What Did You Learn?

Key Terms

real numbers, *p. 2*
rational and irrational numbers, *p. 2*
absolute value, *p. 5*
variables, *p. 6*
algebraic expressions, *p. 6*
Basic Rules of Algebra, *p. 7*
Zero-Factor Property, *p. 8*
exponential form, *p. 12*
scientific notation, *p. 14*
square root, cube root, *p. 15*
conjugate, *p. 18*
polynomial in x, *p. 24*
FOIL Method, *p. 25*
completely factored, *p. 27*
domain, *p. 37*
rational expression, *p. 37*
rectangular coordinate system, *p. 48*
Distance Formula, *p. 50*
Midpoint Formula, *p. 51*
standard form of the equation of a circle, *p. 53*

Key Concepts

P.1 ■ Using the Basic Rules of Algebra

The properties of real numbers are also true for variables and expressions and are called the Basic Rules of Algebra.

P.2 ■ Using the properties of exponents

The properties of exponents can be used when the exponent is an integer or a rational number.

P.2 ■ Simplifying radicals

1. Let a and b be real numbers and let $n \geq 2$ be a positive integer. If $a = b^n$, then b is an nth root of a.
2. An expression involving radicals is in simplest form when all possible factors have been removed from the radical, all fractions have radical-free denominators, and the index of the radical is reduced.
3. If a is a real number and n and m are positive integers such that the principal nth root of a exists, then $a^{m/n} = (a^{1/n})^m = \left(\sqrt[n]{a}\right)^m$ and $a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$.

P.3 ■ Operations with polynomials

1. Add or subtract like terms (terms having the exact same variables to the exact same powers) by adding their coefficients.
2. To find the product of two polynomials, use the left and right Distributive Properties.

P.3 ■ Factoring polynomials

1. Writing a polynomial as a product is called factoring. If a polynomial cannot be factored using integer coefficients, it is prime or irreducible over the integers.
2. Factor a polynomial by removing a common factor, by recognizing special product forms, and/or by grouping.

P.4 ■ Operations with rational expressions

1. To add or subtract rational expressions, first rewrite the expressions with the LCD. Then add or subtract the numerators and place over the LCD.
2. To multiply rational expressions, multiply the numerators, multiply the denominators, and then simplify.
3. To divide two rational expressions, invert the divisor and multiply.

P.5 ■ Plotting points in the Cartesian plane

Each point in the plane corresponds to an ordered pair (x, y) of real numbers x and y, called the coordinates of the point. The x-coordinate represents the directed distance from the y-axis to the point, and the y-coordinate represents the directed distance from the x-axis to the point.

P.5 ■ Using the Distance and Midpoint Formulas

1. The distance d between points (x_1, y_1) and (x_2, y_2) in the plane is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
2. The midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by the Midpoint Formula.

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

P.6 ■ Using line plots, histograms, bar graphs, and line graphs to represent data

1. Use a line plot when ordering small sets of numbers by hand.
2. Use a histogram when organizing large sets of data.
3. Use a bar graph when the labels of the bars are not necessarily numbers. The bars of a bar graph can be horizontal or vertical.
4. Use a line graph to show trends over periods of time.

Review Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

P.1 **In Exercises 1 and 2, determine which numbers are (a) natural numbers, (b) whole numbers, (c) integers, (d) rational numbers, and (e) irrational numbers.**

1. $\{11, -14, -\frac{8}{9}, \frac{5}{2}, \sqrt{6}, 0.4\}$

2. $\{\sqrt{15}, -22, -\frac{10}{3}, 0, 5.2, \frac{3}{7}\}$

In Exercises 3 and 4, use a calculator to find the decimal form of each rational number. If it is a nonterminating decimal, write the repeating pattern. Then plot the numbers on the real number line and place the correct inequality symbol (< or >) between them.

3. (a) $\frac{5}{6}$ (b) $\frac{7}{8}$

4. (a) $\frac{1}{3}$ (b) $\frac{9}{25}$

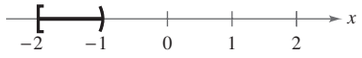
In Exercises 5 and 6, verbally describe the subset of real numbers represented by the inequality. Then sketch the subset on the real number line.

5. $x \leq 7$

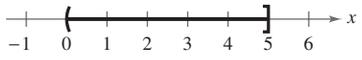
6. $x > 1$

In Exercises 7 and 8, use the interval notation to describe the set.

7.



8.



In Exercises 9 and 10, find the distance between a and b.

9. $a = -74, \quad b = 48$

10. $a = -123, \quad b = -9$

In Exercises 11–14, use absolute value notation to describe the situation.

11. The distance between x and 7 is at least 6.

12. The distance between x and 25 is no more than 10.

13. The distance between y and -30 is less than 5.

14. The distance between y and -16 is greater than 8.

In Exercises 15–18, evaluate the expression for each value of x. (If not possible, state the reason.)

Expression	*Values*	
15. $9x - 2$	(a) $x = -1$	(b) $x = 3$
16. $x^2 - 11x + 24$	(a) $x = -2$	(b) $x = 2$
17. $-2x^2 - x + 3$	(a) $x = 3$	(b) $x = -3$
18. $\frac{4x}{x - 1}$	(a) $x = -1$	(b) $x = 1$

In Exercises 19–22, identify the rule of algebra illustrated by the statement.

19. $2x + (3x - 10) = (2x + 3x) - 10$

20. $\frac{2}{y + 4} \cdot \frac{y + 4}{2} = 1, \quad y \neq -4$

21. $(t^2 + 1) + 3 = 3 + (t^2 + 1)$

22. $0 + (a - 5) = a - 5$

In Exercises 23–28, perform the operation(s). (Write fractional answers in simplest form.)

23. $\frac{2}{3} + \frac{8}{9}$

24. $\frac{3}{4} - \frac{1}{6} + \frac{1}{8}$

25. $\frac{3}{16} \div \frac{9}{2}$

26. $\frac{3}{4} \cdot \frac{2}{9}$

27. $\frac{x}{5} + \frac{7x}{12}$

28. $\frac{9}{x} \div \frac{1}{6}$

P.2 **In Exercises 29–32, simplify each expression.**

29. (a) $(-2z)^3$ (b) $(a^2b^4)(3ab^{-2})$

30. (a) $\frac{(8y)^0}{y^2}$ (b) $\frac{40(b - 3)^5}{75(b - 3)^2}$

31. (a) $\frac{6^2u^3v^{-3}}{12u^{-2}v}$ (b) $\frac{3^{-4}m^{-1}n^{-3}}{9^{-2}mn^{-3}}$

32. (a) $(x^{-1} + y)^{-2}$ (b) $\left(\frac{y^{-2}}{x}\right)^{-1}\left(\frac{x^2}{y^{-2}}\right)$

In Exercises 33–38, write the number in scientific notation.

33. 2,585,000,000

34. $-3{,}250{,}000$

35. 0.000000125

36. -0.000002104

37. *Sales of The Hershey Company in 2006:* \$5,100,000,000 (Source: Value Line)

38. *Number of meters in one foot:* 0.3048

In Exercises 39–44, write the number in decimal notation.

39. 1.28×10^5

40. -4.002×10^2

41. 1.80×10^{-5}

42. -4.02×10^{-2}

43. *Distance between the sun and Jupiter:* 4.836×10^8 miles

44. *Ratio of day to year:* 2.74×10^{-3}

In Exercises 45 and 46, use the properties of radicals to simplify the expression.

45. $\left(\sqrt[4]{78}\right)^4$

46. $\sqrt[5]{8} \cdot \sqrt[5]{4}$

In Exercises 47–52, simplify by removing all possible factors from the radical.

47. $\sqrt{25a^2}$

48. $\sqrt[5]{64x^6}$

49. $\sqrt{\frac{81}{144}}$

50. $\sqrt[3]{\frac{125}{216}}$

51. $\sqrt[3]{\frac{2x^3}{27}}$

52. $\sqrt{\frac{75x^2}{y^4}}$

In Exercises 53–58, simplify the expression.

53. $\sqrt{48} - \sqrt{27}$

54. $3\sqrt{32} + 4\sqrt{98}$

55. $8\sqrt{3x} - 5\sqrt{3x}$

56. $-11\sqrt{36y} - 6\sqrt{y}$

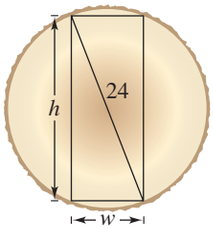
57. $\sqrt{8x^3} + \sqrt{2x}$

58. $3\sqrt{14x^2} - \sqrt{56x^2}$

Strength of a Wooden Beam **In Exercises 59 and 60, use the figure, which shows the rectangular cross section of a wooden beam cut from a log of diameter 24 inches.**

59. Find the area of the cross section when $w = 12\sqrt{2}$ inches and $h = \sqrt{24^2 - \left(12\sqrt{2}\right)^2}$ inches. What is the shape of the cross section? Explain.

60. The rectangular cross section will have a maximum strength when $w = 8\sqrt{3}$ inches and $h = \sqrt{24^2 - \left(8\sqrt{3}\right)^2}$ inches. Find the area of the cross section.



In Exercises 61 and 62, rationalize the denominator of the expression. Then simplify your answer.

61. $\frac{1}{3 - \sqrt{5}}$

62. $\frac{1}{\sqrt{x} - 1}$

In Exercises 63 and 64, rationalize the numerator of the expression. Then simplify your answer.

63. $\frac{\sqrt{20}}{4}$

64. $\frac{\sqrt{2} - \sqrt{11}}{3}$

In Exercises 65–68, simplify the expression.

65. $64^{5/2}$

66. $64^{-2/3}$

67. $(-3x^{2/5})(-2x^{1/2})$

68. $(x - 1)^{1/3}(x - 1)^{-1/4}$

P.3 In Exercises 69 and 70, write the polynomial in standard form. Then identify the degree and leading coefficient of the polynomial.

69. $15x^2 - 2x^5 + 3x^3 + 5 - x^4$

70. $-2x^4 + x^2 - 10 - x + x^3$

In Exercises 71–78, perform the operations and write the result in standard form.

71. $-(3x^2 + 2x) + (1 - 5x)$

72. $8y - [2y^2 - (3y - 8)]$

73. $(2x^3 - 5x^2 + 10x - 7) + (4x^2 - 7x - 2)$

74. $(6x^4 - 4x^3 - x + 3 - 20x^2) - (16 + 9x^4 - 11x^2)$

75. $(a^2 + a - 3)(a^3 + 2)$

76. $(x^3 - 3x)(2x^2 + 3x + 5)$

77. $(y^2 - y)(y^2 + 1)(y^2 + y + 1)$

78. $\left(x - \frac{1}{x}\right)(x + 2)$

In Exercises 79–84, find the special product.

79. $(x + 8)(x - 8)$

80. $(7x + 4)(7x - 4)$

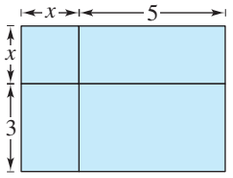
81. $(x - 4)^3$

82. $(2x - 1)^3$

83. $(m - 4 + n)(m - 4 - n)$

84. $(x - y - 6)(x - y + 6)$

85. ***Geometry*** Use the area model to write two different expressions for the area. Then equate the two expressions and name the algebraic property that is illustrated.



86. ***Compound Interest*** After 2 years, an investment of \$2500 compounded annually at an interest rate r will yield an amount of $2500(1 + r)^2$. Write this polynomial in standard form.

In Exercises 87–92, factor out the common factor.

87. $7x + 35$

88. $4b - 12$

89. $x^3 - x$

90. $x(x - 3) + 4(x - 3)$

91. $2x^3 + 18x^2 - 4x$

92. $-6x^4 - 3x^3 + 12x$

93. ***Geometry*** The surface area of a right circular cylinder is $S = 2\pi r^2 + 2\pi rh$.

(a) Draw a right circular cylinder of radius r and height h. Use the figure to explain how the surface area formula is obtained.

(b) Factor the expression for surface area.

94. ***Business*** The revenue for selling x units of a product at a price of p dollars per unit is $R = xp$. For a flat panel television the revenue is $R = 1600x - 0.50x^2$. Factor the expression and determine an expression that gives the price in terms of x.

In Exercises 95–102, factor the expression.

95. $x^2 - 169$

96. $9x^2 - \frac{1}{25}$

97. $x^3 + 216$

98. $64x^3 - 27$

99. $x^2 - 6x - 27$

100. $x^2 - 9x + 14$

101. $2x^2 + 21x + 10$

102. $3x^2 + 14x + 8$

In Exercises 103–106, factor by grouping.

103. $x^3 - 4x^2 - 3x + 12$

104. $x^3 - 6x^2 - x + 6$

105. $2x^2 - x - 15$

106. $6x^2 + x - 12$

P.4 In Exercises 107–110, find the domain of the expression.

107. $-5x^2 - x - 1$

108. $9x^4 + 7, \quad x > 0$

109. $\dfrac{4}{2x - 3}$

110. $\sqrt{x + 12}$

In Exercises 111–114, write the rational expression in simplest form.

111. $\dfrac{4x^2}{4x^3 + 28x}$

112. $\dfrac{6xy}{xy + 2x}$

113. $\dfrac{x^2 - x - 30}{x^2 - 25}$

114. $\dfrac{x^2 - 9x + 18}{8x - 48}$

In Exercises 115–122, perform the operations and simplify your answer.

115. $\dfrac{x^2 - 4}{x^4 - 2x^2 - 8} \cdot \dfrac{x^2 + 2}{x^2}$

116. $\dfrac{2x - 1}{x + 1} \cdot \dfrac{x^2 - 1}{2x^2 - 7x + 3}$

117. $\dfrac{x^2(5x - 6)}{2x + 3} \div \dfrac{5x}{2x + 3}$

118. $\dfrac{4x - 6}{(x - 1)^2} \div \dfrac{2x^2 - 3x}{x^2 + 2x - 3}$

119. $x - 1 + \dfrac{1}{x + 2} + \dfrac{1}{x - 1}$

120. $2x + \dfrac{3}{2(x - 4)} - \dfrac{1}{2(x + 2)}$

121. $\dfrac{1}{x} - \dfrac{x - 1}{x^2 + 1}$

122. $\dfrac{1}{x - 1} + \dfrac{1 - x}{x^2 + x + 1}$

In Exercises 123 and 124, simplify the complex fraction.

123. $\dfrac{\left(\dfrac{1}{x} - \dfrac{1}{y}\right)}{(x^2 - y^2)}$

124. $\dfrac{\left(\dfrac{1}{2x - 3} - \dfrac{1}{2x + 3}\right)}{\left(\dfrac{1}{2x} - \dfrac{1}{2x + 3}\right)}$

P.5 In Exercises 125–128, plot the point in the Cartesian plane and determine the quadrant in which it is located.

125. $(8, -3)$

126. $(-4, -9)$

127. $\left(-\frac{5}{2}, 10\right)$

128. $(-6.5, -0.5)$

In Exercises 129 and 130, determine the quadrant(s) in which (x, y) is located so that the conditions are satisfied.

129. $x > 0$ and $y = -2$

130. $xy = 4$

Revenue **In Exercises 131 and 132, use the table, which shows the operating revenues (in millions of dollars) for the motion picture industry for the years 1998 to 2003.** (Source: U.S. Census Bureau)

Year	Operating revenue (in millions of dollars)
1998	48,002
1999	51,448
2000	54,040
2001	55,937
2002	60,486
2003	64,096

131. Sketch a scatter plot of the data.

132. What statement can be made about the operating revenue for the motion picture industry?

In Exercises 133 and 134, plot the points and find the distance between the points.

133. $(-3, 8), (1, 5)$

134. $(5.6, 0), (0, 8.2)$

In Exercises 135 and 136, plot the points and find the midpoint of the line segment joining the points.

135. $(-12, 5), (4, -7)$

136. $(1.8, 7.4), (-0.6, -14.5)$

In Exercises 137 and 138, write the standard form of the equation of the specified circle.

137. Center: $(3, -1)$; solution point: $(-5, 1)$

138. Endpoints of a diameter: $(-4, 6), (10, -2)$

In Exercises 139 and 140, the polygon is shifted to a new position in the plane. Find the coordinates of the vertices of the polygon in the new position.

139. Original coordinates of vertices:

$(4, 8), (6, 8), (4, 3), (6, 3)$

Shift: three units downward, two units to the left

140. Original coordinates of vertices:

$(0, 1), (3, 3), (0, 5), (-3, 3)$

Shift: five units upward, four units to the right

P.6

141. ***Consumer Awareness*** Use a line plot to organize the following sample of prices (in dollars) of running shoes. Which price occurred with the greatest frequency?

100, 65, 67, 88, 69, 60, 100, 100, 88, 79, 99, 75, 65, 89, 68, 74, 100, 66, 81, 95, 75, 69, 85, 91, 71

142. ***Veterans*** The list shows the numbers of Gulf War veterans (in thousands) in the 50 states and District of Columbia from 1990 to 2004. Use a frequency distribution and a histogram to organize the data. (Source: Department of Veterans Affairs)

AK 18	AL 81	AR 46	AZ 93
CA 361	CO 89	CT 28	DC 6
DE 13	FL 277	GA 179	HI 20
IA 37	ID 28	IL 133	IN 84
KS 43	KY 62	LA 72	MA 54
MD 95	ME 20	MI 116	MN 55
MO 85	MS 49	MT 16	NC 154
ND 10	NE 28	NH 18	NJ 62
NM 32	NV 41	NY 137	OH 155
OK 65	OR 53	PA 134	RI 11
SC 86	SD 13	TN 97	TX 354
UT 29	VA 196	VT 7	WA 122
WI 65	WV 27	WY 11	

143. ***Meteorology*** The normal daily maximum and minimum temperatures (in °F) for each month for the city of Chicago are shown in the table. Construct a double bar graph for the data. (Source: National Climatic Data Center)

Month	Max.	Min.
Jan.	29.6	14.3
Feb.	34.7	19.2
Mar.	46.1	28.5
Apr.	58.0	37.6
May	69.9	47.5
Jun.	79.2	57.2
Jul.	83.5	63.2
Aug.	81.2	62.2
Sep.	73.9	53.7
Oct.	62.1	42.1
Nov.	47.1	31.6
Dec.	34.4	20.4

144. ***Law Enforcement*** The table shows the numbers of people indicted for public corruption in the United States from 1995 to 2003. Construct a line graph for the data and state what information the graph reveals. (Source: U.S. Department of Justice)

Year	Number of indictments
1995	1051
1996	984
1997	1057
1998	1174
1999	1134
2000	1000
2001	1087
2002	1136
2003	1150

TABLE FOR 144

145. ***Basketball*** The list shows the average numbers of points per game for the top 20 NBA players for the 2004–2005 regular NBA season. Organize the data in an appropriate display. Explain your choice of graph. (Source: National Basketball Association)

30.7, 27.6, 27.2, 26.1, 26.0, 25.7, 25.5, 24.6, 24.5, 24.3, 24.1, 23.9, 23.0, 22.9, 22.2, 22.2, 22.2, 22.0, 21.7, 21.7

146. ***Salaries*** The table shows the average salaries (in thousands of dollars) for professors, associate professors, assistant professors, and instructors at public institutions of higher education from 2003 to 2005. Organize the data in an appropriate display. Explain your choice of graph. (Source: American Association of University Professors)

Rank	2003	2004	2005
Professor	84.1	85.8	88.5
Associate Professor	61.5	62.4	64.4
Assistant Professor	51.5	52.5	54.3
Instructor	37.2	37.9	39.4

Synthesis

True or False? **In Exercises 147 and 148, determine whether the statement is true or false. Justify your answer.**

147. $\frac{x^3 - 1}{x - 1} = x^2 + x + 1$ for all values of x.

148. A binomial sum squared is equal to the sum of the terms squared.

Error Analysis **In Exercises 149 and 150, describe the error.**

149. ~~$(2x)^4 = 2x^4$~~

150. ~~$\sqrt{3^2 + 4^2} = 3 + 4$~~

151. ***Writing*** Explain why $\sqrt{5u} + \sqrt{3u} \neq 2\sqrt{2u}$.

P Chapter Test

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

Take this test as you would take a test in class. After you are finished, check your work against the answers in the back of the book.

1. Use < or > to show the relationship between $-\frac{10}{3}$ and $-|-4|$.

2. Find the distance between the real numbers -17 and 39.

3. Identify the rule of algebra illustrated by $(5 - x) + 0 = 5 - x$.

In Exercises 4 and 5, evaluate each expression without using a calculator.

4. (a) $27\left(-\frac{2}{3}\right)$ (b) $\frac{5}{18} \div \frac{15}{8}$ (c) $\left(-\frac{2}{7}\right)^3$ (d) $\left(\frac{3^2}{2}\right)^{-3}$

5. (a) $\sqrt{5} \cdot \sqrt{125}$ (b) $\frac{\sqrt{72}}{\sqrt{2}}$ (c) $\frac{5.4 \times 10^8}{3 \times 10^3}$ (d) $(3 \times 10^4)^3$

In Exercises 6 and 7, simplify each expression.

6. (a) $3z^2(2z^3)^2$ (b) $(u - 2)^{-4}(u - 2)^{-3}$ (c) $\left(\frac{x^{-2}y^2}{3}\right)^{-1}$

7. (a) $9z\sqrt{8z} - 3\sqrt{2z^3}$ (b) $-5\sqrt{16y} + 10\sqrt{y}$ (c) $\sqrt[3]{\frac{16}{v^5}}$

8. Write the polynomial $3 - 2x^5 + 3x^3 - x^4$ in standard form. Identify the degree and leading coefficient.

In Exercises 9–12, perform the operations and simplify.

9. $(x^2 + 3) - [3x + (8 - x^2)]$

10. $(2x - 5)(4x^2 + 3)$

11. $\frac{8x}{x - 3} + \frac{24}{3 - x}$

12. $\frac{\left(\frac{2}{x} - \frac{2}{x + 1}\right)}{\left(\frac{4}{x^2 - 1}\right)}$

In Exercises 13–15, find the special product.

13. $\left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)$

14. $(x - 2)^3$

15. $[(x + y) - z][(x + y) + z]$

In Exercises 16–18, factor the expression completely.

16. $2x^4 - 3x^3 - 2x^2$

17. $x^3 + 2x^2 - 4x - 8$

18. $8x^3 - 27$

19. Rationalize each denominator: (a) $\frac{16}{\sqrt[3]{16}}$, (b) $\frac{6}{1 - \sqrt{3}}$, and (c) $\frac{1}{\sqrt{x + 2} - \sqrt{2}}$.

20. Write an expression for the area of the shaded region in the figure at the right and simplify the result.

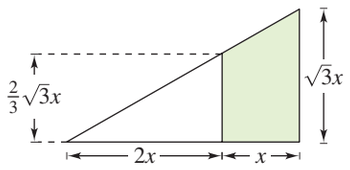


Figure for 20

21. Plot the points $(-2, 5)$ and $(6, 0)$. Find the coordinates of the midpoint of the line segment joining the points and the distance between the points.

22. The numbers (in millions) of votes cast for the Democratic candidates for president in 1980, 1984, 1988, 1992, 1996, 2000, and 2004 were 35.5, 37.5, 41.7, 44.9, 47.4, 51.0, and 58.9, respectively. Construct a bar graph for the data. (Source: Office of the Clerk, U.S. House of Representatives)

Proofs in Mathematics

What does the word *proof* mean to you? In mathematics, the word *proof* is used to mean simply a valid argument. When you are proving a statement or theorem, you must use facts, definitions, and accepted properties in a logical order. You can also use previously proved theorems in your proof. For instance, the Distance Formula is used in the proof of the Midpoint Formula below. There are several different proof methods, which you will see in later chapters.

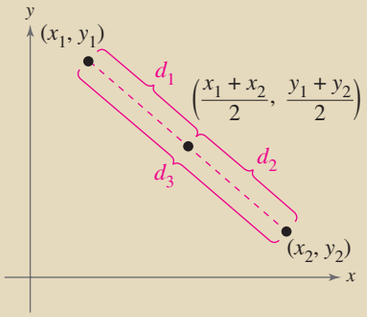
The Midpoint Formula (p. 52)

The midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by the Midpoint Formula

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Proof

Using the figure, you must show that $d_1 = d_2$ and $d_1 + d_2 = d_3$.



By the Distance Formula, you obtain

$$d_1 = \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2}$$

$$= \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_2 = \sqrt{\left(x_2 - \frac{x_1 + x_2}{2}\right)^2 + \left(y_2 - \frac{y_1 + y_2}{2}\right)^2}$$

$$= \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_3 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

So, it follows that $d_1 = d_2$ and $d_1 + d_2 = d_3$.

The Cartesian Plane

The Cartesian plane was named after the French mathematician René Descartes (1596–1650). While Descartes was lying in bed, he noticed a fly buzzing around on the square ceiling tiles. He discovered that the position of the fly could be described by which ceiling tile the fly landed on. This led to the development of the Cartesian plane. Descartes felt that a coordinate plane could be used to facilitate description of the positions of objects.