ are polynomials.

1. Simplify f, if possible. Any restrictions on the domain of f not in the simplified function should be listed.
2. Find and plot the y-intercept (if any) by evaluating $f(0)$.
3. Find the zeros of the numerator (if any) by setting the numerator equal to zero. Then plot the corresponding x-intercepts.
4. Find the zeros of the denominator (if any) by setting the denominator equal to zero. Then sketch the corresponding vertical asymptotes using dashed vertical lines and plot the corresponding holes using open circles.
5. Find and sketch any other asymptotes of the graph using dashed lines.
6. Plot at least one point *between* and one point *beyond* each x-intercept and vertical asymptote.
7. Use smooth curves to complete the graph between and beyond the vertical asymptotes, excluding any points where f is not defined.

What you should learn

- Analyze and sketch graphs of rational functions.
- Sketch graphs of rational functions that have slant asymptotes.
- Use rational functions to model and solve real-life problems.

Why you should learn it

The graph of a rational function provides a good indication of the future behavior of a mathematical model. Exercise 86 on page 316 models the population of a herd of elk after their release onto state game lands.

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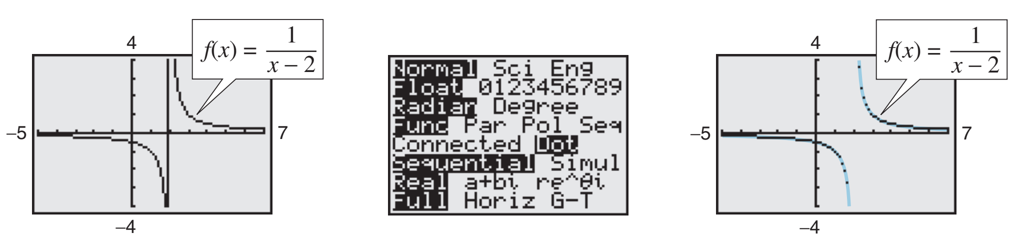
TECHNOLOGY SUPPORT For instructions on how to use the *connected* mode and the *dot* mode, see Appendix A; for specific keystrokes, go to this textbook's *Online Study Center*.

TECHNOLOGY TIP Some graphing utilities have difficulty graphing rational functions that have vertical asymptotes. Often, the utility will connect parts of the graph that are not supposed to be connected. Notice that the graph in Figure 3.51(a) should consist of two *unconnected* portions—one to the left of $x = 2$ and the other to the right of $x = 2$. To eliminate this problem, you can try changing the *mode* of the graphing utility to *dot mode* [see Figure 3.51(b)]. The problem with this mode is that the graph is then represented as a collection of dots rather than as a smooth curve, as shown in Figure 3.51(c). In this text, a blue curve is placed behind the graphing utility's display to indicate where the graph should appear. [See Figure 3.51(c).]



(a) *Connected* mode (b) Mode screen (c) *Dot* mode

Figure 3.51

Example 1 Sketching the Graph of a Rational Function

Sketch the graph of $g(x) = \dfrac{3}{x-2}$ by hand.

Solution

y-Intercept: $\left(0, -\frac{3}{2}\right)$, because $g(0) = -\frac{3}{2}$

x-Intercept: None, because $3 \neq 0$

Vertical Asymptote: $x = 2$, zero of denominator

Horizontal Asymptote: $y = 0$, because degree of $N(x) <$ degree of $D(x)$

Additional Points:

x	-4	1	2	3	5
$g(x)$	-0.5	-3	Undefined	3	1

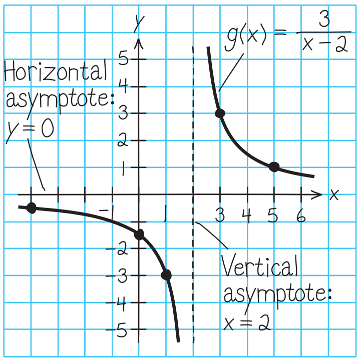


Figure 3.52

By plotting the intercept, asymptotes, and a few additional points, you can obtain the graph shown in Figure 3.52. Confirm this with a graphing utility.

CHECKPOINT Now try Exercise 9.

STUDY TIP

Note in the examples in this section that the vertical asymptotes are included in the tables of additional points. This is done to emphasize numerically the behavior of the graph of the function.

Note that the graph of g in Example 1 is a vertical stretch and a right shift of the graph of

$$f(x) = \frac{1}{x}$$

because

$$g(x) = \frac{3}{x-2} = 3\left(\frac{1}{x-2}\right) = 3f(x-2).$$

Example 2 Sketching the Graph of a Rational Function

Sketch the graph of $f(x) = \dfrac{2x-1}{x}$ by hand.

Solution

y-Intercept: None, because $x = 0$ is not in the domain

x-Intercept: $\left(\frac{1}{2}, 0\right)$, because $2x - 1 = 0$

Vertical Asymptote: $x = 0$, zero of denominator

Horizontal Asymptote: $y = 2$, because degree of $N(x) =$ degree of $D(x)$

Additional Points:

x	-4	-1	0	$\frac{1}{4}$	4
$f(x)$	2.25	3	Undefined	-2	1.75

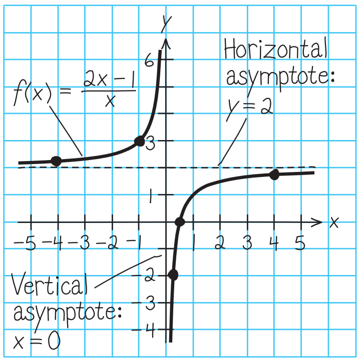


Figure 3.53

By plotting the intercept, asymptotes, and a few additional points, you can obtain the graph shown in Figure 3.53. Confirm this with a graphing utility.

CHECKPOINT Now try Exercise 13.

Example 3 Sketching the Graph of a Rational Function

Sketch the graph of $f(x) = \dfrac{x}{x^2 - x - 2}$.

Solution

Factor the denominator to determine more easily the zeros of the denominator.

$$f(x) = \frac{x}{x^2 - x - 2} = \frac{x}{(x+1)(x-2)}.$$

y-Intercept: (0, 0), because $f(0) = 0$

x-Intercept: (0, 0)

Vertical Asymptotes: $x = -1$, $x = 2$, zeros of denominator

Horizontal Asymptote: $y = 0$, because degree of $N(x) <$ degree of $D(x)$

Additional Points:

x	-3	-1	-0.5	1	2	3
$f(x)$	-0.3	Undefined	0.4	-0.5	Undefined	0.75

The graph is shown in Figure 3.54.

CHECKPOINT Now try Exercise 21.

Example 4 Sketching the Graph of a Rational Function

Sketch the graph of $f(x) = \dfrac{x^2 - 9}{x^2 - 2x - 3}$.

Solution

By factoring the numerator and denominator, you have

$$f(x) = \frac{x^2 - 9}{x^2 - 2x - 3} = \frac{(x-3)(x+3)}{(x-3)(x+1)} = \frac{x+3}{x+1}, \quad x \neq 3.$$

y-Intercept: (0, 3), because $f(0) = 3$

x-Intercept: $(-3, 0)$

Vertical Asymptote: $x = -1$, zero of (simplified) denominator

Hole: $\left(3, \frac{3}{2}\right)$, f is not defined at $x = 3$

Horizontal Asymptote: $y = 1$, because degree of $N(x) =$ degree of $D(x)$

Additional Points:

x	-5	-2	-1	-0.5	1	3	4
$f(x)$	0.5	-1	Undefined	5	2	Undefined	1.4

The graph is shown in Figure 3.55.

CHECKPOINT Now try Exercise 23.

Exploration

Use a graphing utility to graph

$$f(x) = 1 + \frac{1}{x - \frac{1}{x}}.$$

Set the graphing utility to *dot* mode and use a decimal viewing window. Use the *trace* feature to find three "holes" or "breaks" in the graph. Do all three holes represent zeros of the denominator

$$x - \frac{1}{x}?$$

Explain.

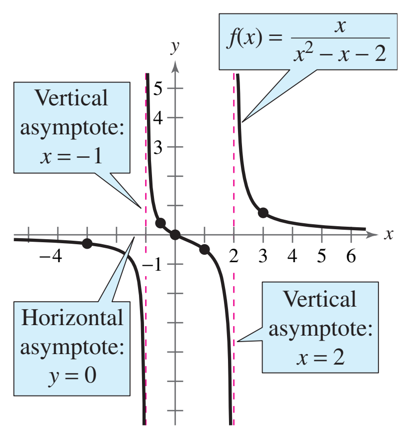


Figure 3.54

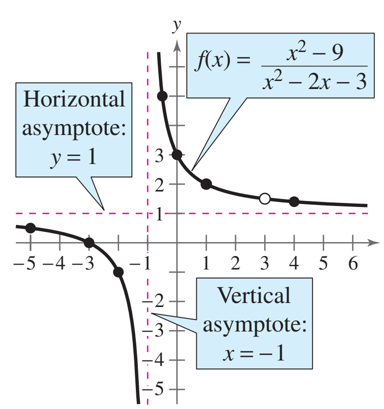


Figure 3.55 ***Hole at $x = 3$***

Slant Asymptotes

Consider a rational function whose denominator is of degree 1 or greater. If the degree of the numerator is exactly *one more* than the degree of the denominator, the graph of the function has a **slant** (or **oblique**) **asymptote.** For example, the graph of

$$f(x) = \frac{x^2 - x}{x + 1}$$

has a slant asymptote, as shown in Figure 3.56. To find the equation of a slant asymptote, use long division. For instance, by dividing $x + 1$ into $x^2 - x$, you have

$$f(x) = \frac{x^2 - x}{x + 1} = \underbrace{x - 2}_{\substack{\text{Slant asymptote} \\ (y = x - 2)}} + \frac{2}{x + 1}.$$

As x increases or decreases without bound, the remainder term $2/(x + 1)$ approaches 0, so the graph of f approaches the line $y = x - 2$, as shown in Figure 3.56.

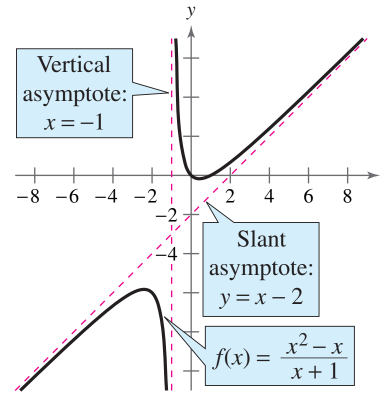


Figure 3.56

Exploration

Do you think it is possible for the graph of a rational function to cross its horizontal asymptote or its slant asymptote? Use the graphs of the following functions to investigate this question. Write a summary of your conclusion. Explain your reasoning.

$$f(x) = \frac{x}{x^2 + 1}$$

$$g(x) = \frac{2x}{3x^2 - 2x + 1}$$

$$h(x) = \frac{x^3}{x^2 + 1}$$

Example 5 A Rational Function with a Slant Asymptote

Sketch the graph of $f(x) = \dfrac{x^2 - x - 2}{x - 1}$.

Solution

First write $f(x)$ in two different ways. Factoring the numerator

$$f(x) = \frac{x^2 - x - 2}{x - 1} = \frac{(x - 2)(x + 1)}{x - 1}$$

enables you to recognize the x-intercepts. Long division

$$f(x) = \frac{x^2 - x - 2}{x - 1} = x - \frac{2}{x - 1}$$

enables you to recognize that the line $y = x$ is a slant asymptote of the graph.

y-Intercept: $(0, 2)$, because $f(0) = 2$

x-Intercepts: $(-1, 0)$ and $(2, 0)$

Vertical Asymptote: $x = 1$, zero of denominator

Horizontal Asymptote: None, because degree of $N(x) >$ degree of $D(x)$

Slant Asymptote: $y = x$

Additional Points:

x	-2	0.5	1	1.5	3
$f(x)$	-1.33	4.5	Undefined	-2.5	2

The graph is shown in Figure 3.57.

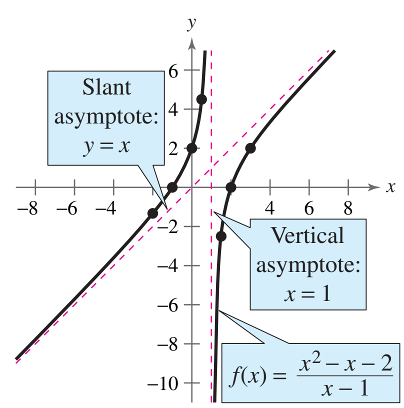


Figure 3.57

CHECKPOINT Now try Exercise 45.

Application

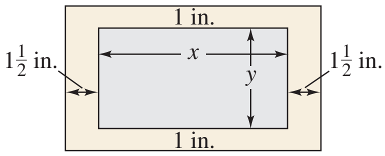


Figure 3.58

Example 6 Finding a Minimum Area

A rectangular page is designed to contain 48 square inches of print. The margins on each side of the page are $1\frac{1}{2}$ inches wide. The margins at the top and bottom are each 1 inch deep. What should the dimensions of the page be so that the minimum amount of paper is used?

Graphical Solution

Let A be the area to be minimized. From Figure 3.58, you can write

$$A = (x + 3)(y + 2).$$

The printed area inside the margins is modeled by $48 = xy$ or $y = 48/x$. To find the minimum area, rewrite the equation for A in terms of just one variable by substituting $48/x$ for y.

$$A = (x + 3)\left(\frac{48}{x} + 2\right) = \frac{(x + 3)(48 + 2x)}{x}, \quad x > 0$$

The graph of this rational function is shown in Figure 3.59. Because x represents the width of the printed area, you need consider only the portion of the graph for which x is positive. Using the *minimum* feature or the *zoom* and *trace* features of a graphing utility, you can approximate the minimum value of A to occur when $x \approx 8.5$ inches. The corresponding value of y is $48/8.5 \approx 5.6$ inches. So, the dimensions should be

$$x + 3 \approx 11.5 \text{ inches by } y + 2 \approx 7.6 \text{ inches.}$$

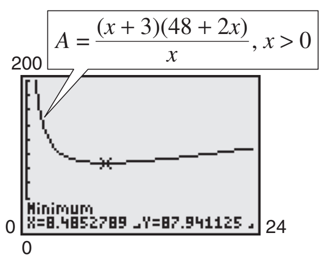


Figure 3.59

CHECKPOINT Now try Exercise 79.

Numerical Solution

Let A be the area to be minimized. From Figure 3.58, you can write

$$A = (x + 3)(y + 2).$$

The printed area inside the margins is modeled by $48 = xy$ or $y = 48/x$. To find the minimum area, rewrite the equation for A in terms of just one variable by substituting $48/x$ for y.

$$A = (x + 3)\left(\frac{48}{x} + 2\right) = \frac{(x + 3)(48 + 2x)}{x}, \quad x > 0$$

Use the *table* feature of a graphing utility to create a table of values for the function

$$y_1 = \frac{(x + 3)(48 + 2x)}{x}$$

beginning at $x = 1$. From the table, you can see that the minimum value of y_1 occurs when x is somewhere between 8 and 9, as shown in Figure 3.60. To approximate the minimum value of y_1 to one decimal place, change the table to begin at $x = 8$ and set the table step to 0.1. The minimum value of y_1 occurs when $x \approx 8.5$, as shown in Figure 3.61. The corresponding value of y is $48/8.5 \approx 5.6$ inches. So, the dimensions should be $x + 3 \approx 11.5$ inches by $y + 2 \approx 7.6$ inches.

X	Y1
6	90
7	88.571
8	88
9	88
10	88.4
11	89.091
12	90

X=8

Figure 3.60

X	Y1
8.2	87.961
8.3	87.949
8.4	87.943
8.5	87.941
8.6	87.944
8.7	87.952
8.8	87.964

X=8.5

Figure 3.61

If you go on to take a course in calculus, you will learn an analytic technique for finding the exact value of x that produces a minimum area in Example 6. In this case, that value is $x = 6\sqrt{2} \approx 8.485$.

3.6 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

Fill in the blanks.

1. For the rational function $f(x) = N(x)/D(x)$, if the degree of $N(x)$ is exactly one more than the degree of $D(x)$, then the graph of f has a _______ (or oblique) _______ .
2. The graph of $f(x) = 1/x$ has a _______ asymptote at $x = 0$.

In Exercises 1–4, use a graphing utility to graph $f(x) = 2/x$ and the function g in the same viewing window. Describe the relationship between the two graphs.

1. $g(x) = f(x) + 1$
2. $g(x) = f(x - 1)$
3. $g(x) = -f(x)$
4. $g(x) = \frac{1}{2}f(x + 2)$

In Exercises 5–8, use a graphing utility to graph $f(x) = 2/x^2$ and the function g in the same viewing window. Describe the relationship between the two graphs.

5. $g(x) = f(x) - 2$
6. $g(x) = -f(x)$
7. $g(x) = f(x - 2)$
8. $g(x) = \frac{1}{4}f(x)$

In Exercises 9–26, sketch the graph of the rational function by hand. As sketching aids, check for intercepts, vertical asymptotes, horizontal asymptotes, and holes. Use a graphing utility to verify your graph.

9. $f(x) = \dfrac{1}{x + 2}$
10. $f(x) = \dfrac{1}{x - 6}$
11. $C(x) = \dfrac{5 + 2x}{1 + x}$
12. $P(x) = \dfrac{1 - 3x}{1 - x}$
13. $f(t) = \dfrac{1 - 2t}{t}$
14. $g(x) = \dfrac{1}{x + 2} + 2$
15. $f(x) = \dfrac{x^2}{x^2 - 4}$
16. $g(x) = \dfrac{x}{x^2 - 9}$
17. $f(x) = \dfrac{x}{x^2 - 1}$
18. $f(x) = -\dfrac{1}{(x - 2)^2}$
19. $g(x) = \dfrac{4(x + 1)}{x(x - 4)}$
20. $h(x) = \dfrac{2}{x^2(x - 3)}$
21. $f(x) = \dfrac{3x}{x^2 - x - 2}$
22. $f(x) = \dfrac{2x}{x^2 + x - 2}$
23. $f(x) = \dfrac{x^2 + 3x}{x^2 + x - 6}$
24. $g(x) = \dfrac{5(x + 4)}{x^2 + x - 12}$
25. $f(x) = \dfrac{x^2 - 1}{x + 1}$
26. $f(x) = \dfrac{x^2 - 16}{x - 4}$

In Exercises 27–36, use a graphing utility to graph the function. Determine its domain and identify any vertical or horizontal asymptotes.

27. $f(x) = \dfrac{2 + x}{1 - x}$
28. $f(x) = \dfrac{3 - x}{2 - x}$
29. $f(t) = \dfrac{3t + 1}{t}$
30. $h(x) = \dfrac{x - 2}{x - 3}$
31. $h(t) = \dfrac{4}{t^2 + 1}$
32. $g(x) = -\dfrac{x}{(x - 2)^2}$
33. $f(x) = \dfrac{x + 1}{x^2 - x - 6}$
34. $f(x) = \dfrac{x + 4}{x^2 + x - 6}$
35. $f(x) = \dfrac{20x}{x^2 + 1} - \dfrac{1}{x}$
36. $f(x) = 5\left(\dfrac{1}{x - 4} - \dfrac{1}{x + 2}\right)$

Exploration **In Exercises 37–42, use a graphing utility to graph the function. What do you observe about its asymptotes?**

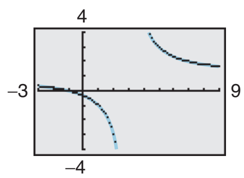
37. $h(x) = \dfrac{6x}{\sqrt{x^2 + 1}}$
38. $f(x) = -\dfrac{x}{\sqrt{9 + x^2}}$
39. $g(x) = \dfrac{4|x - 2|}{x + 1}$
40. $f(x) = -\dfrac{8|3 + x|}{x - 2}$
41. $f(x) = \dfrac{4(x - 1)^2}{x^2 - 4x + 5}$
42. $g(x) = \dfrac{3x^4 - 5x + 3}{x^4 + 1}$

In Exercises 43–50, sketch the graph of the rational function by hand. As sketching aids, check for intercepts, vertical asymptotes, and slant asymptotes.

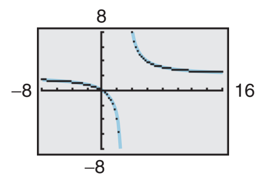
43. $f(x) = \dfrac{2x^2 + 1}{x}$
44. $g(x) = \dfrac{1 - x^2}{x}$
45. $h(x) = \dfrac{x^2}{x - 1}$
46. $f(x) = \dfrac{x^3}{x^2 - 1}$
47. $g(x) = \dfrac{x^3}{2x^2 - 8}$
48. $f(x) = \dfrac{x^2 - 1}{x^2 + 4}$
49. $f(x) = \dfrac{x^3 + 2x^2 + 4}{2x^2 + 1}$
50. $f(x) = \dfrac{2x^2 - 5x + 5}{x - 2}$

Graphical Reasoning **In Exercises 51–54, use the graph to estimate any *x*-intercepts of the rational function. Set $y = 0$ and solve the resulting equation to confirm your result.**

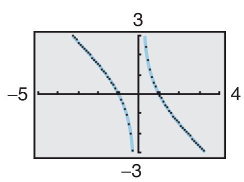
51. $y = \dfrac{x+1}{x-3}$



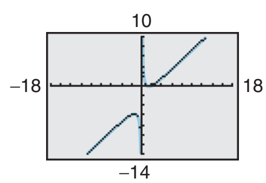
52. $y = \dfrac{2x}{x-3}$



53. $y = \dfrac{1}{x} - x$



54. $y = x - 3 + \dfrac{2}{x}$



In Exercises 55–58, use a graphing utility to graph the rational function. Determine the domain of the function and identify any asymptotes.

55. $y = \dfrac{2x^2 + x}{x + 1}$

56. $y = \dfrac{x^2 + 5x + 8}{x + 3}$

57. $y = \dfrac{1 + 3x^2 - x^3}{x^2}$

58. $y = \dfrac{12 - 2x - x^2}{2(4 + x)}$

In Exercises 59–64, find all vertical asymptotes, horizontal asymptotes, slant asymptotes, and holes in the graph of the function. Then use a graphing utility to verify your result.

59. $f(x) = \dfrac{x^2 - 5x + 4}{x^2 - 4}$

60. $f(x) = \dfrac{x^2 - 2x - 8}{x^2 - 9}$

61. $f(x) = \dfrac{2x^2 - 5x + 2}{2x^2 - x - 6}$

62. $f(x) = \dfrac{3x^2 - 8x + 4}{2x^2 - 3x - 2}$

63. $f(x) = \dfrac{2x^3 - x^2 - 2x + 1}{x^2 + 3x + 2}$

64. $f(x) = \dfrac{2x^3 + x^2 - 8x - 4}{x^2 - 3x + 2}$

Graphical Reasoning **In Exercises 65–76, use a graphing utility to graph the function and determine any *x*-intercepts. Set $y = 0$ and solve the resulting equation to confirm your result.**

65. $y = \dfrac{1}{x + 5} + \dfrac{4}{x}$

66. $y = \dfrac{2}{x + 1} - \dfrac{3}{x}$

67. $y = \dfrac{1}{x + 2} + \dfrac{2}{x + 4}$

68. $y = \dfrac{2}{x + 2} - \dfrac{3}{x - 1}$

69. $y = x - \dfrac{6}{x - 1}$

70. $y = x - \dfrac{9}{x}$

71. $y = x + 2 - \dfrac{1}{x + 1}$

72. $y = 2x - 1 + \dfrac{1}{x - 2}$

73. $y = x + 1 + \dfrac{2}{x - 1}$

74. $y = x + 2 + \dfrac{2}{x + 2}$

75. $y = x + 3 - \dfrac{2}{2x - 1}$

76. $y = x - 1 - \dfrac{2}{2x - 3}$

77. ***Concentration of a Mixture*** A 1000-liter tank contains 50 liters of a 25% brine solution. You add x liters of a 75% brine solution to the tank.

(a) Show that the concentration C, the proportion of brine to the total solution, of the final mixture is given by

$$C = \frac{3x + 50}{4(x + 50)}.$$

(b) Determine the domain of the function based on the physical constraints of the problem.

(c) Use a graphing utility to graph the function. As the tank is filled, what happens to the rate at which the concentration of brine increases? What percent does the concentration of brine appear to approach?

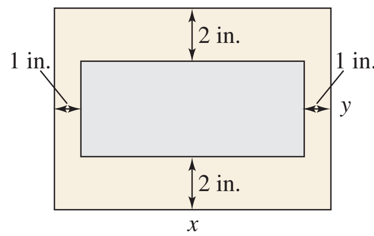
78. ***Geometry*** A rectangular region of length x and width y has an area of 500 square meters.

(a) Write the width y as a function of x.

(b) Determine the domain of the function based on the physical constraints of the problem.

(c) Sketch a graph of the function and determine the width of the rectangle when $x = 30$ meters.

79. ***Page Design*** A page that is x inches wide and y inches high contains 30 square inches of print. The margins at the top and bottom are 2 inches deep and the margins on each side are 1 inch wide (see figure).



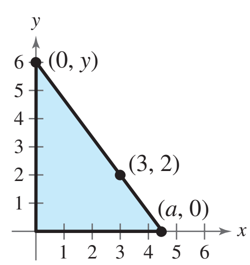
(a) Show that the total area A of the page is given by

$$A = \frac{2x(2x + 11)}{x - 2}.$$

(b) Determine the domain of the function based on the physical constraints of the problem.

(c) Use a graphing utility to graph the area function and approximate the page size such that the minimum amount of paper will be used. Verify your answer numerically using the *table* feature of a graphing utility.

80. ***Geometry*** A right triangle is formed in the first quadrant by the x-axis, the y-axis, and a line segment through the point (3, 2) (see figure).



(a) Show that an equation of the line segment is given by

$$y = \frac{2(a - x)}{a - 3}, \quad 0 \le x \le a.$$

(b) Show that the area of the triangle is given by

$$A = \frac{a^2}{a - 3}.$$

(c) Use a graphing utility to graph the area function and estimate the value of a that yields a minimum area. Estimate the minimum area. Verify your answer numerically using the *table* feature of a graphing utility.

81. ***Cost*** The ordering and transportation cost C (in thousands of dollars) for the components used in manufacturing a product is given by

$$C = 100\left(\frac{200}{x^2} + \frac{x}{x + 30}\right), \quad x \ge 1$$

where x is the order size (in hundreds). Use a graphing utility to graph the cost function. From the graph, estimate the order size that minimizes cost.

82. ***Average Cost*** The cost C of producing x units of a product is given by $C = 0.2x^2 + 10x + 5$, and the average cost per unit is given by

$$\overline{C} = \frac{C}{x} = \frac{0.2x^2 + 10x + 5}{x}, \quad x > 0.$$

Sketch the graph of the average cost function, and estimate the number of units that should be produced to minimize the average cost per unit.

83. ***Medicine*** The concentration C of a chemical in the bloodstream t hours after injection into muscle tissue is given by

$$C = \frac{3t^2 + t}{t^3 + 50}, \quad t \ge 0.$$

(a) Determine the horizontal asymptote of the function and interpret its meaning in the context of the problem.

(b) Use a graphing utility to graph the function and approximate the time when the bloodstream concentration is greatest.

(c) Use a graphing utility to determine when the concentration is less than 0.345.

84. ***Numerical and Graphical Analysis*** A driver averaged 50 miles per hour on the round trip between Baltimore, Maryland and Philadelphia, Pennsylvania, 100 miles away. The average speeds for going and returning were x and y miles per hour, respectively.

(a) Show that $y = \dfrac{25x}{x - 25}$.

(b) Determine the vertical and horizontal asymptotes of the function.

(c) Use a graphing utility to complete the table. What do you observe?

x	30	35	40	45	50	55	60
y							

(d) Use a graphing utility to graph the function.

(e) Is it possible to average 20 miles per hour in one direction and still average 50 miles per hour on the round trip? Explain.

85. ***Comparing Models*** The numbers of people A (in thousands) attending women's NCAA Division I college basketball games from 1990 to 2004 are shown in the table. Let t represent the year, with $t = 0$ corresponding to 1990. (Source: NCAA)

Year	Attendance, A (in thousands)
1990	2,777
1991	3,013
1992	3,397
1993	4,193
1994	4,557
1995	4,962
1996	5,234
1997	6,734
1998	7,387
1999	8,010
2000	8,698
2001	8,825
2002	9,533
2003	10,164
2004	10,016

(a) Use the *regression* feature of a graphing utility to find a linear model for the data. Use a graphing utility to plot the data and graph the model in the same viewing window.

(b) Find a rational model for the data. Take the reciprocal of A to generate the points $(t, 1/A)$. Use the *regression* feature of a graphing utility to find a linear model for this data. The resulting line has the form $1/A = at + b$. Solve for A. Use a graphing utility to plot the data and graph the rational model in the same viewing window.

(c) Use the *table* feature of a graphing utility to create a table showing the predicted attendance based on each model for each of the years in the original table. Which model do you prefer? Why?

86. ***Elk Population*** A herd of elk is released onto state game lands. The expected population P of the herd can be modeled by the equation $P = (10 + 2.7t)/(1 + 0.1t)$, where t is the time in years since the initial number of elk were released.

(a) State the domain of the model. Explain your answer.

(b) Find the initial number of elk in the herd.

(c) Find the populations of elk after 25, 50, and 100 years.

(d) Is there a limit to the size of the herd? If so, what is the expected population?

Use a graphing utility to confirm your results for parts (a) through (d).

Synthesis

True or False? **In Exercises 87 and 88, determine whether the statement is true or false. Justify your answer.**

87. If the graph of a rational function f has a vertical asymptote at $x = 5$, it is possible to sketch the graph without lifting your pencil from the paper.

88. The graph of a rational function can never cross one of its asymptotes.

Think About It **In Exercises 89 and 90, use a graphing utility to graph the function. Explain why there is no vertical asymptote when a superficial examination of the function might indicate that there should be one.**

89. $h(x) = \dfrac{6 - 2x}{3 - x}$

90. $g(x) = \dfrac{x^2 + x - 2}{x - 1}$

Think About It **In Exercises 91 and 92, write a rational function satisfying the following criteria. (There are many correct answers.)**

91. Vertical asymptote: $x = -2$
Slant asymptote: $y = x + 1$
Zero of the function: $x = 2$

92. Vertical asymptote: $x = -4$
Slant asymptote: $y = x - 2$
Zero of the function: $x = 3$

Skills Review

In Exercises 93–96, simplify the expression.

93. $\left(\dfrac{x}{8}\right)^{-3}$

94. $(4x^2)^{-2}$

95. $\dfrac{3^{7/6}}{3^{1/6}}$

96. $\dfrac{(x^{-2})(x^{1/2})}{(x^{-1})(x^{5/2})}$

In Exercises 97–100, use a graphing utility to graph the function and find its domain and range.

97. $f(x) = \sqrt{6 + x^2}$

98. $f(x) = \sqrt{121 - x^2}$

99. $f(x) = -|x + 9|$

100. $f(x) = -x^2 + 9$

101. ***Make a Decision*** To work an extended application analyzing the total manpower of the Department of Defense, visit this textbook's *Online Study Center*. (Data Source: U.S. Department of Defense)