P.5 The Cartesian Plane

The Cartesian Plane

Just as you can represent real numbers by points on a real number line, you can represent ordered pairs of real numbers by points in a plane called the **rectangular coordinate system,** or the **Cartesian plane,** after the French mathematician René Descartes (1596–1650).

The Cartesian plane is formed by using two real number lines intersecting at right angles, as shown in Figure P.10. The horizontal real number line is usually called the ***x*-axis,** and the vertical real number line is usually called the ***y*-axis.** The point of intersection of these two axes is the **origin,** and the two axes divide the plane into four parts called **quadrants.**

What you should learn

- Plot points in the Cartesian plane and sketch scatter plots.
- Use the Distance Formula to find the distance between two points.
- Use the Midpoint Formula to find the midpoint of a line segment.
- Find the equation of a circle.
- Translate points in the plane.

Why you should learn it

The Cartesian plane can be used to represent relationships between two variables. For instance, Exercise 85 on page 58 shows how to represent graphically the numbers of recording artists inducted into the Rock and Roll Hall of Fame from 1986 to 2006.

Alex Bartel/Stock Boston

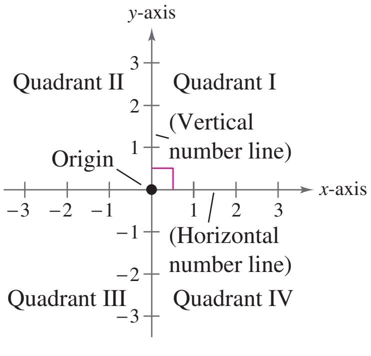


Figure P.10 ***The Cartesian Plane***

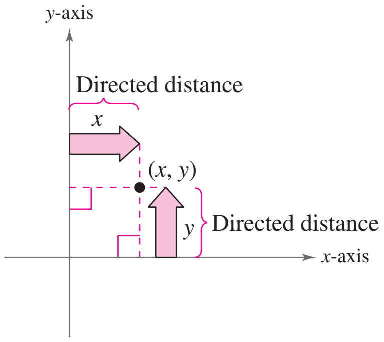
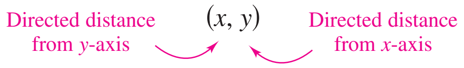


Figure P.11 ***Ordered Pair (x, y)***

Each point in the plane corresponds to an **ordered pair** (x, y) of real numbers x and y, called **coordinates** of the point. The ***x*-coordinate** represents the directed distance from the y-axis to the point, and the ***y*-coordinate** represents the directed distance from the x-axis to the point, as shown in Figure P.11.



The notation (x, y) denotes both a point in the plane and an open interval on the real number line. The context will tell you which meaning is intended.

Example 1 Plotting Points in the Cartesian Plane

Plot the points $(-1, 2)$, $(3, 4)$, $(0, 0)$, $(3, 0)$, and $(-2, -3)$.

Solution

To plot the point $(-1, 2)$, imagine a vertical line through -1 on the x-axis and a horizontal line through 2 on the y-axis. The intersection of these two lines is the point $(-1, 2)$. This point is one unit to the left of the y-axis and two units up from the x-axis. The other four points can be plotted in a similar way (see Figure P.12).

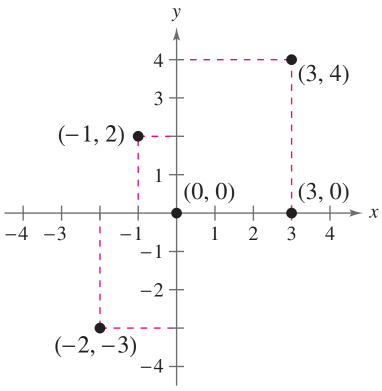


Figure P.12

✓CHECKPOINT Now try Exercise 3.

The beauty of a rectangular coordinate system is that it enables you to see relationships between two variables. It would be difficult to overestimate the importance of Descartes's introduction of coordinates to the plane. Today, his ideas are in common use in virtually every scientific and business-related field.

In the next example, data is represented graphically by points plotted on a rectangular coordinate system. This type of graph is called a **scatter plot.**

Example 2 Sketching a Scatter Plot

The amounts A (in millions of dollars) spent on archery equipment in the United States from 1999 to 2004 are shown in the table, where t represents the year. Sketch a scatter plot of the data by hand. (Source: National Sporting Goods Association)

Year, t	Amount, A
1999	262
2000	259
2001	276
2002	279
2003	281
2004	282

Solution

Before you sketch the scatter plot, it is helpful to represent each pair of values by an ordered pair (t, A), as follows.

$$(1999, 262), (2000, 259), (2001, 276), (2002, 279), (2003, 281), (2004, 282)$$

To sketch a scatter plot of the data shown in the table, first draw a vertical axis to represent the amount (in millions of dollars) and a horizontal axis to represent the year. Then plot the resulting points, as shown in Figure P.13. Note that the break in the t-axis indicates that the numbers 0 through 1998 have been omitted.

STUDY TIP

In Example 2, you could have let $t = 1$ represent the year 1999. In that case, the horizontal axis of the graph would not have been broken, and the tick marks would have been labeled 1 through 6 (instead of 1999 through 2004).

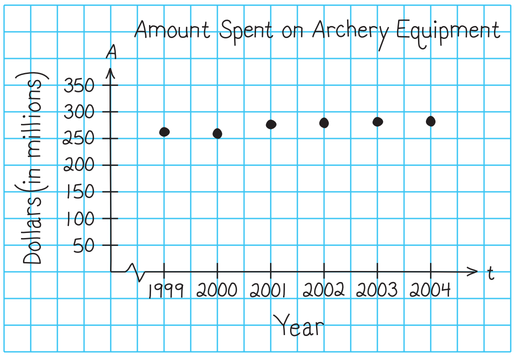
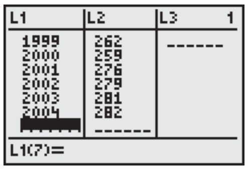


Figure P.13

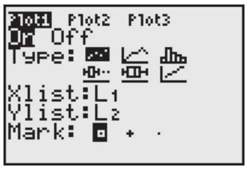
CHECKPOINT Now try Exercise 21.

TECHNOLOGY TIP You can use a graphing utility to graph the scatter plot in Example 2. First, enter the data into the graphing utility's *list editor* as shown in Figure P.14. Then use the *statistical plotting* feature to set up the scatter plot, as shown in Figure P.15. Finally, display the scatter plot (use a viewing window in which $1998 \leq x \leq 2005$ and $0 \leq y \leq 300$), as shown in Figure P.16.



```
L1      L2      L3     1
 1999    262    ------
 2000    259
 2001    276
 2002    279
 2003    281
 2004    282
        ------
L1(7)=
```

Figure P.14



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Plot1 Plot2 Plot3
On  Off
Type:
Xlist:L1
Ylist:L2
Mark: ▫  +  ·
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Figure P.15

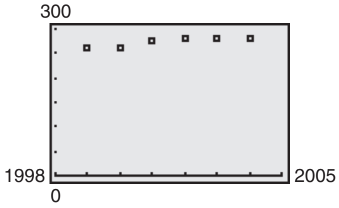
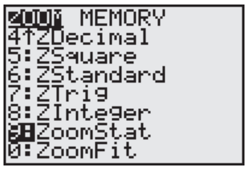


Figure P.16

Some graphing utilities have a *ZoomStat* feature, as shown in Figure P.17. This feature automatically selects an appropriate viewing window that displays all the data in the list editor, as shown in Figure P.18.



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ZOOM MEMORY
4↑ZDecimal
5:ZSquare
6:ZStandard
7:ZTrig
8:ZInteger
9:ZoomStat
0:ZoomFit
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Figure P.17

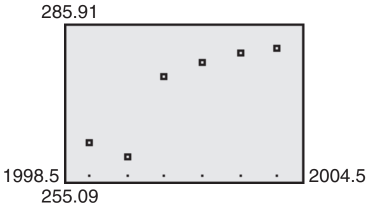


Figure P.18

TECHNOLOGY SUPPORT

For instructions on how to use the *list editor*, see Appendix A; for specific keystrokes, go to this textbook's *Online Study Center*.

The Distance Formula

Recall from the Pythagorean Theorem that, for a right triangle with hypotenuse of length c and sides of lengths a and b, you have $a^2 + b^2 = c^2$, as shown in Figure P.19. (The converse is also true. That is, if $a^2 + b^2 = c^2$, then the triangle is a right triangle.)

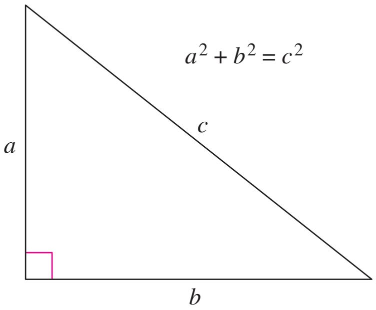


Figure P.19

Suppose you want to determine the distance d between two points (x_1, y_1) and (x_2, y_2) in the plane. With these two points, a right triangle can be formed, as shown in Figure P.20. The length of the vertical side of the triangle is $|y_2 - y_1|$, and the length of the horizontal side is $|x_2 - x_1|$. By the Pythagorean Theorem,

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This result is called the **Distance Formula.**

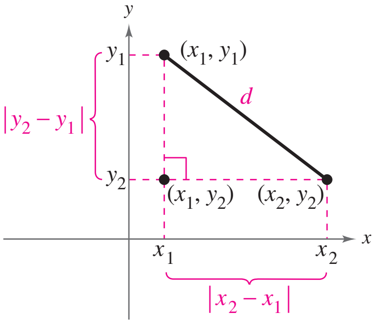


Figure P.20

The Distance Formula

The distance d between the points (x_1, y_1) and (x_2, y_2) in the plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Example 3 Finding a Distance

Find the distance between the points $(-2, 1)$ and $(3, 4)$.

Algebraic Solution

Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (3, 4)$. Then apply the Distance Formula as follows.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{[3 - (-2)]^2 + (4 - 1)^2} && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2. \\ &= \sqrt{(5)^2 + (3)^2} && \text{Simplify.} \\ &= \sqrt{34} \approx 5.83 && \text{Simplify.} \end{aligned}$$

So, the distance between the points is about 5.83 units.

You can use the Pythagorean Theorem to check that the distance is correct.

$$\begin{aligned} d^2 &\overset{?}{=} 3^2 + 5^2 && \text{Pythagorean Theorem} \\ \left(\sqrt{34}\right)^2 &\overset{?}{=} 3^2 + 5^2 && \text{Substitute for } d. \\ 34 &= 34 && \text{Distance checks. } \checkmark \end{aligned}$$

CHECKPOINT Now try Exercise 23.

Graphical Solution

Use centimeter graph paper to plot the points $A(-2, 1)$ and $B(3, 4)$. Carefully sketch the line segment from A to B. Then use a centimeter ruler to measure the length of the segment.

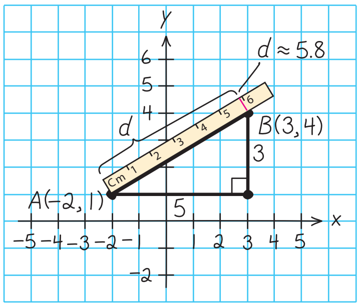


Figure P.21

The line segment measures about 5.8 centimeters, as shown in Figure P.21. So, the distance between the points is about 5.8 units.

Example 4 Verifying a Right Triangle

Show that the points $(2, 1)$, $(4, 0)$, and $(5, 7)$ are the vertices of a right triangle.

Solution

The three points are plotted in Figure P.22. Using the Distance Formula, you can find the lengths of the three sides as follows.

$$\begin{aligned} d_1 &= \sqrt{(5 - 2)^2 + (7 - 1)^2} = \sqrt{9 + 36} = \sqrt{45} \\ d_2 &= \sqrt{(4 - 2)^2 + (0 - 1)^2} = \sqrt{4 + 1} = \sqrt{5} \\ d_3 &= \sqrt{(5 - 4)^2 + (7 - 0)^2} = \sqrt{1 + 49} = \sqrt{50} \end{aligned}$$

Because $(d_1)^2 + (d_2)^2 = 45 + 5 = 50 = (d_3)^2$, you can conclude that the triangle must be a right triangle.

CHECKPOINT Now try Exercise 37.

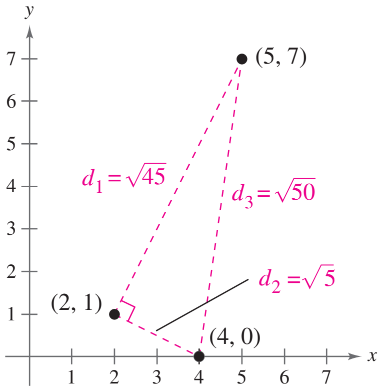


Figure P.22

The Midpoint Formula

To find the **midpoint** of the line segment that joins two points in a coordinate plane, find the average values of the respective coordinates of the two endpoints using the **Midpoint Formula.**

An overhead projector is useful for showing how to plot points and equations. Try projecting a grid onto the chalkboard, or try using overhead markers and graph directly on the transparency. A viewscreen, a device used with an overhead projector to project a graphing calculator's screen image, is also useful.

The Midpoint Formula (See the proof on page 74.)

The midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by the Midpoint Formula

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Exercises 57 and 58 on page 57 help develop a general understanding of the Midpoint Formula.

Example 5 Finding a Line Segment's Midpoint

Find the midpoint of the line segment joining the points $(-5, -3)$ and $(9, 3)$.

Solution

Let $(x_1, y_1) = (-5, -3)$ and $(x_2, y_2) = (9, 3)$.

$$\begin{aligned}\text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) && \text{Midpoint Formula}\\ &= \left(\frac{-5 + 9}{2}, \frac{-3 + 3}{2}\right) && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.\\ &= (2, 0) && \text{Simplify.}\end{aligned}$$

The midpoint of the line segment is $(2, 0)$, as shown in Figure P.23.

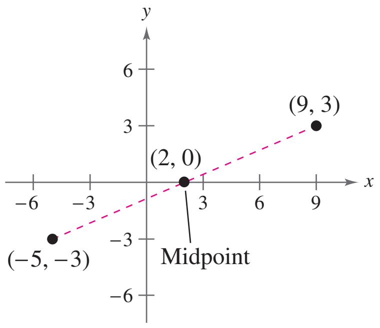


Figure P.23

CHECKPOINT Now try Exercise 49.

Example 6 Estimating Annual Sales

Kraft Foods Inc. had annual sales of \$29.71 billion in 2002 and \$32.17 billion in 2004. Without knowing any additional information, what would you estimate the 2003 sales to have been? (Source: Kraft Foods Inc.)

Solution

One solution to the problem is to assume that sales followed a *linear* pattern. With this assumption, you can estimate the 2003 sales by finding the midpoint of the line segment connecting the points $(2002, 29.71)$ and $(2004, 32.17)$.

$$\begin{aligned}\text{Midpoint} &= \left(\frac{2002 + 2004}{2}, \frac{29.71 + 32.17}{2}\right)\\ &= (2003, 30.94)\end{aligned}$$

So, you would estimate the 2003 sales to have been about \$30.94 billion, as shown in Figure P.24. (The actual 2003 sales were \$31.01 billion.)

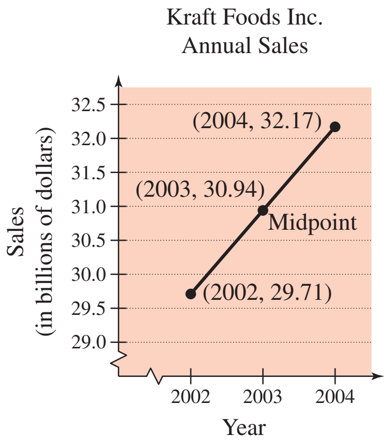


Figure P.24

CHECKPOINT Now try Exercise 55.

The Equation of a Circle

The Distance Formula provides a convenient way to define circles. A **circle of radius *r*** with center at the point (h, k) is shown in Figure P.25. The point (x, y) is on this circle if and only if its distance from the center (h, k) is r. This means that a **circle** in the plane consists of all points (x, y) that are a given positive distance r from a fixed point (h, k). Using the Distance Formula, you can express this relationship by saying that the point (x, y) lies on the circle if and only if

$$\sqrt{(x-h)^2+(y-k)^2}=r.$$

By squaring each side of this equation, you obtain the **standard form of the equation of a circle.**

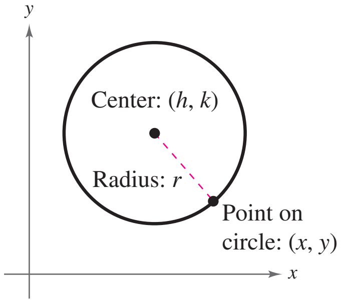


Figure P.25

Standard Form of the Equation of a Circle

The **standard form of the equation of a circle** is

$$(x-h)^2+(y-k)^2=r^2.$$

The point (h, k) is the **center** of the circle, and the positive number r is the **radius** of the circle. The standard form of the equation of a circle whose center is the origin, $(h, k) = (0, 0)$, is $x^2 + y^2 = r^2$.

Activities

1. Set up a Cartesian plane and plot the points $(3, 0)$ and $(-4, 1)$.
2. Find x such that the distance between $(x, 5)$ and $(-1, 2)$ is 5.
   Answer: $x = 3, -5$
3. Find the midpoint of the line segment joining the points $(-1, -4)$ and $(3, -2)$.
   Answer: $(1, -3)$
4. Write the standard form of the equation of the circle with center at $(-3, 5)$ and radius 2.
   Answer: $(x + 3)^2 + (y - 5)^2 = 4$

Example 7 Writing the Equation of a Circle

The point $(3, 4)$ lies on a circle whose center is at $(-1, 2)$, as shown in Figure P.26. Write the standard form of the equation of this circle.

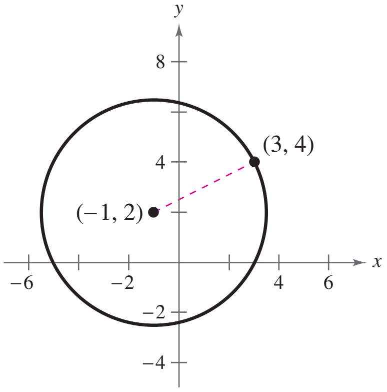


Figure P.26

Solution

The radius r of the circle is the distance between $(-1, 2)$ and $(3, 4)$.

$$\begin{aligned} r &= \sqrt{[3-(-1)]^2+(4-2)^2} && \text{Substitute for } x, y, h, \text{ and } k.\\ &= \sqrt{16+4} && \text{Simplify.}\\ &= \sqrt{20} && \text{Radius} \end{aligned}$$

Using $(h, k) = (-1, 2)$ and $r = \sqrt{20}$, the equation of the circle is

$$\begin{aligned} (x-h)^2+(y-k)^2 &= r^2 && \text{Equation of circle}\\ [x-(-1)]^2+(y-2)^2 &= \left(\sqrt{20}\right)^2 && \text{Substitute for } h, k, \text{ and } r.\\ (x+1)^2+(y-2)^2 &= 20. && \text{Standard form} \end{aligned}$$

✓CHECKPOINT Now try Exercise 61.

Example 8 Translating Points in the Plane

The triangle in Figure P.27 has vertices at the points $(-1, 2)$, $(1, -4)$, and $(2, 3)$. Shift the triangle three units to the right and two units upward and find the vertices of the shifted triangle, as shown in Figure P.28.

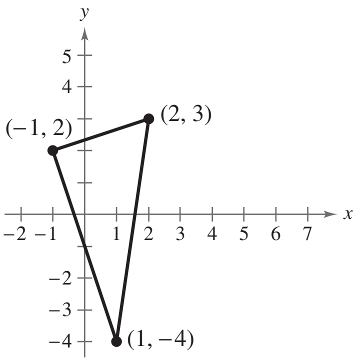


Figure P.27

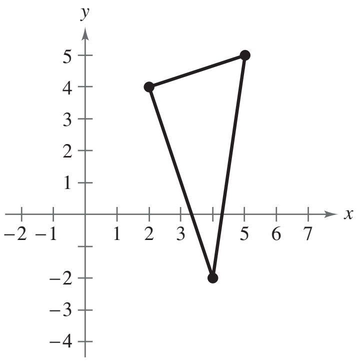


Figure P.28

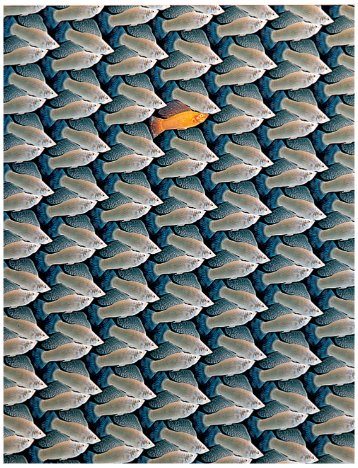
Solution

To shift the vertices three units to the right, add 3 to each of the x-coordinates. To shift the vertices two units upward, add 2 to each of the y-coordinates.

Original Point	*Translated Point*
$(-1, 2)$	$(-1 + 3, 2 + 2) = (2, 4)$
$(1, -4)$	$(1 + 3, -4 + 2) = (4, -2)$
$(2, 3)$	$(2 + 3, 3 + 2) = (5, 5)$

Plotting the translated points and sketching the line segments between them produces the shifted triangle shown in Figure P.28.

CHECKPOINT Now try Exercise 79.



Paul Morrell

Much of computer graphics, including this computer-generated goldfish tessellation, consists of transformations of points in a coordinate plane. One type of transformation, a translation, is illustrated in Example 8. Other types of transformations include reflections, rotations, and stretches.

Example 8 shows how to translate points in a coordinate plane. The following transformed points are related to the original points as follows.

Original Point	*Transformed Point*	
(x, y)	$(-x, y)$	$(-x, y)$ is a reflection of the original point in the y-axis.
(x, y)	$(x, -y)$	$(x, -y)$ is a reflection of the original point in the x-axis.
(x, y)	$(-x, -y)$	$(-x, -y)$ is a reflection of the original point through the origin.

The figures provided with Example 8 were not really essential to the solution. Nevertheless, it is strongly recommended that you develop the habit of including sketches with your solutions, even if they are not required, because they serve as useful problem-solving tools.

P.5 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Vocabulary Check

1. Match each term with its definition.

(a) x-axis

(b) y-axis

(c) origin

(d) quadrants

(e) x-coordinate

(f) y-coordinate

(i) point of intersection of vertical axis and horizontal axis

(ii) directed distance from the x-axis

(iii) horizontal real number line

(iv) four regions of the coordinate plane

(v) directed distance from the y-axis

(vi) vertical real number line

In Exercises 2–5, fill in the blanks.

2. An ordered pair of real numbers can be represented in a plane called the rectangular coordinate system or the _______ plane.

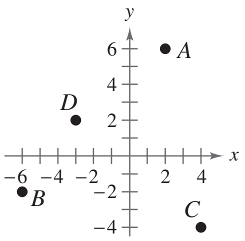
3. The _______ is a result derived from the Pythagorean Theorem.

4. Finding the average values of the respective coordinates of the two endpoints of a line segment in a coordinate plane is also known as using the _______ .

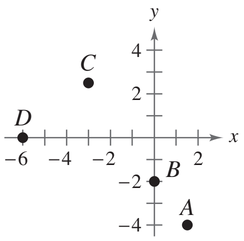
5. The standard form of the equation of a circle is _______ , where the point (h, k) is the _______ of the circle and the positive number r is the _______ of the circle.

In Exercises 1 and 2, approximate the coordinates of the points.

1.



2.



In Exercises 3–6, plot the points in the Cartesian plane.

3. $(-4, 2), (-3, -6), (0, 5), (1, -4)$

4. $(4, -2), (0, 0), (-4, 0), (-5, -5)$

5. $(3, 8), (0.5, -1), (5, -6), (-2, -2.5)$

6. $\left(1, -\frac{1}{2}\right), \left(-\frac{3}{4}, 2\right), (3, -3), \left(\frac{3}{2}, \frac{4}{3}\right)$

In Exercises 7–10, find the coordinates of the point.

7. The point is located five units to the left of the y-axis and four units above the x-axis.

8. The point is located three units below the x-axis and two units to the right of the y-axis.

9. The point is located six units below the x-axis and the coordinates of the point are equal.

10. The point is on the x-axis and 10 units to the left of the y-axis.

In Exercises 11–20, determine the quadrant(s) in which (x, y) is located so that the condition(s) is (are) satisfied.

11. $x > 0$ and $y < 0$

12. $x < 0$ and $y < 0$

13. $x = -4$ and $y > 0$

14. $x > 2$ and $y = 3$

15. $y < -5$

16. $x > 4$

17. $x < 0$ and $-y > 0$

18. $-x > 0$ and $y < 0$

19. $xy > 0$

20. $xy < 0$

In Exercises 21 and 22, sketch a scatter plot of the data shown in the table.

21. ***Sales*** The table shows the sales y (in millions of dollars) for Apple Computer, Inc. for the years 1997–2006. (Source: Value Line)

Year	Sales, y (in millions of dollars)
1997	7,081
1998	5,941
1999	6,134
2000	7,983
2001	5,363
2002	5,742
2003	6,207
2004	8,279
2005	13,900
2006	16,600

22. ***Meteorology*** The table shows the lowest temperature on record y (in degrees Fahrenheit) in Duluth, Minnesota for each month x, where $x = 1$ represents January. (Source: NOAA)

Month, x	Temperature, y
1	−39
2	−39
3	−29
4	−5
5	17
6	27
7	35
8	32
9	22
10	8
11	−23
12	−34

In Exercises 23–32, find the distance between the points algebraically and verify graphically by using centimeter graph paper and a centimeter ruler.

23. $(6, -3), (6, 5)$

24. $(1, 4), (8, 4)$

25. $(-3, -1), (2, -1)$

26. $(-3, -4), (-3, 6)$

27. $(-2, 6), (3, -6)$

28. $(8, 5), (0, 20)$

29. $\left(\frac{1}{2}, \frac{4}{3}\right), (2, -1)$

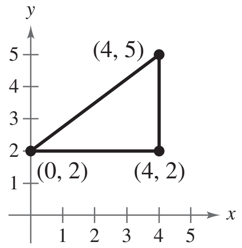
30. $\left(-\frac{2}{3}, 3\right), \left(-1, \frac{5}{4}\right)$

31. $(-4.2, 3.1), (-12.5, 4.8)$

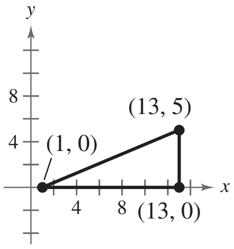
32. $(9.5, -2.6), (-3.9, 8.2)$

In Exercises 33–36, (a) find the length of each side of the right triangle and (b) show that these lengths satisfy the Pythagorean Theorem.

33.



34.



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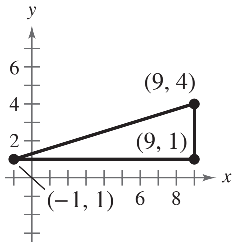
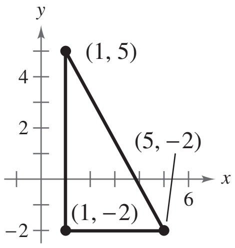


36.



In Exercises 37–44, show that the points form the vertices of the polygon.

37. Right triangle: $(4, 0), (2, 1), (-1, -5)$

38. Right triangle: $(-1, 3), (3, 5), (5, 1)$

39. Isosceles triangle: $(1, -3), (3, 2), (-2, 4)$

40. Isosceles triangle: $(2, 3), (4, 9), (-2, 7)$

41. Parallelogram: $(2, 5), (0, 9), (-2, 0), (0, -4)$

42. Parallelogram: $(0, 1), (3, 7), (4, 4), (1, -2)$

43. Rectangle: $(-5, 6), (0, 8), (-3, 1), (2, 3)$ (*Hint:* Show that the diagonals are of equal length.)

44. Rectangle: $(2, 4), (3, 1), (1, 2), (4, 3)$ (*Hint:* Show that the diagonals are of equal length.)

In Exercises 45–54, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points.

45. $(1, 1), (9, 7)$

46. $(1, 12), (6, 0)$

47. $(-4, 10), (4, -5)$

48. $(-7, -4), (2, 8)$

49. $(-1, 2), (5, 4)$

50. $(2, 10), (10, 2)$

51. $\left(\frac{1}{2}, 1\right), \left(-\frac{5}{2}, \frac{4}{3}\right)$

52. $\left(-\frac{1}{3}, -\frac{1}{3}\right), \left(-\frac{1}{6}, -\frac{1}{2}\right)$

53. $(6.2, 5.4), (-3.7, 1.8)$

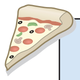
54. $(-16.8, 12.3), (5.6, 4.9)$

Revenue **In Exercises 55 and 56, use the Midpoint Formula to estimate the annual revenues (in millions of dollars) for Wendy's Intl., Inc. and Papa John's Intl. in 2003. The revenues for the two companies in 2000 and 2006 are shown in the tables. Assume that the revenues followed a linear pattern.** (Source: Value Line)

55. Wendy's Intl., Inc.

Year	Annual revenue (in millions of dollars)
2000	2237
2006	3950

56. Papa John's Intl.



Year	Annual revenue (in millions of dollars)
2000	945
2006	1005

57. ***Exploration*** A line segment has (x_1, y_1) as one endpoint and (x_m, y_m) as its midpoint. Find the other endpoint (x_2, y_2) of the line segment in terms of x_1, y_1, x_m, and y_m. Use the result to find the coordinates of the endpoint of a line segment if the coordinates of the other endpoint and midpoint are, respectively,

(a) $(1, -2), (4, -1)$

(b) $(-5, 11), (2, 4)$

58. ***Exploration*** Use the Midpoint Formula three times to find the three points that divide the line segment joining (x_1, y_1) and (x_2, y_2) into four parts. Use the result to find the points that divide the line segment joining the given points into four equal parts.

(a) $(1, -2), (4, -1)$

(b) $(-2, -3), (0, 0)$

In Exercises 59–72, write the standard form of the equation of the specified circle.

59. Center: $(0, 0)$; radius: 3

60. Center: $(0, 0)$; radius: 6

61. Center: $(2, -1)$; radius: 4

62. Center: $\left(0, \frac{1}{3}\right)$; radius: $\frac{1}{3}$

63. Center: $(-1, 2)$; solution point: $(0, 0)$

64. Center: $(3, -2)$; solution point: $(-1, 1)$

65. Endpoints of a diameter: $(0, 0), (6, 8)$

66. Endpoints of a diameter: $(-4, -1), (4, 1)$

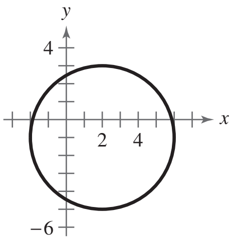
67. Center: $(-2, 1)$; tangent to the x-axis

68. Center: $(3, -2)$; tangent to the y-axis

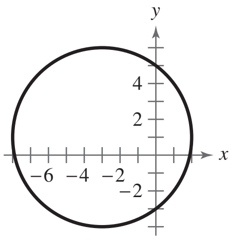
69. The circle inscribed in the square with vertices $(7, -2)$, $(-1, -2)$, $(-1, -10)$, and $(7, -10)$

70. The circle inscribed in the square with vertices $(-12, 10)$, $(8, 10)$, $(8, -10)$, and $(-12, -10)$

71.



72.



In Exercises 73–78, find the center and radius, and sketch the circle.

73. $x^2 + y^2 = 25$

74. $x^2 + y^2 = 16$

75. $(x - 1)^2 + (y + 3)^2 = 4$

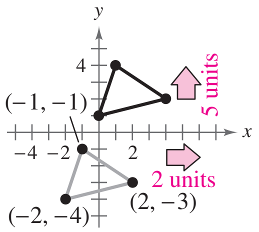
76. $x^2 + (y - 1)^2 = 49$

77. $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$

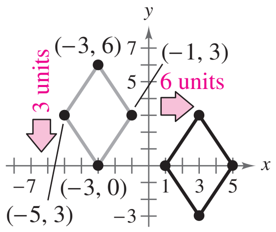
78. $\left(x - \frac{2}{3}\right)^2 + \left(y + \frac{1}{4}\right)^2 = \frac{25}{9}$

In Exercises 79–82, the polygon is shifted to a new position in the plane. Find the coordinates of the vertices of the polygon in the new position.

79.



80.



81. Original coordinates of vertices:

$(0, 2), (3, 5), (5, 2), (2, -1)$

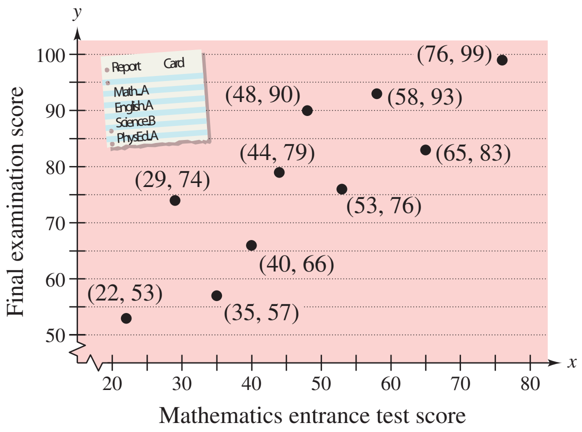
Shift: three units upward, one unit to the left

82. Original coordinates of vertices:

$(1, -1), (3, 2), (1, -2)$

Shift: two units downward, three units to the left

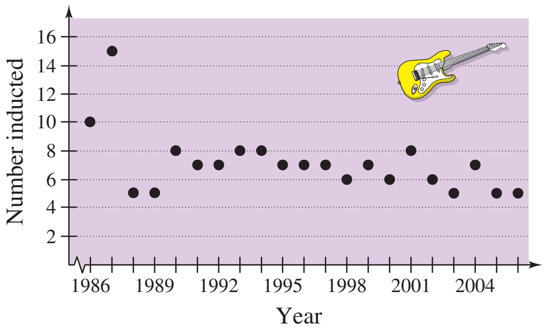
Analyzing Data **In Exercises 83 and 84, refer to the scatter plot, which shows the mathematics entrance test scores *x* and the final examination scores *y* in an algebra course for a sample of 10 students.**



83. Find the entrance exam score of any student with a final exam score in the 80s.

84. Does a higher entrance exam score necessarily imply a higher final exam score? Explain.

85. ***Rock and Roll Hall of Fame*** The graph shows the numbers of recording artists inducted into the Rock and Roll Hall of Fame from 1986 to 2006.

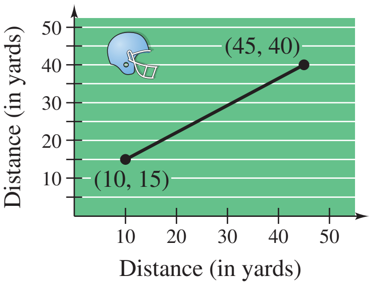


(a) Describe any trends in the data. From these trends, predict the number of artists that will be elected in 2007.

(b) Why do you think the numbers elected in 1986 and 1987 were greater than in other years?

86. ***Flying Distance*** A jet plane flies from Naples, Italy in a straight line to Rome, Italy, which is 120 kilometers north and 150 kilometers west of Naples. How far does the plane fly?

87. ***Sports*** In a football game, a quarterback throws a pass from the 15-yard line, 10 yards from the sideline, as shown in the figure. The pass is caught on the 40-yard line, 45 yards from the same sideline. How long is the pass?



88. ***Sports*** A major league baseball diamond is a square with 90-foot sides. Place a coordinate system over the baseball diamond so that home plate is at the origin and the first base line lies on the positive x-axis (see figure). Let one unit in the coordinate plane represent one foot. The right fielder fields the ball at the point (300, 25). How far does the right fielder have to throw the ball to get a runner out at home plate? How far does the right fielder have to throw the ball to get a runner out at third base? (Round your answers to one decimal place.)

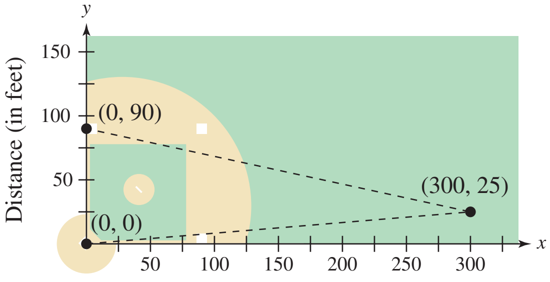


Figure for 88

89. ***Boating*** A yacht named Beach Lover leaves port at noon and travels due north at 16 miles per hour. At the same time another yacht, The Fisherman, leaves the same port and travels west at 12 miles per hour.

(a) Using graph paper, plot the coordinates of each yacht at 2 P.M. and 4 P.M. Let the port be at the origin of your coordinate system.

(b) Find the distance between the yachts at 2 P.M. and 4 P.M. Are the yachts twice as far from each other at 4 P.M. as they were at 2 P.M.?

90. ***Make a Conjecture*** Plot the points (2, 1), $(-3, 5)$, and $(7, -3)$ on a rectangular coordinate system. Then change the signs of the indicated coordinate(s) of each point and plot the three new points on the same rectangular coordinate system. Make a conjecture about the location of a point when each of the following occurs.

(a) The sign of the x-coordinate is changed.

(b) The sign of the y-coordinate is changed.

(c) The signs of both the x- and y-coordinates are changed.

91. Show that the coordinates (2, 6), $\left(2 + 2\sqrt{3}, 0\right)$, and $\left(2 - 2\sqrt{3}, 0\right)$ form the vertices of an equilateral triangle.

92. Show that the coordinates $(-2, -1)$, (4, 7), and $(2, -4)$ form the vertices of a right triangle.

Synthesis

True or False? **In Exercises 93–95, determine whether the statement is true or false. Justify your answer.**

93. In order to divide a line segment into 16 equal parts, you would have to use the Midpoint Formula 16 times.

94. The points $(-8, 4)$, (2, 11) and $(-5, 1)$ represent the vertices of an isosceles triangle.

95. If four points represent the vertices of a polygon, and the four sides are equal, then the polygon must be a square.

96. ***Think About It*** What is the y-coordinate of any point on the x-axis? What is the x-coordinate of any point on the y-axis?

97. ***Think About It*** When plotting points on the rectangular coordinate system, is it true that the scales on the x- and y-axes must be the same? Explain.